These Notes Accompany the Spring, 2025 Midterm Test: Click Here for Original Test

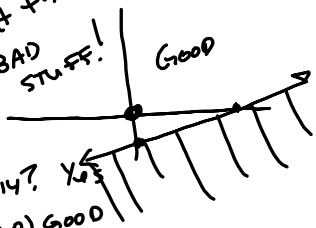
1 (10 pts) Find feasible Region for the system of inequalities.

$2x-7y \le 14$
$3x+2y < 6$
$x \le 7$
$y \le 3$

$2x-7y \le 14$

x	y
0	-2
7	0

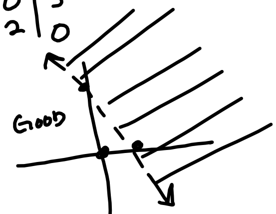
Scratch out the BAD STUFF!



$0 \le 14$? Yes
(0,0) GOOD

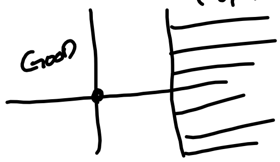
$3x+2y < 6$

x	y
0	3
2	0



$0 < 6$? Yes
(0,0) Goo

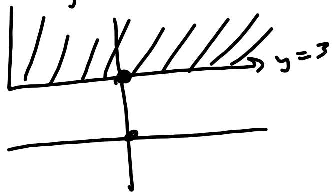
$x \le 7$



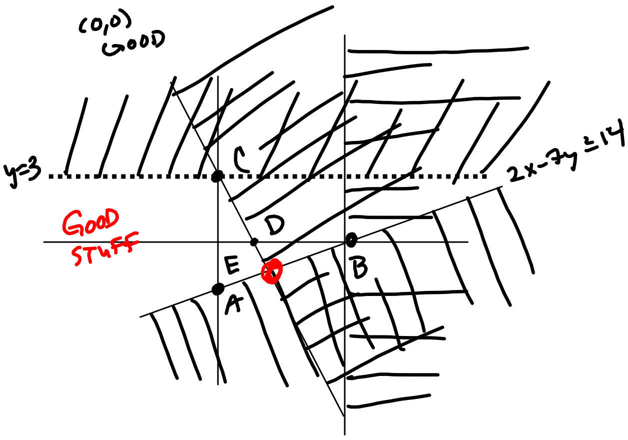
$0 \le 7$
Yes
(0,0)
GOOD

x=7

$y \le 3$



$0 \le 3$?
Yes.
(0,0) GOOD



A = (0,-2)
B = (7,0)
C = (0,3) Corner
D = (2,0)
E = $\left(\frac{14}{5}, -\frac{6}{5}\right)$ CORNER

E is last corner point
(We already have the other one @ (0,3))

Solve $3x+2y=6$
One way: $2x-7y=14$

$2y = 6-3x$ $\qquad$ $-7y = 14-2x$

$y = \frac{6-3x}{2}$ $\qquad$ $y = \frac{14-2x}{-7}$

$\Longrightarrow \frac{6-3x}{2} = \frac{14-2x}{-7}$

$-7(6-3x) = \underline{-42+21x} = 2(14-2x) \underline{= 28-4x}$

$-42+21x = 28-4x$
$\underline{+42+4x = +42+4x}$

$25x = 70$
$x = \frac{70}{25} = \boxed{\frac{14}{5} = x}$

E

$y = \frac{6-3x}{2}$
$= \frac{6-\left(\frac{14}{5}\right)(3)}{2} = \frac{30-14(3)}{10} = \frac{30-42}{10} = \frac{-12}{10} = \boxed{-\frac{6}{5} = y}$

#1 Another pass at making this video, with a slightly different approach. This might be more efficient for you.

Find feasible Region for the system of inequalities.

