1340 Week 3 written SOLUTIONS H. MILLS

(1) We solve $\sqrt{7x} + x = 0$ Context 1pt (makes 6pts, but that's ok.)

$\Rightarrow \sqrt{7x} = -x$ (1pt) I need to see this, uncluttered & clear

$\Rightarrow \sqrt{7x}^2 = (-x)^2$ I don't need to see this. Change 0.4 (correct without hurting) But they should not clutter $(\sqrt{7x})^2 = (-x)^2$ step with DO Need to see this

$\Rightarrow 7x = (-x)^2 = x^2$ (1pt)

$\Rightarrow x^2 - 7x = 0$ (1pt*) This or equivalent (more than 1 way) the squaring step, like I just did

$\Rightarrow x(x-7) = 0$

$\Rightarrow x \in \{0, 7\}$ (1pt) or $x = 0, 7$ is OK

Check: $\sqrt{0} + 0 = 0$ ✓

$\sqrt{49} + 7 = 14 \neq 0$ ✗

$x = 7$ is extraneous

Soln is $\boxed{x = 0}$ (1pt) (support needed)

* Don't care if they factor, complete the square or use quadratic formula, if they execute them properly

(2) (5pts) $2(x-5)^2 - 13(x-5) - 7 = 0$ is quadratic in $w = x - 5$ Context 1pt

$\Rightarrow 2w^2 - 13w - 7 = 0$ (1pt)

$\Rightarrow (2w+1)(w-7) = 0$

$\Rightarrow w = -\frac{1}{2} = x - 5 \Rightarrow x = 5 - \frac{1}{2} = \boxed{\frac{9}{2} = x}$ or

(1pt) or $w = 7 = x - 5 \Rightarrow x = 5 + 7 = \boxed{12 = x}$ 20ts

or write: $\boxed{x \in \{\frac{9}{2}, 12\}}$ is stylin'

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#s 3-6 We find all real solutions of the equation.

(3) (5pts) $x^5 - 18x^4 + 6x^3 = 0$ →

$x^3(x^2 - 18x + 6) = 0$

→ $x = 0$ OR $x^2 - 18x + 6 = 0$ → $a=1, b=-18, c=6$ (1pt)

$b^2 - 4ac = 18^2 - 4(1)(6)$

$= 324 - 24 = 300$

Quadratic technique 1pt

$x^2 - 18x + 9^2 - 81 + 6$

$= (x-9)^2 - 75 = 0$

$x - 9 = \pm\sqrt{75} = 5\sqrt{3}$

$x = 9 \pm 5\sqrt{3}$

2	300
2	150
3	75
5	25
5	

$\Rightarrow \sqrt{300} = 2 \cdot 5\sqrt{3}$

$= 10\sqrt{3}$ Simplify

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{18 \pm 10\sqrt{3}}{2} = 9 \pm 5\sqrt{3} = x$ 1pt

FINAL ANS: $x \in \{0, 9 \pm 5\sqrt{3}\}$ 2pts OR $x = 0, 9 \pm 5\sqrt{3}$

(4) (5pts) $2x^3 + 7x^2 - 8x - 28 = 0$ →

$x^2(2x+7) - 4(2x+7)$ 1pt Write eq'n.

$= (2x+7)(x^2 - 4)$

$= (2x+7)(x-2)(x+2) = 0$ → 2pts FACTOR EVERYTHING

$\Rightarrow x \in \{-\frac{7}{2}, \pm 2\}$ → 2pts Final answer

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(5) (5pts) $\frac{x+6}{x^2+6} = \frac{2}{x+2}$ LCD $= (x+2)(x^2+6)$

$\Rightarrow (x+2)(x^2+6)\left(\frac{x+6}{x^2+6} = \frac{2}{x+2}\right)$ or cross-multiply.

$\Rightarrow (x+6)(x+2) = 2(x^2+6)$ OR equivalent (2pts)

$\Rightarrow x^2+8x+12 = 2x^2+12$

$\Rightarrow x^2-8x = 0$ (1pt)

$\Rightarrow x(x-8) = 0$

$\Rightarrow x \in \{0, 8\}$ (2pts) or $x = 0, 8$

Check: $\frac{0+6}{0^2+6} \stackrel{?}{=} \frac{2}{0+2}$ ✓

$\frac{8+6}{64+6} \stackrel{?}{=} \frac{2}{8+2}$

$\frac{1}{5} = \frac{7}{35} = \frac{14}{70} = \frac{2}{10} = \frac{1}{5}$ ✓

(6) (5pts) $\frac{x+\frac{4}{x}}{\frac{4}{x}+5} = 2x$

$\Rightarrow \frac{x\left(x+\frac{4}{x}\right)}{x\left(\frac{4}{x}+5\right)} = \frac{x^2+4}{4+5x} = \frac{x^2+4}{5x+4} = 2x$

$\Rightarrow x^2+4 = 2x(5x+4) = 10x^2+8x$

$\Rightarrow 9x^2+8x-4 = 0$ (3pts)

$(-4)(9) = (-)(2)(2)(3)(3)$

$\Rightarrow a=9, b=8, c=-4$

$\Rightarrow b^2-4ac = 8^2-4(9)(-4)$

$= 64+144$

$= 208$

when I say 2pts for an answer, that includes the proper support & support that "reads" Needs to be organized, top to bottom, left to right. (2pts)

2 | 208
2 | 104
2 | 52
2 | 26
13

$\sqrt{208} = 2^2\sqrt{13} = 4\sqrt{13}$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-8 \pm 4\sqrt{13}}{2(9)} = \boxed{\frac{-4 \pm 2\sqrt{13}}{9} = x}$

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(7 pts) We solve $\frac{x+6}{x^2+6} \geq \frac{2}{x+2}$

By #5, we know they're equal @ $x=0,8$
But there's a lot more to it than that. We also have a sign change on the right-hand side @ $x=-2$.
This is how to beat these. Use $x=0,8$ as a check, but the idea is to get everything on one side & then analyze a SIGN PATTERN.

