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WEEK 6 WRITTEN SOLUTIONS

H. MILLS

1) Let $s(x) = x^2 - 5x$. 1 pt

2) 5 pts The net change in s from $x=1$ to $x=5$ is Context 1 pt

$$s(5) - s(1) = 5^2 - 5(5) - (1^2 - 5(1)) = -1 + 5 + 4 = s(5) - s(1)$$

3) 5 pts The average rate of change from $x=1$ to $x=5$ is 2 pts

$$\frac{s(5) - s(1)}{5 - 1} = \frac{4}{4} = 1$$

4) 5 pts We simplify the difference quotient Context - 1 pt

$$\frac{s(a+h) - s(a)}{h} = \frac{(a+h)^2 - 5(a+h) - (a^2 - 5a)}{h}$$

$$= \frac{a^2 + 2ah + h^2 - 5a - 5h - a^2 + 5a}{h}$$

$$= \frac{2ah + h^2 - 5h}{h} = \frac{h(2a + h - 5)}{h} = 2a + h - 5 \quad (h \neq 0)$$

1 Bonus

= Difference quotient.

2) 5 pts $f(x) = \frac{1}{x-3}$ 1 pt

$$\frac{f(a+h) - f(a)}{h} = \frac{\frac{1}{a+h-3} - \frac{1}{a-3}}{h} = \frac{\left(\frac{1}{a+h-3}\right)\left(\frac{a-3}{a-3}\right) - \left(\frac{1}{a-3}\right)\left(\frac{a+h-3}{a+h-3}\right)}{h}$$

$$= \frac{\frac{a-3 - (a+h-3)}{(a+h-3)(a-3)}}{h} = \frac{1}{h} \left[\frac{a-3 - a-h+3}{(a+h-3)(a-3)} \right]$$

$$= \frac{1}{h} \left[\frac{-h}{(a+h-3)(a-3)} \right] = \frac{-1}{(a+h-3)(a-3)} \quad (h \neq 0)$$

1 Bonus

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MILLS

3 (5 pts) The table gives the population P in a small town from 2002-2020. Figures are for January 1st each year

Year	2002	2004	2006	2008	2010	2012	2014	2016
Pop	3200	3645	4357	4869	5871	6375	6288	5318

Year	2018	2020
Pop	4921	4636

At least enough of the table to cover 2010 & 2014

Yr	2010	2014
Pop	5871	6288

2 pts content

The average rate of change from 2010-2014 is

$$\frac{P(2014) - P(2010)}{2014 - 2010} = \frac{6288 - 5871}{4} = \frac{417}{4}$$

3 pts as long as it's supported

4 (5 pts) The avg. rate of change for $f(x) = 2\sqrt{x}$ between $x = a+h$ & $x = a$ is

$$\frac{f(a+h) - f(a)}{h} = \frac{2\sqrt{a+h} - 2\sqrt{a}}{h}$$

Content - 1 pt

$$= \left(\frac{2\sqrt{a+h} - 2\sqrt{a}}{h} \right) \left(\frac{2\sqrt{a+h} + 2\sqrt{a}}{2\sqrt{a+h} + 2\sqrt{a}} \right) = \frac{4(a+h) - 4a}{h(2\sqrt{a+h} + 2\sqrt{a})}$$

1 pt

$$= \frac{4a + 4h - 4a}{h(2\sqrt{a+h} + 2\sqrt{a})} = \frac{4h}{h(2\sqrt{a+h} + 2\sqrt{a})} = \frac{4}{2\sqrt{a+h} + 2\sqrt{a}}$$

1 pt

$$= \frac{4}{2(\sqrt{a+h} + \sqrt{a})} = \frac{2}{\sqrt{a+h} + \sqrt{a}}$$

1 pt

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CONTEXT - .5

5) A pond is filling with H₂O at a rate of 10 ^{MILLS} $\frac{\text{gallons}}{\text{min}}$. Initially, there are 500 gal of H₂O in the pond, or words to their effect.

2) 5pts We model $V = \text{Volume of H}_2\text{O in the pond}$ (in gallons) as a function of $t = \text{time (in minutes)}$

Lexicon: words of units for variables
1pt

$$V = \text{initial} + (\text{rate of filling})(\text{time})$$

$$= 500 + 10t \quad 3.5 \text{pts}$$

b) 5pts If the pond holds 2000 gallons, how long will it take to fill it? Context .5pts

We solve $V(t) = 2000 \rightarrow$

$$500 + 10t = 2000 \rightarrow 2 \text{pts}$$

$$10t = 1500 \rightarrow 2.5 \text{pts}$$

$$t = 150 \text{ minutes}$$

d) 5pts Assume you stop when the pond is full. What's the domain of $V(t)$? Context 1pt

$$D(V) = [0, 150]$$

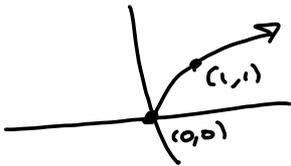
$$R(V) = [500, 2000] \quad 4 \text{pts}$$