$2x-7y \le 14$

$3x+2y < 6$

$x \le 7$

$y \le 3$

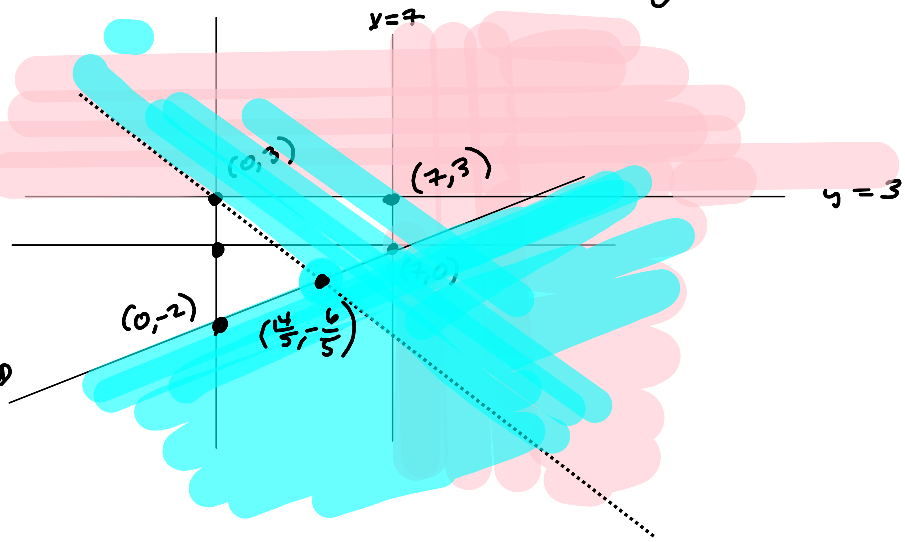
$y \le 3$

$0 \le 3$?

Yes. $(0,0)$ GOOD

$0 \le 7$? Yes

$(0,0)$ GOOD



INTERCEPT METHOD

$2x-7y \le 14$

x	y
0	-2
7	0

$-7y = 14 \rightarrow y = \frac{14}{-7} = -2$

$2x = 14 \Rightarrow x = \frac{14}{2} = 7$

$0 \le 14$

$3x+2y < 6$

x	y
0	3
2	0

$\frac{6}{2} = 3$

$\frac{6}{3} = 2$

$0 < 6$?

Yes

$(0,0)$ GOOD

CORNER POINT:

$(2x-7y = 14)(-3)$

$(3x+2y = 6)(2)$

$$
\begin{aligned}
-6x + 21y &= -42 \\
6x + 4y &= 12 \\
\hline
25y &= -30
\end{aligned}
$$

$y = \frac{-30}{25} = \boxed{\frac{-6}{5} = y}$

$2x-7y = 2x-7(-\frac{6}{5})$

$= (2x+\frac{42}{5} = 14)(5)$

$$
\begin{aligned}
10x + 42 &= 70 \\
-42 &= -42 \\
\hline
\frac{10x}{10} &= \frac{28}{10}
\end{aligned}
$$

$x = \frac{28}{10} = \frac{14}{5} = \boxed{\frac{14}{5} = x}$

(2) (10 pts) Let $A=(4,5), B=(-1,2), C=(10,-5)$ be 3 points in the plane.

We apply the converse of the Pythagorean Theorm to prove triangle ABC is a right triangle.

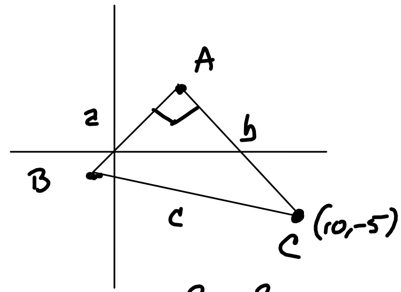
Lexicon: (Worth 2 or 3 out of 10 and 1 or 2 out of 5 points)

$d_1 =$ ***d*(*A*, *B*) be the distance from *A* to *B*.** $= a$

$d_2 = d(A,C)$ " " " " A to C. $= b$

$d_3 = d(B,C)$ " " " " B to C. $= c$

$A=(4,5), B=(-1,2), C=(10,-5)$



$d_1^2 = (-1-4)^2 + (2-5)^2 = a^2$

$= (-5)^2 + (-3)^2$

$= 25 + 9 = \boxed{34 = a^2}$

Want $a^2 + b^2 = c^2$

$d_2^2 = (10-4)^2 + (-5-5)^2$

$= 6^2 + 10^2$

$= 36 + 100 = \boxed{136 = b^2}$

Pythagorus:

Right $\triangle \Rightarrow a^2+b^2=c^2$

converse:

$a^2+b^2=c^2 \Rightarrow$ Right $\triangle$

$d_3^2 = ((10-(-1))^2 + (-5-2)^2$

$= 11^2 + 7^2 = 121 + 49 = 170 = c^2$

$a^2+b^2 = 34+136 = 170 = c^2 \Rightarrow$ Right triangle.

We compute the squares of these distances and see if a Pythagorean relationship exists between the 3 distances.

