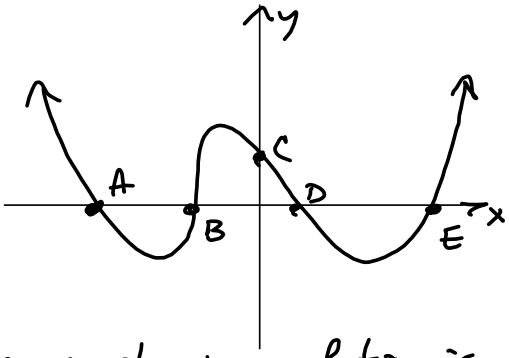
1340 WEEK 4 SOLUTIONS H. MILLS

(1) Use the graph of $f(x) = x^4 + 4x^3 - 34x^2 - 76x + 105$ to answer the following:

(a) (5pts) Solve $f(x) = 0$



Sketch Key:

$A = (-7, 0)$

$B = (-3, 0)$

$C = (0, 105)$ y-int

$D = (1, 0)$

$E = (5, 0)$

From the graph, the solution is $x = -7, -3, 1, 5$. The solution <u>set</u> is $\{-7, -3, 1, 5\}$. We write

$x \in \underbrace{\{-7, -3, 1, 5\}}$ & $x = -7, -3, 1, 5$, interchangeably.

This is a set

This is a condition x must satisfy

Condition for club membership and the club, itself, are not quite the same thing.

(b) (5pts) Solve $f(x) > 0$. We report intervals of x where the graph's y-values are above the x-axis. This is true

for $x < -7$ OR $-3 < x < 1$ OR $x > 5$

$(-\infty, -7) \cup (-3, 1) \cup (5, \infty)$

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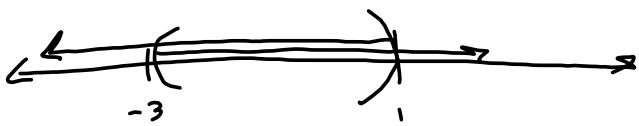
"OR" means "UNION."

"AND" means "INTERSECTION."

This page is more instruction than assignment.

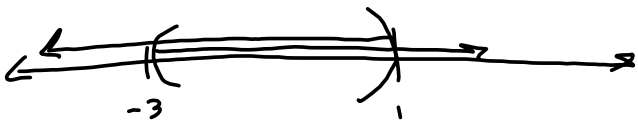
$-3 < x < 1$ means

$-3 < x$ AND $x < 1$. The <u>intersection</u> is the set contained in BOTH. It's the overlap

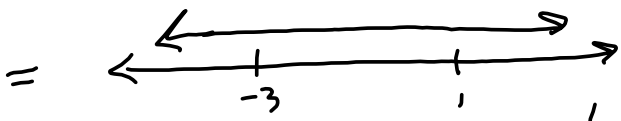


AND

$-3 < x$ OR $x < 1$ looks like



OR



It's the whole real line!

MIND YOUR AND'S and OR'S.

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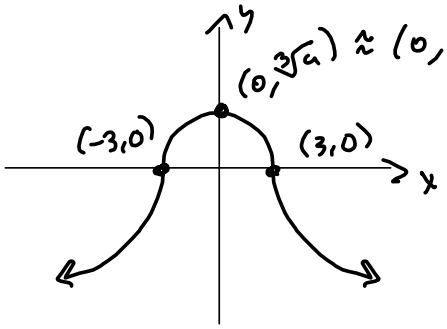
(2) Let $y = f(x) = \sqrt[3]{9-x^2}$, we employ a graphing utility to answer the following

(a) (5pts) A hand sketch with x- & y-intercepts shown.



$\approx (0, 2.08008382305)$ from Desmos.

(b) (5pts) we check for symmetry

$y = \sqrt[3]{9-x^2}$

$\sqrt[3]{9-(-x)^2} = \sqrt[3]{9-x^2} = y$, so $\boxed{\text{symmetric in or about the y-axis. The graph supports this.}}$

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(3) We solve the eq'n $\sqrt{3x+22}+2=x$ in two ways:

(a) (5pts) with a graphing utility:

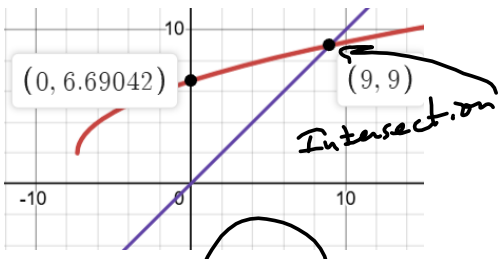
Two ways:

① Graph $\sqrt{3x+22}+2=y$ & $y=x$, separately and find the intersection(s).

