1340 Harry Mills

Harry's my 1st name, so it's what would be on the roster, so that's how I print my name. I don't mind cursive, but I don't want a cryptic signature.

1 We graph the region for the system of inequalities:

$x > 1$

$y \geq 0$

$2x + 3y \leq 6$

Context of question

I use "scratch out the bad stuff" method. Note key details I include and busy work I do not include (Like tick marks).

Get things generally right. Label things the way I do & get an intuition for what matters and what doesn't. This is the highest order of human intelligence.

Final graph on Page 3. I included a lot of extra explanation...

Show work

$x = 1$ is a vertical line with x-intercept $\boxed{(1,0) \text{ x-int}}$ No y-int

$x > 1$ is all points to the right of the line $x = 1$.

$y = 0$ is the x-axis!

$y \geq 0$ is all points on or above the x-axis. Its x- & y-ints are just $\boxed{(0,0)}$

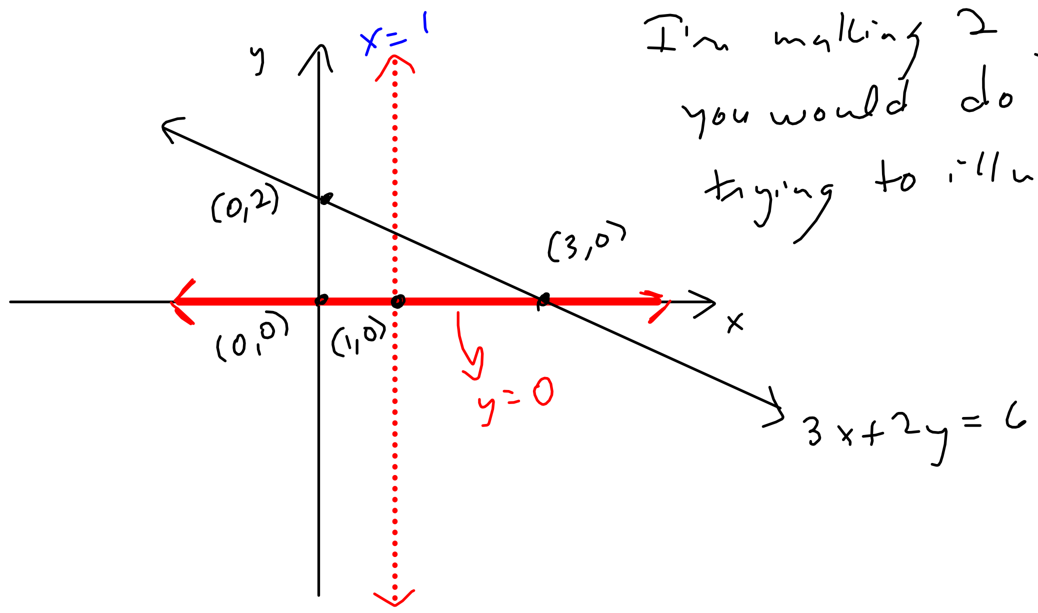
Show work

$2x + 3y = 6$:

x	y
0	2
3	0

$2(0) + 3y = 6 \Rightarrow y = \frac{6}{3} = 2 \rightsquigarrow$ (0,2) y-int

$2x + 3(0) = 6 \Rightarrow y = \frac{6}{2} = 3 \rightsquigarrow$ (3,0) x-int

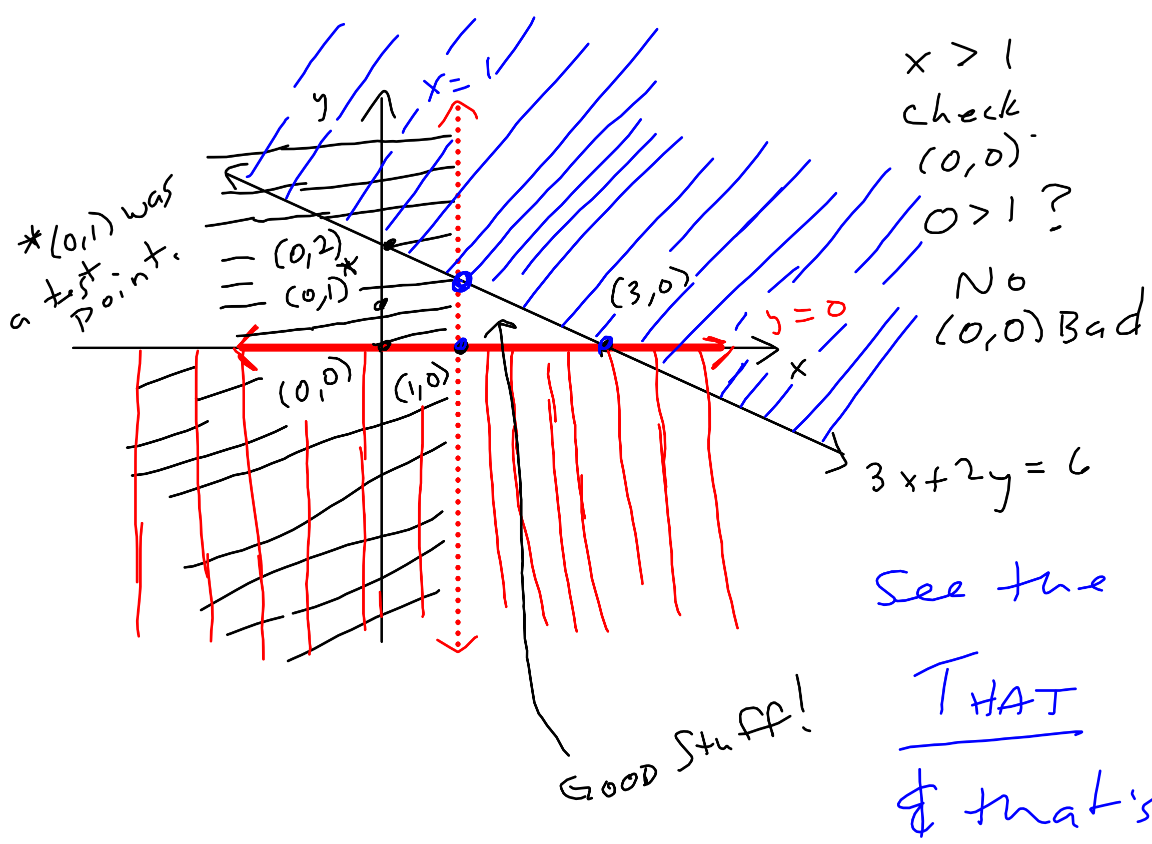


I'm making 2 graphs, but you would do it on one graph. I'm trying to illustrate the process.

FINAL GRAPH ON PAGE 3

"scratch out the bad stuff"

After building the lines and labeling them, as above, I would then add the shading layers for the inequalities.



$x > 1$
check $(0,0)$:
$0 > 1$?
No
$(0,0)$ Bad

$y \geq 0$
check any point not on y-axis:
$(0,1)$:
$1 \geq 0$?
Yes.
$(0,1)$ Good

$3x+2y \leq 6$
check $(0,0)$:
$3(0)+2(0) \leq 6$?
$0 \leq 6$?
Yes
$(0,0)$ Good

See the triangle in the middle?
THAT is the "feasible region."
& that's what we want!

we have 2 of the 3 corners ;
(1,0) & (3,0).

The 3^{rd} corner is where $x=1$ and intersect:

$x=1 \Longrightarrow$ $2x+3y = 2(1)+3y = 6$

$\Longrightarrow 3y = 4$

$\Longrightarrow y = \frac{4}{3}$

So $(1, 4/3)$ is that top corner.

Best way to present it?

I'd give all key points a letter label.