$d_1^2 = \left(\sqrt{(4-(-1))^2 + (5-2)^2}\right)^2 = 5^2 + 3^2 = 25+9 = 34$

$d_2^2 = (4-10)^2 + (5-(-5))^2 = 6^2 + 10^2 = 36+100 = 136$

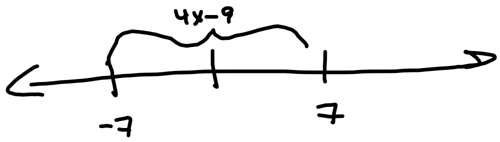
$d_3^2 = (-1-10)^2 + (2-(-5))^2 = 11^2 + 7^2 = 121+49 = 170.$

$d_1^2 + d_2^2 = 34 + 136 = 170 = d_3^2$

$\Rightarrow$ ABC is a right triangle.

3. Solve the absolute-value inequalities.

   a. (5 pts) $|4x-9| \le 7$



AND

$|\cdot| \le 7 \longrightarrow$

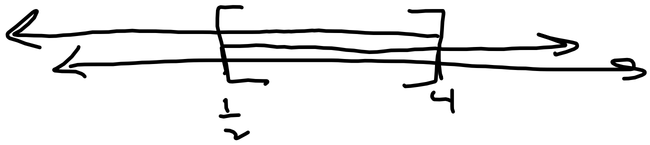
$\cdot \le 7$ AND $\cdot \ge -7$

$4x-9 \le 7$ AND $4x-9 \ge -7$

$$\begin{array}{r} 4x-9 \ge -7 \\ +9 = +9 \\ \hline 4x \ge 2 \end{array}$$

$4x \le 16$

$x \le \frac{16}{4} = 4$ AND $x \ge \frac{2}{4} = \frac{1}{2}$



AND Keep Both happy

$= [\frac{1}{2}, 4]$

$|4x-9| \le 7 \longrightarrow$

$$\begin{array}{r} -7 \le 4x-9 \le 7 \\ +9 = +9 = +9 \\ \hline 2 \le 4x \le 16 \end{array}$$

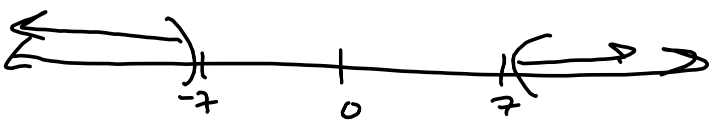
$\boxed{\text{WORKS for the "AND"}}$

NOT FOR the "OR"!

$\frac{1}{2} = \frac{2}{4} \le x \le \frac{16}{4}$

$\frac{1}{2} \le x \le 4$

$\longrightarrow \{x \mid \frac{1}{2} \le x \le 4\}$



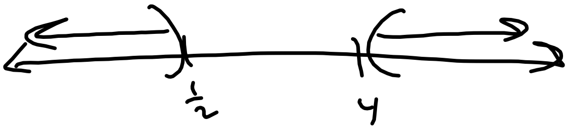
b. (5 pts) $|4x-9|>7$

$4x-9>7$ OR $4x-9<-7$

$4x>16$ $\quad$ $4x<2$

$x>\frac{16}{4}=4$ OR $x<\frac{2}{4}=\frac{1}{2}$ $\quad$ $-7>4x-9>7$

$\{x \mid \frac{1}{2}>x>4\}=\emptyset$



OR = "$\cup$" = "$\vee$" = union = $\boxed{\{x \mid x<\frac{1}{2} \text{ OR } x>4\}}$

No overlap.

Disjoint sets. Need "union symbol."

$\boxed{x \in (-\infty,\frac{1}{2}) \cup (4,\infty)}$

c. (5 pts) $|4x-9| \le 0$

$-0 \le 4x-9 \le 0$

$9 \le 4x \le 9$

$\frac{9}{4} \le x \le \frac{9}{4}$

$\boxed{x=\frac{9}{4}}$

d. (5 pts) $|4x-9|>-7$ $\quad$ $\boxed{(-\infty,\infty)}$

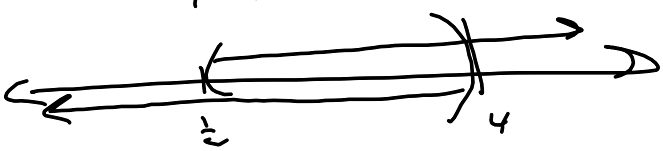
$|5x+3|<-7<0$

$\emptyset$

$4x-9>-7$ OR $4x-9<7$

$4x>2$ $\quad$ $4x<16$

$x>\frac{2}{4}=\frac{1}{2}$ OR $x<\frac{16}{4}=4$



"OR"