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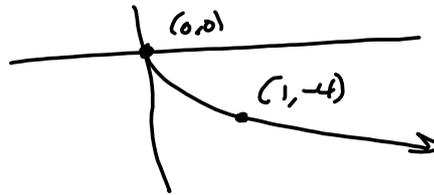
MILLS

⑥ **10pts BONUS** We graph $g(x) = -4\sqrt{-5x-20} + 64$ by transforming the graph of $f(x) = \sqrt{x}$:

① $f(x) = \sqrt{x}$

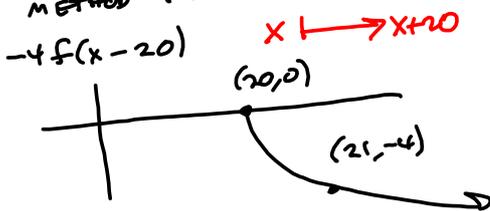


① $-4f(x) = -4\sqrt{x}$ $y \mapsto -4y$

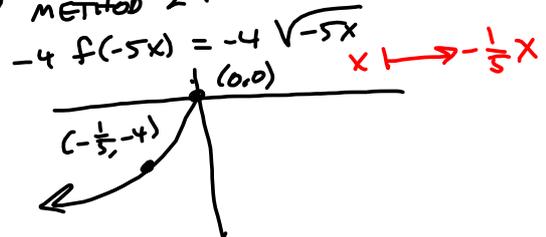


Scratch: $g(x) = -4\sqrt{-5(x+4)} + 64$

② METHOD 1:

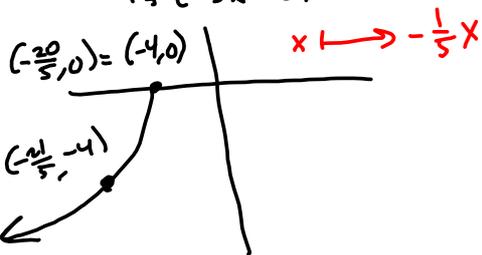


② METHOD 2:



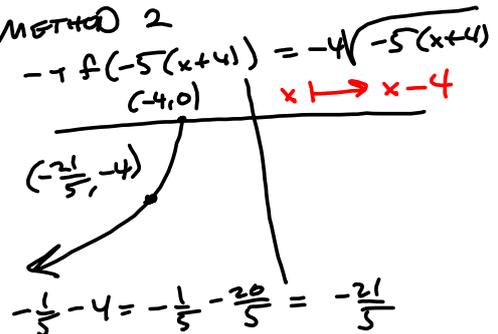
③

METHOD 1

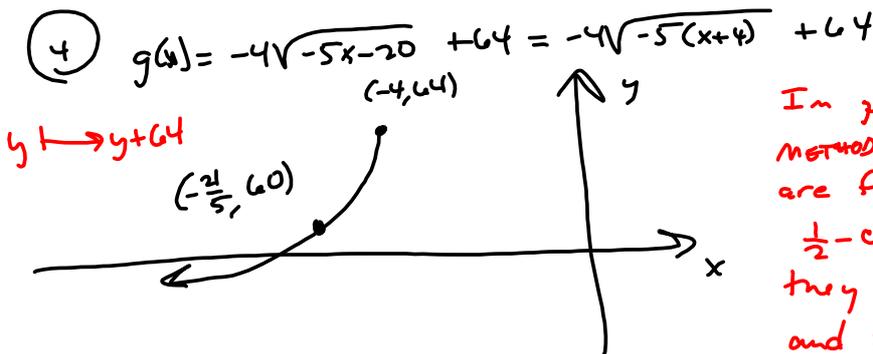


③

METHOD 2



④



In general **2pts per sketch** METHOD 1 or METHOD 2 are fine. $\frac{1}{2}$ -credit **(5pts)** if they get the last graph and show the endpoint. Try to give credit for correct moves. This makes it tough to grade if they make early mistakes, but do other moves correctly.

7 Spts Bonus $g(x)$ has no y -int.

$$g(x) = 0 \Rightarrow$$

$$-4\sqrt{-5x-20} + 64 = 0$$

$$\Rightarrow -4\sqrt{-5x-20} = -64$$

$$\Rightarrow \sqrt{-5x-20} = \frac{-64}{-4} = 16$$

$$\Rightarrow -5x-20 = 16^2 = 256$$

$$\Rightarrow -5x = 276$$

$$\Rightarrow x = \frac{276}{-5} \Rightarrow \left(-\frac{276}{5}, 0\right) \text{ is } x\text{-int}$$

Also an ok answer, but prefer the ordered pair.

The $x \mapsto x+20$, etc.

are instructive notes for students looking at the work, later.

Not required on student papers.

TAKE OFF 3 points if they call everything " $f(x)$," willy-nilly.

$f(x) = \sqrt{x}$. Everything else is a transformed

$f(x)$, like $-4f(x)$, $-4f(-5x)$, $-4f(x-20)$,

$f(-5(x+4))$, $-4f(-5x-20)$, $-4f(-5x-20) + 64 = g(x)$

They can just write " $g(x)$ " on last one
 may.