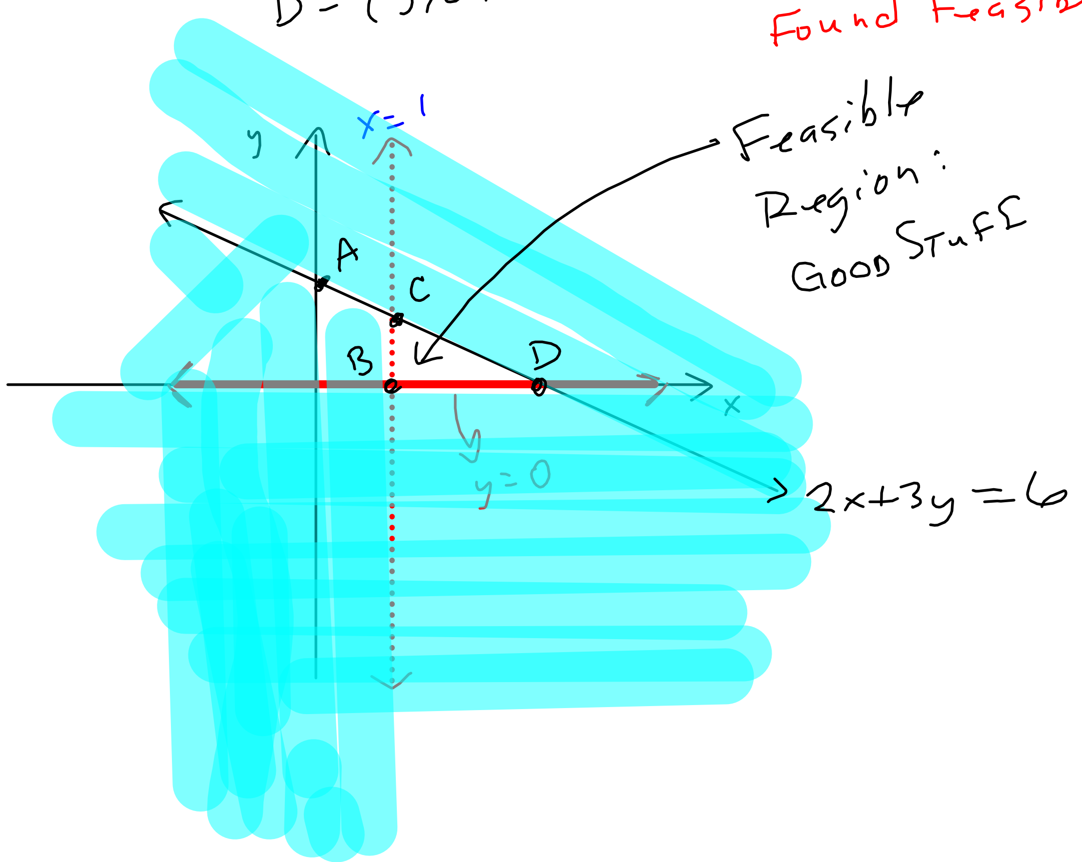
$A = (0,2)$

$B = (1,0)$

$C = (1, \frac{4}{3})$

$D = (3,0)$

Intercepts 1pt
Corners 1pt
Labels 1pt
Found Feasible Region 2pts



Shade the bad stuff & the good stuff is revealed.

Finished Graph.

1340

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(2) The distance between $(x_1,y_1) = (2,-3)$ & $(x_2,y_2) = (-7,2)$ is

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2} = \sqrt{(2-(-7))^2+(-3-2)^2} = \sqrt{9^2+5^2} = \sqrt{81+25}$$

$$\boxed{=\sqrt{106}}$$

$2 \,|\, \underline{106}$
$\quad 53$

Context – 1 pt
SETUP – 3 pts
ANSWER – 2 pts

(3) The midpoint of $(x_1,y_1) = (\frac{\pi}{2}, \sqrt{3})$ & $(x_2,y_2) = (\frac{\pi}{3},-2)$ is

Context – 1 pt
SETUP – 2 pt
Final – 2 pt

$$\text{midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{\frac{\pi}{2}+\frac{\pi}{3}}{2}, \frac{\sqrt{3}-2}{2}\right)$$

$$= \left(\frac{\frac{3\pi+2\pi}{6}}{2}, \frac{\sqrt{3}-2}{2}\right) = \boxed{\left(\frac{5\pi}{12}, \frac{\sqrt{3}-2}{2}\right) = \text{midpoint}}$$

$$\approx (1.308996939, -0.133974596216) \approx \boxed{(1.3090, -0.1340)}$$

to 4 places

(4) We use Pythagorus to prove that the points $A=(1,2)$, $B=(-5,0)$, & $C=(-4,-3)$ form the vertices of a right triangle, i.e., the sum of the squares of the two shorter sides equals the square of the longest side.