$=(-\infty,\infty)$

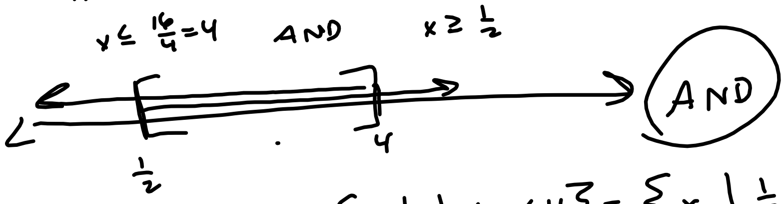
3. Solve the absolute-value inequalities.

a. (5 pts) $|4x-9| \le 7$

$4x-9 \le 7$ AND $4x-9 \ge -7$

$4x \le 16$ $\qquad$ $4x \ge 2$

$x \le \frac{16}{4} = 4$ AND $x \ge \frac{1}{2}$



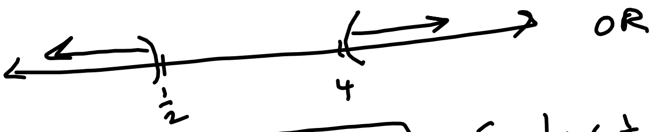
$= [\frac{1}{2}, 4] = \{x \mid \frac{1}{2} < x < 4\} = \{x \mid \frac{1}{2} < x \text{ AND } x < 4\}$

b. (5 pts) $|4x-9| > 7$

$4x-9 > 7$ OR $4x-9 < -7$

$4x > 16$ $\qquad$ $4x < 2$

$x > \frac{16}{4} = 4$ OR $x < \frac{2}{4} = \frac{1}{2}$



$= \boxed{(-\infty, \frac{1}{2}) \cup (4, \infty)} = \{x \mid x < \frac{1}{2} \text{ OR } x > 4\}$

$\frac{1}{2} > x > 4$ No!

c. (5 pts) $|4x-9| \le 0$

$4x-9 = 0$

$4x = 9$

$x = \frac{9}{4}$

$x \in \{\frac{9}{4}\}$

d. (5 pts) $|4x-9| > -7$

$|4x-9| \ge 0 > -7$

$x \in (-\infty, \infty)$

Final words on solving inequalities.

$$|-3x+21| < 5$$

$-3x+21<5$ AND $-3x+21>-5$

$-3x<-16$ $\qquad$ $-3x>-26$

$x > \frac{-16}{-3} = \frac{16}{3}$ AND $x < \frac{-26}{-3} = \frac{26}{3}$

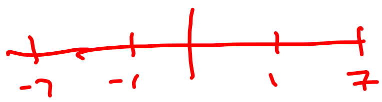
$$\boxed{x \in \left(\frac{16}{3}, \frac{26}{3}\right)}$$

When people go wrong!

$-3x < -16$

$\frac{-3x}{-3} > \frac{-16}{-3}$

$x > \frac{-16}{-3}$



$1 < 7$

$-1 > -7$

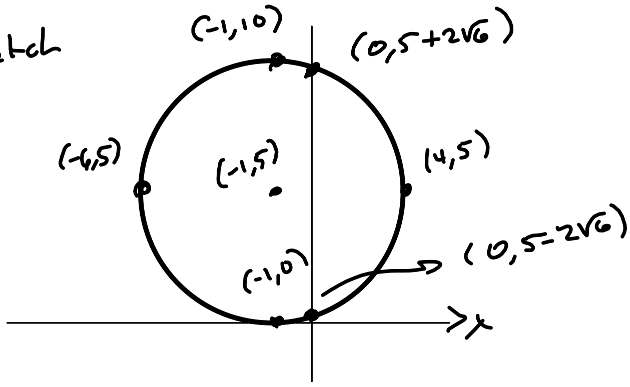
4 a) $x^2+y^2+2x-10y=-1$

$x^2+2x+1^2 \quad +y^2-10y+(5)^2=-1 \quad +1^2+25$

$(x+1)^2+(y-5)^2=25$

$$\boxed{\begin{array}{c}(h,k)=(-1,5)\\ r=5\end{array}}$$

b) sketch



c) Find x- & y-ints:

x-int : $(-1,0)$

y-int : $x=0 \Rightarrow$

$(0+1)^2+(y-5)^2=25$

$1+(y-5)^2=25$

$(y-5)^2=24$

$y-5=\pm\sqrt{24} \quad =2\sqrt{6}$

$y=5\pm2\sqrt{6}$

(5) $f(x) = \frac{1}{x-8}$, $g(x) = \sqrt{x-3}$, $h(x) = x^2 + 2x$

(2) Domains:

$\frac{\text{stuff}}{0}$ $\quad$ $\sqrt{\text{negative}}$ $\Big\}$ $\frac{\text{Stuff}}{\sqrt{\text{negative}}}$

$\frac{\text{STUFF}}{\text{JUNK}}$ Need JUNK $\neq 0$

$\sqrt{\text{JUNK}}$ " " ≥ 0

$\Big\}$ $\frac{\text{STUFF}}{\sqrt{\text{JUNK}}}$ Need JUNK > 0

$f(x)$: Need $x - 8 \neq 0$

$x \neq 8$

ALL legit $\{x \mid x \neq 8\} = (-\infty, 8) \cup (8, \infty) = \boxed{\mathbb{R} \setminus \{8\} = \mathcal{D}(f)}$

$g(x)$: Need $x - 3 \geq 0$

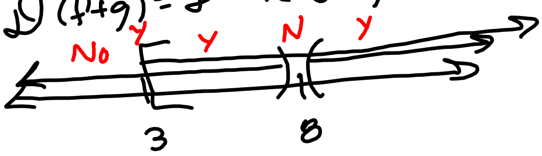
$\Rightarrow x \geq 3$

$\Rightarrow x \in [3, \infty) = \{x \mid x \geq 3\} = \mathcal{D}(g)$

$h(x)$: Polynomial $\Rightarrow \boxed{\mathcal{D}(h) = \mathbb{R}} = (-\infty, \infty) = \{x \mid x \text{ is real}\}$

(3) $\boxed{f + g = \frac{1}{x-8} + \sqrt{x-3}}$

$\mathcal{D}(f+g) = \mathcal{D}(f) \cap \mathcal{D}(g) = \{x \mid x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g)\}$



$= \boxed{[3, 8) \cup (8, \infty) = \mathcal{D}(f+g)} = \{x \mid 3 \leq x < 8 \text{ OR } 8 < x\}$

(c) $g \circ h = g(h(x)) = \sqrt{h(x) - 3} = \sqrt{x^2 + 2x - 3}$

Need $x^2 + 2x - 3 \geq 0$

$(x+3)(x-1) \geq 0$

Y Y N Y Y

$+ \quad - \quad +$ ≥ 0

$-3 \quad 1$

$=0 \quad =0$

$= \boxed{(-\infty, -3] \cup [1, \infty)} = \{x \mid x \leq -3 \text{ OR } x \geq 1\}$

(d) $f \circ h = f(h(x)) = \dfrac{1}{h(x) - 8} = \dfrac{1}{x^2 + 2x - 8}$

$\sim(A \text{ OR } B)$

$= \sim A \text{ AND } \sim B$

Need $x^2 + 2x - 8 \neq 0$

$(x+4)(x-2)$

NOT (THIS OR THAT) means NOT THIS and NOT THAT

$-4 \quad 2$

$\mathcal{D}(f \circ h) = \mathbb{R} \setminus \{-4, 2\} = \{x \mid x \neq -4 \text{ or } 2\} = \{x \mid x \neq -4 \text{ AND } x \neq 2\}$

$= (-\infty, -4) \cup (-4, 2) \cup (2, \infty)$

NOT (A AND B)

$\Longleftrightarrow$ NOT A OR NOT B

$\sim(A \wedge B)$

$\Longleftrightarrow \sim A \vee \sim B$

(a) $A = (3,-5) = (x_1, y_1)$

$B = (7,8) = (x_2, y_2)$

Line defined by A & B

$y = m(x - x_1) + y_1$

$= m(x-3) - 5$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-(-5)}{7-3} = \frac{13}{4}$

$\boxed{y = \frac{13}{4}(x-3) - 5}$

(b) Parallel thru $(-11,7)$

$\boxed{y = \frac{13}{4}(x+11) + 7}$

(c) Perpendicular, thru $(-11,7)$

$\boxed{y = -\frac{4}{13}(x+11) + 7}$

$y = m(x - x_1) + y_1$

$y - y_1 = m(x - x_1)$

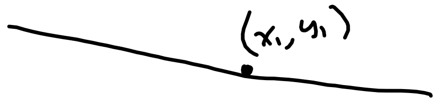
$m = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{RISE}}{\text{RUN}}$