② Graph $y=\sqrt{3x+22}+2-x$ and find the x-intercepts.

Method 1: Graph both sides, separately

$y=\sqrt{3x+22}+2$ & $y=x$.

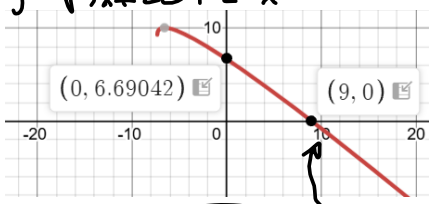


Intersection

$\Rightarrow x=9$

METHOD 2: Get everything on one side

$y=\sqrt{3x+22}+2-x$



x-intercept

$\Rightarrow x=9$

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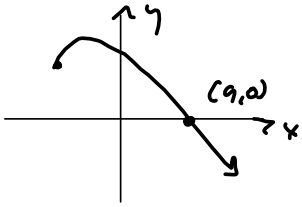
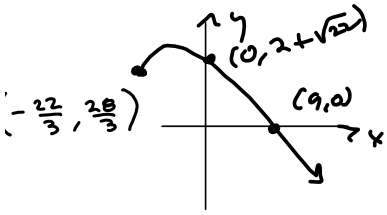
Teacher's notes on #3a. #3b on next page.

A hand sketch, with key point & general "shape" of the graphs, will suffice, here.

$y=\sqrt{3x+22}+2-x$



would've been OK



This would be BONUS LAND for excellence.

$3x+22=0$

$3x=-22$

$x=-\frac{22}{3} \Longrightarrow$

$y=\sqrt{3(-\frac{22}{3})+22}+2-(-\frac{22}{3})$

$=2+\frac{22}{3}=\frac{6+22}{3}=\frac{28}{3}$

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(b) (5pts) Algebraic Solution:

$$\sqrt{3x+22} + 2 = x \longrightarrow$$

Need to see $\rightarrow$ $\sqrt{3x+22} = x - 2 \longrightarrow$

$$\longrightarrow \left(\sqrt{3x+22}\right)^2 = (x-2)^2 \longrightarrow$$ OPTIONAL TO SHOW. If you must see this step, write an extra line, as I did. Don't cram it into the previous step.

Need to see $\rightarrow$ $3x+22 = x^2 - 4x + 4$

$$-3x - 22 = -3x - 22$$

$$0 = x^2 - 7x - 18$$

$$= (x-9)(x+2) = 0 \longrightarrow$$

$x = 9$ OR $x = -2$

Check: $\sqrt{3x+22} + 2 = x$

$$\sqrt{3(9)+22} + 2 \stackrel{?}{=} 9$$

$$\sqrt{27+22} + 2 \stackrel{?}{=} 9$$

$$\sqrt{49} + 2 = 7 + 2 = 9 \checkmark$$

$$\sqrt{3(-2)+22} + 2 = \sqrt{-6+22} + 2 = \sqrt{16} + 2 = 4 + 2 = 6 \neq -2$$

$x = -2$ is extraneous solution

$\boxed{x = 9 \text{ is the solution}}$

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(4) If S is proportional to the product of x and the square of y and inversely proportional to the square root of z, then

$$\boxed{S = k\frac{xy^2}{\sqrt{z}}}, \text{ for some } k > 0.$$

If $S=3$, when $x=3, y=2$, and $z=16$, then

$$S = 3 = \frac{3(2)^2}{\sqrt{16}}k = \frac{3(4)}{4}k = 3 = S$$

$$\Longrightarrow 3k = 3$$

$$\Longrightarrow \boxed{k=1}\ !$$

What is $S(2,3,4)$? I.e., what is S when $x=2, y=3$, and $z=4$?

$$S = k\left(\frac{2(3)^2}{\sqrt{4}}\right) = \frac{2(9)}{2} = \boxed{9 = S}$$