$$d(A,B)=\sqrt{(1-(-5))^2+(2-0)^2}=\sqrt{6^2+2^2}=\sqrt{36+4}\ \sqrt{40}=\sqrt{4\cdot 10}=2\sqrt{10}$$
$$=2\sqrt{10}$$

$$d(A,C)=\sqrt{(1-(-4))^2+(2+3)^2}=\sqrt{5^2+5^2}=\sqrt{50}=5\sqrt{2}=d(A,C)$$

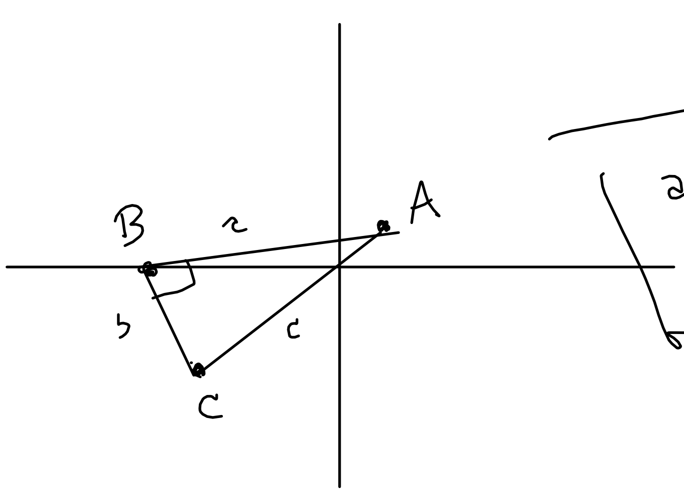
$$d(B,C)=\sqrt{(-5-(-4))^2+(0+3)^2}=\sqrt{1^2+3^2}=\sqrt{10}$$

Context -1pt
Find Distances - 2pts
check pythagorus - 1pt
conclude -1pt

I shouldn't have simplified the radicals, yet.

$d(A,B)=\sqrt{40}$, $\underbrace{d(A,C)=\sqrt{50}}_{\text{Longest}}$, $d(B,C)=\sqrt{10}$

$$d(A,C)^2=d(A,B)^2+d(B,C)^2\ ?$$
$$\sqrt{50}^2=\sqrt{40}^2+\sqrt{10}^2\ ?$$
$$50=40+10\ ?\ \text{Yep.}\ \longrightarrow\ \boxed{\text{It's a right triangle}}\ !$$



$a^2+b^2=c^2 \longrightarrow$ Right triangle

This is the converse of the Pythagorean Theorem, of which the textbook speaks

1340

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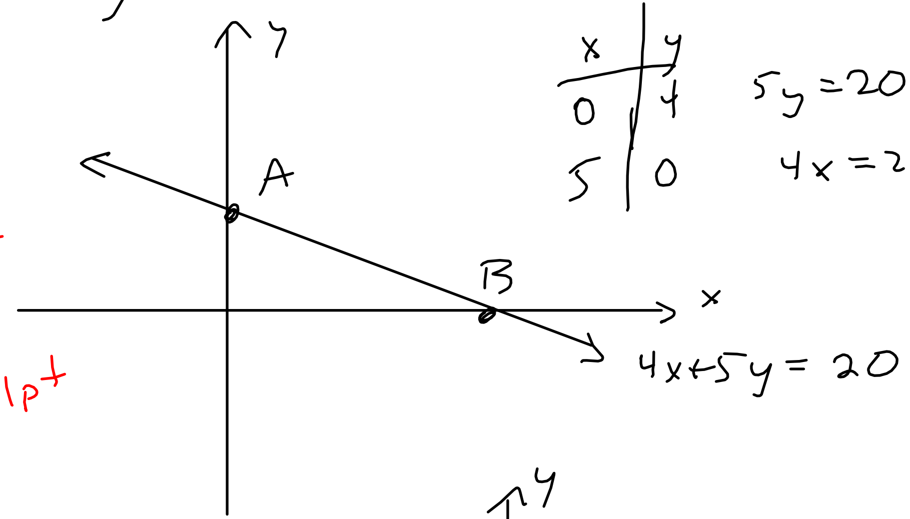
(5) We graph $4x+5y=20$ by the intercept method