$\frac{y_2 - y_1}{x_2 - x_1} = m$ If (x,y) is any point on the line, then

$\frac{y - y_1}{x - x_1} = m$

$y - y_1 = m(x - x_1)$

$\boxed{y = f(x) = m(x - x_1) + y_1 = y}$



$y = -\frac{2}{3}(x - x_1) + y_1$

⑦ $f(x) = 2x^2 - 3x - 1$

ⓐ Solve $f(x) = 0$ w/ Quadratic Formula

$$ax^2+bx+c=0 \Longrightarrow$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\Longrightarrow a = 2, b = -3, c = -1$$

$$\Longrightarrow \cancel{b^2-4ac = -3^2 - 4(2)(-1)} = 9 + 8 = 17$$

NO

$$b^2-4ac = 3^2 - 4(2)(-1) = 9 + 8 = 17$$

$$\Longrightarrow x = \frac{3 \pm \sqrt{17}}{2(2)} = \boxed{\frac{3 \pm \sqrt{17}}{4} = x}$$

ⓑ By completing the square.
(See below.)

$$\left(x - \frac{3}{4}\right)^2 - \frac{17}{16} = 0 \Longrightarrow$$

$$\left(x - \frac{3}{4}\right)^2 = \frac{17}{16} \Longrightarrow$$

$$x - \frac{3}{4} = \pm\sqrt{\frac{17}{16}} = \pm\frac{\sqrt{17}}{4}$$

$$\boxed{x = \frac{3 \pm \sqrt{17}}{4}}$$

$$\sqrt{\left(x - \frac{3}{4}\right)^2} = \sqrt{\frac{17}{16}}$$

$$\left|x - \frac{3}{4}\right| = \sqrt{\frac{17}{16}} = \frac{\sqrt{17}}{4}$$

(c) Write $f(x)$ in standard form $\quad a(x-h)^2+K=f(x)$

$(h,K)=$ Vertex.

$f(x)=2x^2-3x-1$

$\frac{f(x)}{2}=x^2-\frac{3}{2}x-\frac{1}{2}$

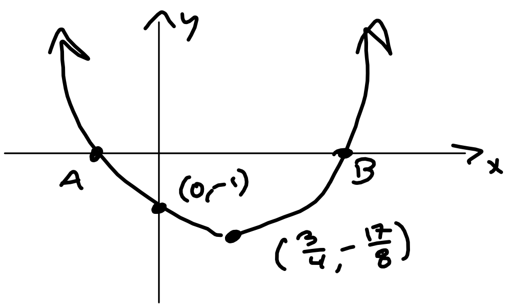
$= \underbrace{x^2-\frac{3}{2}x+\left(\frac{3}{4}\right)^2}-\frac{9}{16}-\frac{1}{2}\cdot\frac{8}{8}$

$\frac{3}{2}\div 2=\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)=\frac{3}{4} \qquad \frac{-9-8}{16}=-\frac{17}{16}$

$\frac{f(x)}{2}=\left(x-\frac{3}{4}\right)^2-\frac{17}{16}$

$\rightarrow \boxed{f(x)=2\left(x-\frac{3}{4}\right)^2-\frac{17}{8}}$

(d)



$(h,K)=\left(\frac{3}{4},-\frac{17}{8}\right)$

y-int: $(0,-1)$

x-ints:

$B=\left(\frac{3+\sqrt{17}}{4},0\right)$

$A=\left(\frac{3-\sqrt{17}}{4},0\right)$

(e) Domain & Range
(Graph helps)

$D=\mathbb{R}$

$R=\left[-\frac{17}{8},\infty\right)$

8 FACTOR $140x^2 - 111x - 198$

We learn to factor so we can use the principle of zero products to solve quadratic equations and polynomial equations in general.

But we can also find zeros to reveal how a polynomial factors.

This is the Factor Theorem at work.

Sledgehammer method:

1. Find the zeros with quadratic formula.

2. Use the zeros to write the factorization.

FACTOR $140x^2 - 111x - 198$

$a = 140,\ b = -111,\ c = -198$

$b^2 - 4ac = 111^2 - 4(140)(-198) = 123201 \to 351$

$3 \overline{)231}$ $7\overline{)77}$ 11 140

$\to x = \dfrac{111 \pm 351}{2(140)}$

$\dfrac{111+351}{2(140)} = \dfrac{462}{280} = \dfrac{\cancel{231}^{77}}{\cancel{140}_{20}} = \dfrac{77}{20}$