$\frac{x+6}{x^2+6} - \frac{2}{x+2} \geq 0$ LCD $= (x+2)(x^2+6)$, as in #5. But here, we ADD FRACTIONS

$\left(\frac{x+6}{x^2+6}\right)\left(\frac{x+2}{x+2}\right) - \left(\frac{2}{x+2}\right)\left(\frac{x^2+6}{x^2+6}\right)$

Can't just cross-multiply, because variable expressions change their sign!

$= \frac{x^2+8x+12-2x^2-12}{(x+2)(x^2+6)} = \frac{-x^2+8x}{(x+2)(x^2+6)}$

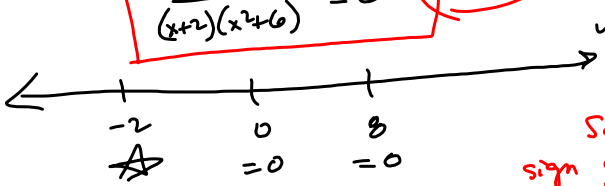
$= \frac{-x(x-8)}{(x+2)(x^2+6)} = 0$ when $x=0,8$ ✓

Also, $=$ ☆ (Blow-up!) when $x=-2$

$x=-2,0,8$ break up the # line into 3 intervals

We want $\frac{x+6}{x^2+6} - \frac{2}{x+2} \geq 0$ so we want

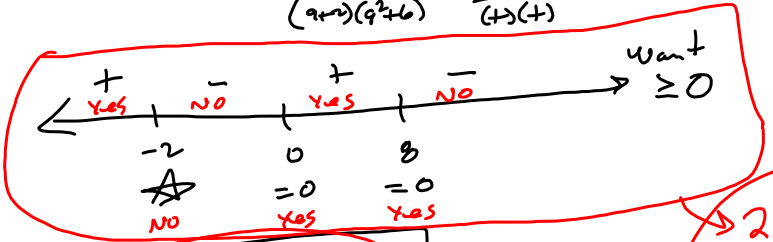
$\frac{-x(x-8)}{(x+2)(x^2+6)} \geq 0$ (2 pts) & how they got there.



Some form of sign analysis. Test values isn't the only one. But they need to back it up.

Interval	Test		
$(-\infty,-2)$	-3	$\frac{-(-3)(-3-8)}{(-3+2)((-3)^2+6)} = \frac{(+)(-)}{(-)(+)}$	$= + > 0$
$(-2,0)$	-1	$\frac{-(-1)(-1-8)}{(-1+2)((-1)^2+6)} = \frac{(+)(-)}{(+)(+)}$	$= - < 0$
$(0,8)$	1	$\frac{-(1)(1-8)}{(1+2)(1^2+6)} = \frac{(-)(-)}{(+)(+)}$	$= + > 0$
$(8,\infty)$	9	$\frac{(-9)(9-8)}{(9+2)(9^2+6)} = \frac{(-)(+)}{(+)(+)}$	$= - < 0$

I use end behavior & sign changes at the Boundaries.



2 pts for sign pattern of some sort.

$x \in (-\infty,-2) \cup [0,8]$ (1 pt)

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(8) We solve the absolute-value inequalities. (1pt for all 3 parts.)

(a) (5pts) $|2x-3| \geq 7 \rightarrow$ KEY

$2x-3 \geq 7$ OR $2x-3 \leq -7$ (2pts)

$2x \geq 10$ $2x \leq -4$ (2pts)

$x \geq 5$ OR $x \leq -2$

Solution-Set Answer

$(-\infty, -2] \cup [5, \infty)$

or WRITE

$\{x \mid x \geq 5 \text{ OR } x \leq -2\}$ (1pt Bonus)

$-7 > 2x-3 > 7$ is NOT OK.

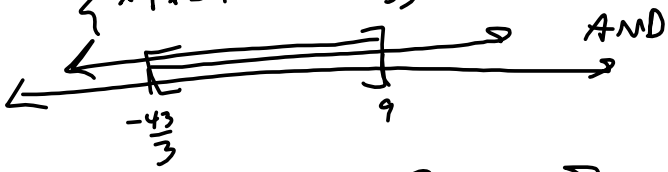
(b) (5pts) $|3x+8| \leq 35 \rightarrow$ (2pts)

$3x+8 \leq 35$ and $3x+8 \geq -35$

$3x \leq 27$ $3x \geq -43$

$x \leq 9$ and $x \geq -\frac{43}{3}$ (2pts)

$\{x \mid x \leq 9 \text{ and } x \geq -\frac{43}{3}\}$



$= \left[-\frac{43}{3}, 9\right]$ (1pt Bonus)

$-35 \leq 3x+8 \leq 35$ is OK. for the "AND" situation.

(c) (5pts) $|2x-3| > -7$

$\rightarrow$ $x =$ any real # (ALWAYS) (5pts)

(d) (5pts) $|3x+8| \leq -35$ (NEVAH!) (5pts)

No Soc'N

The last 2, start taking off points if their technique sprays falsehoods.