x	y
0	4
5	0

$5y=20 \Rightarrow y=4$

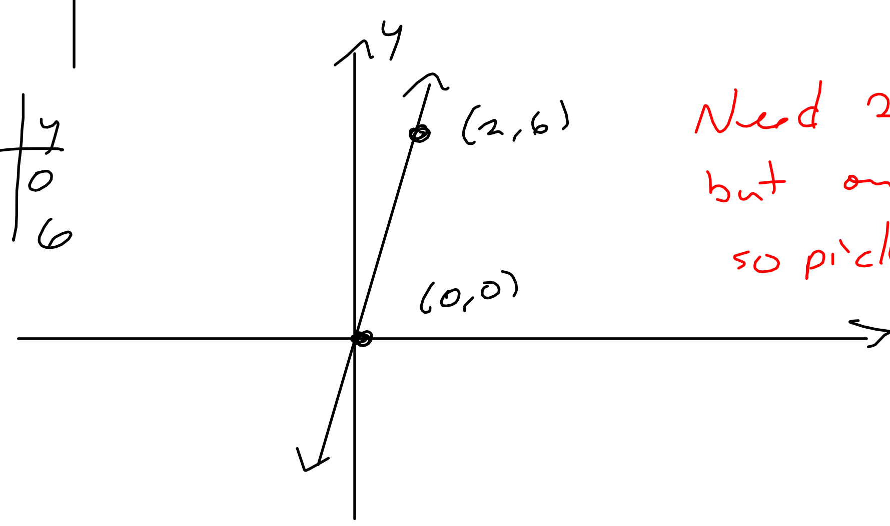
$4x=20 \Rightarrow x=5$

$\boxed{A=(0,4) \quad B=(5,0)}$

Context – 1pt
Method – 2pts
Label key pts – 1pt
Don't waste time on tickmarks – 1pt

(6) Graph $y=3x$:

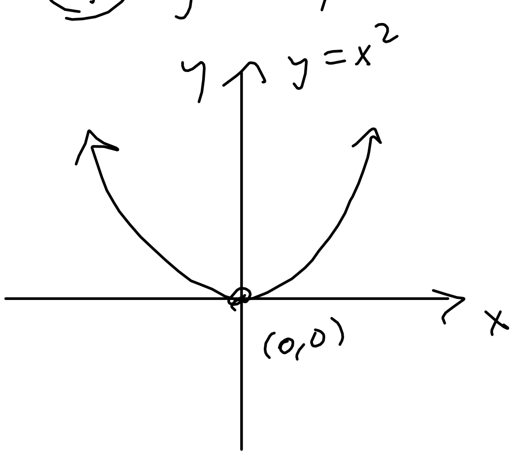
x	y
0	0
2	6

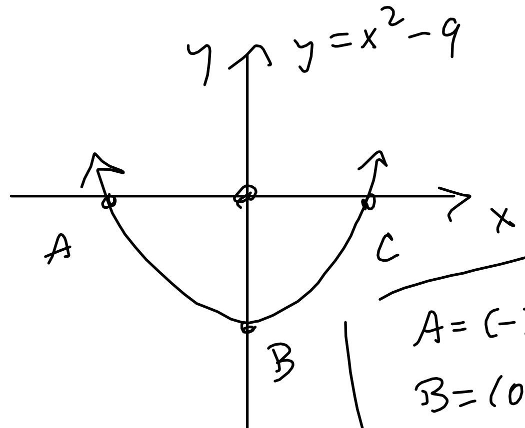


Need 2 points, but only one intercept, so pick a random 2^{nd} point

Intercept – 1pt
Labels – 2pts
No ticks – 1pt
Context – 1pt

(7) $y=x^2-9$





$\boxed{A=(-3,0) \quad B=(0,-9) \quad C=(3,0)}$

$y(0) = 0^2-9 = -9 \rightsquigarrow \boxed{(0,-9) = B}$

$y=0 \Rightarrow$

$x^2-9=0 \Rightarrow$

$(x-3)(x+3)=0 \Rightarrow$

$x \in \{\pm 3\} \rightsquigarrow \boxed{(3,0)=C \quad (-3,0)=A}$

Alternate:

Quadratic Formula:

$x^2-9=0 \Rightarrow$

$a=1, b=0, c=-9$

$\Rightarrow b^2-4ac = 0^2-4(1)(-9) = 36$

$\Rightarrow x = \dfrac{-0 \pm \sqrt{36}}{2(1)} = \pm \dfrac{6}{2} = \boxed{\pm 3 = x}$

Alternate:

Square Root Property

$x^2=9$

$\Rightarrow \boxed{x=\pm 3}$

Context – 1pt
Correct graph shape & location – 2pts
Intercepts – 2pts

1340

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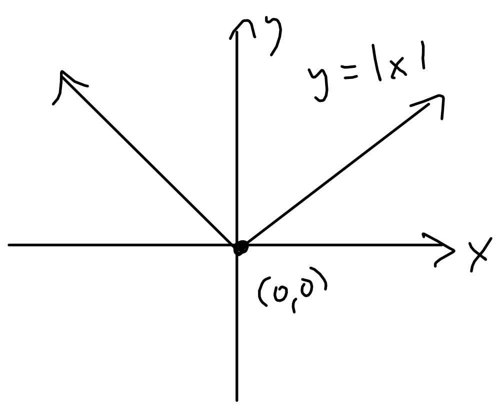
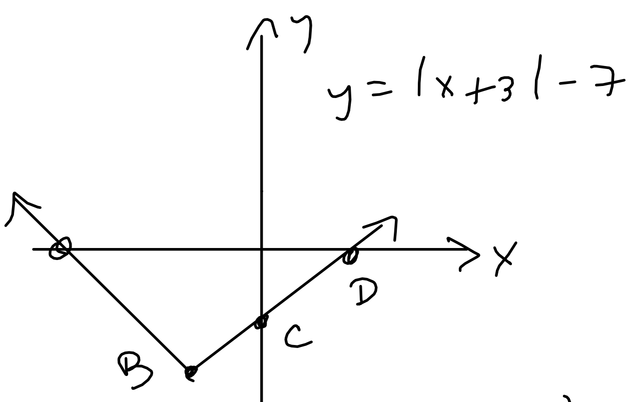
(8) we graph $y = |x+3| - 7$ by shifting $y = |x|$

Left 3   Down 7



More on shifting and stretching basic functions, later.

Context



$(-3,-7)$

Left 3   Down 7

$A = (-10, 0)$

$B = (-3, -7)$

$C = (0, -4)$

$D = (4, 0)$

$y(0) = |3| - 7 = -4$

$(0, -4)$

$y = 0$

$|x+3| - 7 = 0$

$|x+3| = 7$

$x + 3 = \pm 7$

$x = -3 \pm 7$

$-3 + 7 = 4 \rightsquigarrow (4, 0)$

$-3 - 7 = -10 \rightsquigarrow (-10, 0)$

Context - 1pt

Vertex - 2pts

x & y intercepts - 1pt

Label key pts - 1pt

(9) we graph the $\frac{1}{2}$-circle $y = -\sqrt{16-x^2}$

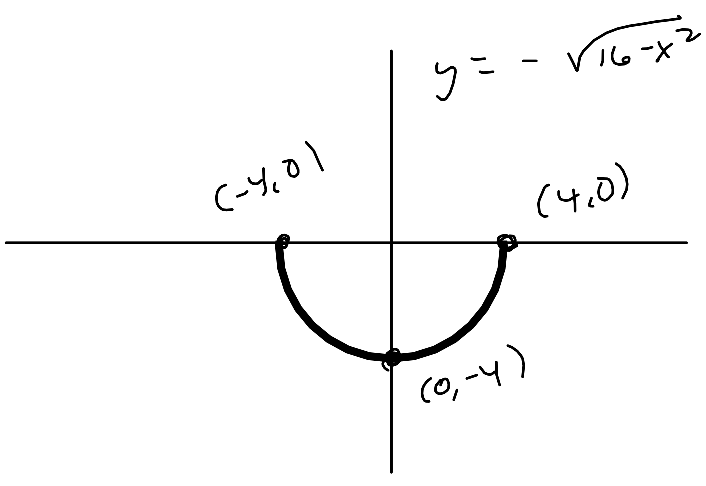
I really want you to recognize that square root structure as a circle.

$y = -\sqrt{16-x^2} \Longrightarrow$ bottom half of circle.