$\dfrac{111-351}{280} = \dfrac{-240}{280} = -\dfrac{6}{7}$

$(7)(20)\left(x + \frac{6}{7}\right)\left(x - \frac{77}{20}\right)$

$\boxed{(7x+6)(20x-77) = f(x)}$

9 Solve $\dfrac{x}{x-1} = \dfrac{-6}{x-7}$ LCM $= (x-1)(x-7) =$ LCD

$$x(x-7) = -6(x-1)$$

$$x^2 - 7x = -6x + 6$$

$$-6 \quad +6x = +6x - 6$$

$$x^2 - x - 6 = 0$$

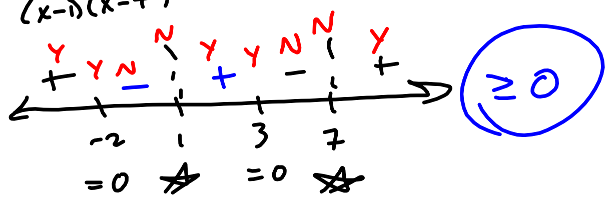
$$(x-3)(x+2) = 0$$

$$\Rightarrow \boxed{x \in \{3, -2\}}$$

10 Solve $\dfrac{x}{x-1} \geq \dfrac{-6}{x-7}$

$$\frac{x^2 - 7x}{(x-1)(x-7)} \geq \frac{-6(x-1)}{(x-7)(x-1)}$$

$$\Rightarrow \frac{x^2 - x - 6}{(x-1)(x-7)} = \frac{(x-3)(x+2)}{(x-1)(x-7)} \geq 0$$



$$\Rightarrow \boxed{x \in [-\infty, -2] \cup (1, 3] \cup (7, \infty).}$$

(1) $f(x) = \frac{1}{x-2}$

(a) Show $f(x)$ is 1-to-1

1-to-1 means:

If $f(x_1) = f(x_2) \Longrightarrow x_1 = x_2$

If $x_1 \neq x_2 \longrightarrow f(x_1) \neq f(x_2)$

$A \Rightarrow B$ is equivalent to $\Longleftrightarrow$ Not $B \Rightarrow$ Not A

$\S$ $f(x_1) = f(x_2)$

$\Rightarrow \frac{1}{x_1-2} = \frac{1}{x_2-2}$

$\Rightarrow x_2 - 2 = x_1 - 2$

$\Rightarrow x_2 = x_1 \Rightarrow$ 1-to-1.

(b) find f^{-1}:

$y = \frac{1}{x-2}$

$x = \frac{1}{y-2}$

$\Rightarrow x(y-2) = xy - 2x = 1$

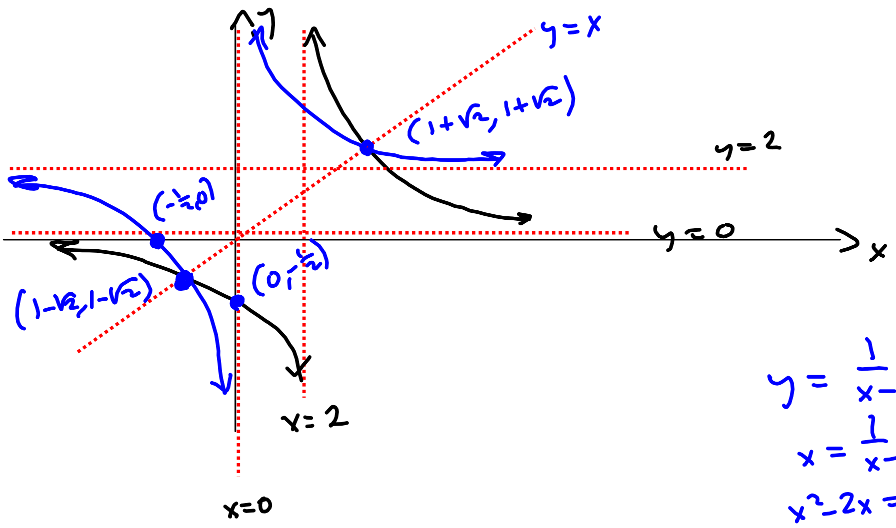
$\Rightarrow xy = 2x + 1$

$\Rightarrow y = \boxed{\frac{2x+1}{x} = f^{-1}(x)}$

(c) Graph f & f^{-1} on same graph. Make f "fat"