$y^2 = 16 - x^2 \Longrightarrow$

$x^2 + y^2 = 16$ so it's evidently a circle of radius $r=4$

Now $y = \sqrt{16-x^2}$ is its top half, so...



context - 1pt
Label Low Point - 1pt
Find x-ints & Label - 2pts
Shape of graph - 1pt

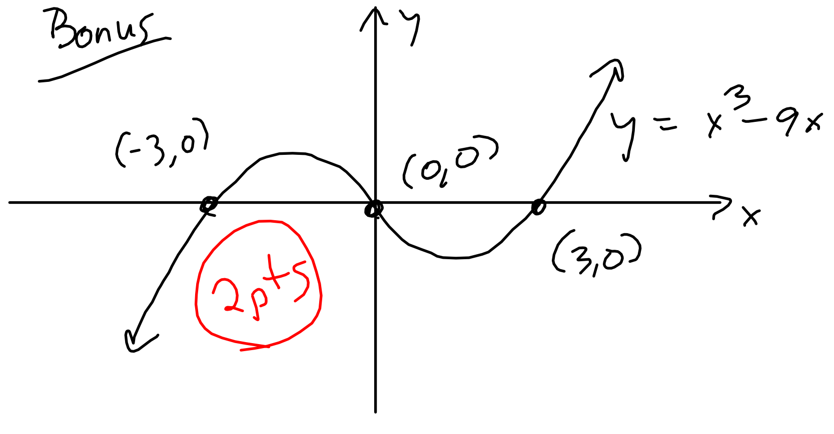
(10) Check $f(x) = x^3 - 9x$ for symmetry

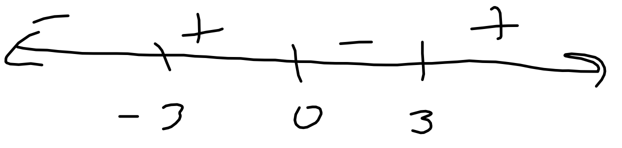
$f(-x) = (-x)^3 - 9(-x) = -x^3 + 9x = -(x^3 - 9x) = -f(x)$

This function is $\boxed{\text{symmetric wrt the origin.}}$

$x^3 - 9x = x(x^2-9) = x(x-3)(x+3)$

Bonus





1pt - Context
1pt - Sub "$-x$" for "x"
1pt - simplify
2pts Conclude $f(-x) = -f(x) \Rightarrow$ odd
$\Rightarrow$ symmetric w.r.t origin

1340 MILLS, H

(10) $x^2+y^2-6x+4y=23 \longrightarrow$

$x^2-6x \quad +y^2+4y \quad =23$

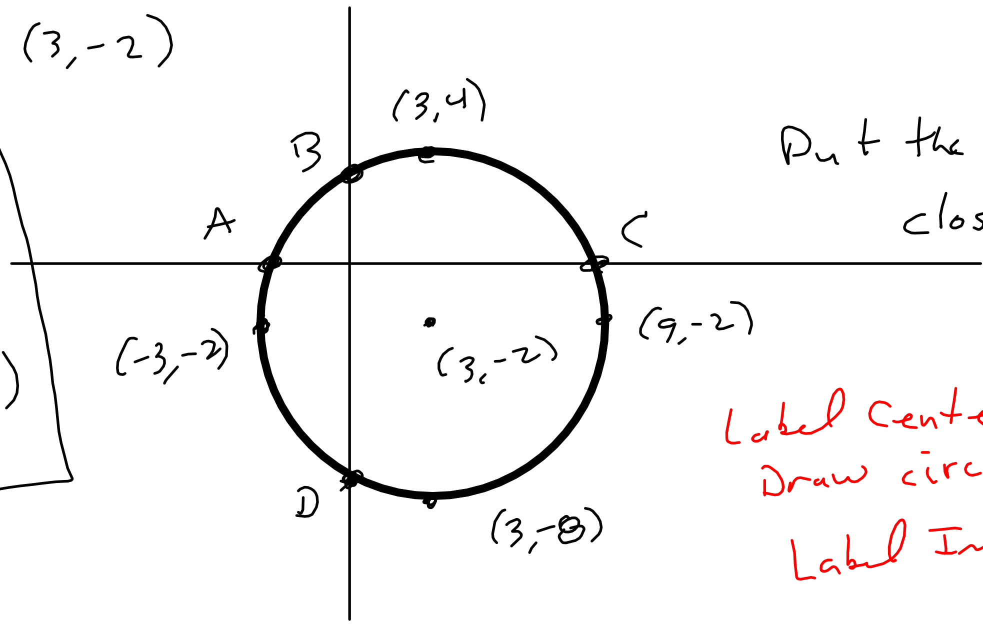
$x^2-6x+3^2+y^2+4y+2^2=23+9+4$

$(x-3)^2+(y+2)^2=36$ Circle of radius $6=r$, centered at

$(h,k)=(3,-2)$

(2) Complete the square - 3pts
Conclude - 2pts

$A=(3-4\sqrt{2},0)$
$B=(0,-2+3\sqrt{3})$
$C=(3+4\sqrt{2},0)$
$D=(0,-2-3\sqrt{3})$



Put the letter labels close to the graph

Label Center & Poles & equator 3pts
Draw circle -2pts
Label Intercepts - 2pts Bonus.

x-int; $y=0$:

$(x-3)^2+(y+2)^2=36$

$(x-3)^2+(2)^2=36$

$(x-3)^2+4=36$

$(x-3)^2=32$

$2 \mid 32,\ 2 \mid 16,\ 2 \mid 8,\ 2 \mid 4,\ 2$ $\quad \sqrt{2^5}$

$x-3=\pm\sqrt{32}=\sqrt{2^5}=\sqrt{2^4\cdot 2}=2^2\sqrt{2}=4\sqrt{2}$

$x=3\pm 4\sqrt{2}$

$(3+4\sqrt{2},0)=C$

$(3-4\sqrt{2},0)=A$

Bonus

y-int: $x=0$

$(-3)^2+(y+2)^2=36$

$9+(y+2)^2=36$

$(y+2)^2=27$

$3 \mid 27,\ 3 \mid 9,\ 3$

$y+2=\pm\sqrt{27}=\pm 3\sqrt{3}$

$y=-2\pm 3\sqrt{3}$

$(0,-2+3\sqrt{3})=B$

$(0,-2-3\sqrt{3})=D$

1340

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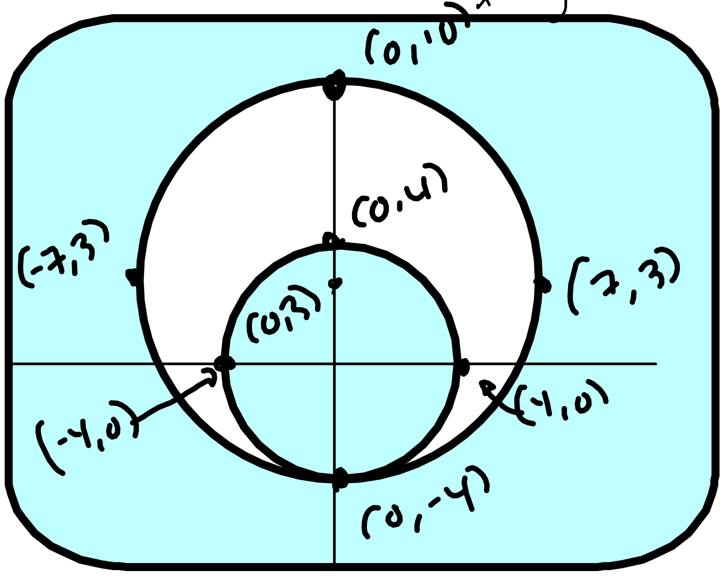
(12) Find area OUTSIDE $x^2+y^2=16$ and INSIDE $x^2+(y-3)^2=49$

$x^2+y^2>16$ $\qquad$ $x^2+(y-3)^2<49$



If you shade the bad stuff, the good stuff is clean.

By the graph, the little circle is entirely contained in the big circle, so

$$\text{Area} = \text{Area BIG} - \text{Area little.}$$
$$= \pi(7)^2 - \pi(4)^2$$
$$= \pi(49-16) = \pi(33) \boxed{= 33\pi = \text{AREA}}$$

Context - 1pt

Figure out the picture - 1pt

Compute area of the 2 circles - 2pts

Final Ans - 1pt