$f = \frac{1}{x-2}$ & $\frac{2x+1}{x} = f^{-1}$

$(-\frac{1}{2}, 0)$



$y = \frac{1}{x-2}$

$x = \frac{1}{x-2}$

$x^2 - 2x = 1$ $\quad x^2 - 2x - 1 = 0$

$x^2 - 2x + 1^2 - 1^2 - 1 = 0$

$(x-1)^2 = 2$

$x = 1 \pm \sqrt{2}$

(d) Find $\mathcal{D}(f)$, $\mathcal{R}(f)$, $\mathcal{D}(f^{-1})$, $\mathcal{R}(f^{-1})$

$$\boxed{\begin{array}{l} \mathcal{D}(f) = \mathbb{R} \setminus \{2\} = \mathcal{R}(f^{-1}) \\ \mathcal{D}(f^{-1}) = \mathbb{R} \setminus \{0\} = \mathcal{R}(f) \end{array}}$$

$\frac{1}{x-2} \Rightarrow x \neq 2$

$\frac{2x+1}{x} \Rightarrow x \neq 0$

12 $f(x) = x^4 - 7x^3 + 21x^2 - 37x + 30$

(a) Find possible rational zeros

$\dfrac{\text{factors of } 30}{\text{factors of } 1}$: $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

(b) Synthetic Division to find all rational zeros & split f as far as we can with them.

$$\begin{array}{r|rrrrr} 1 & 1 & -7 & 21 & -37 & 30 \\ & & 1 & -6 & 15 & -18 \\ \hline & 1 & -6 & 15 & -18 & 12 \end{array}$$

$$\begin{array}{r|rrrrr} -1 & 1 & -7 & 21 & -37 & 30 \\ & & -1 & 8 & -29 & \\ \hline & 1 & -8 & 29 & -66 & \end{array}$$ Nope

$$\begin{array}{r|rrrrr} 2 & 1 & -7 & 21 & -37 & 30 \\ & & 2 & -10 & 22 & -30 \\ \hline & 1 & -5 & 11 & -15 & 0 \end{array}$$ sweet!

$(x-2)\underbrace{(x^3 - 5x^2 + 11x - 15)}_{\text{Depressed.}}$

$$\begin{array}{r|rrrr} 2 & 1 & -5 & 11 & -15 \\ & & 2 & -6 & 10 \\ \hline & 1 & -3 & 5 & -5 \end{array}$$ Nope

$$\begin{array}{r|rrrr} -2 & 1 & -5 & 11 & -15 \\ & & -2 & 14 & \\ \hline & 1 & -7 & 25 & \end{array}$$ None

$$\begin{array}{r|rrrr} 3 & 1 & -5 & 11 & -15 \\ & & 3 & -6 & 15 \\ \hline & 1 & -2 & 5 & 0 \end{array}$$ sweet!

$\boxed{(x-2)(x-3)(x^2 - 2x + 5)}$

$x^2 - 2x + 5$ irreducible?

$a = 1,\ b = -2,\ c = 5$

$b^2 - 4ac = 2^2 - 4(1)(5) = 4 - 20 < 0$ yes.

(c) Find nonreal zeros

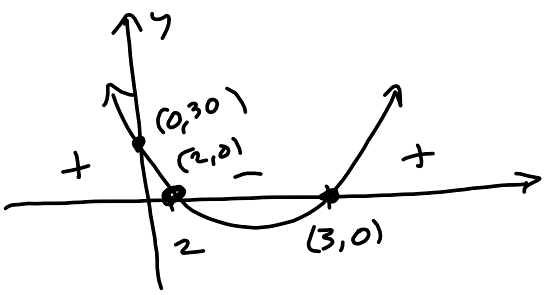
$b^2-4ac = -16 \rightsquigarrow \sqrt{-16} = 4i$

$x = \frac{2 \pm 4i}{2} = \boxed{1 \pm 2i = x}$

(d) Split f into linear factors

$$\boxed{f(x) = (x-2)(x-3)(x-(1+2i))(x-(1-2i))}$$

(e) Sketch f. $f(0) = 30$



x^4 ↖ … ↗

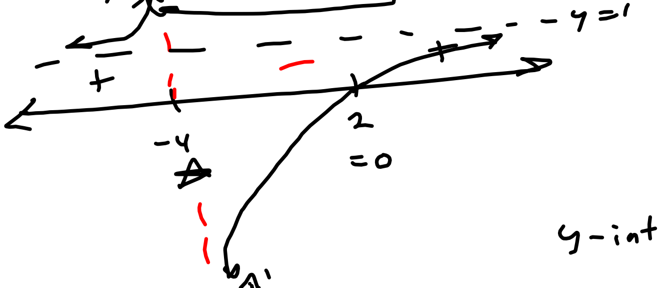
(13) Sketch $R(x) = \frac{(x-2)^1}{(x+4)^1} = 0 \Rightarrow x = 2 \rightarrow \boxed{(2,0)\ x\text{-int}}$

$=0$

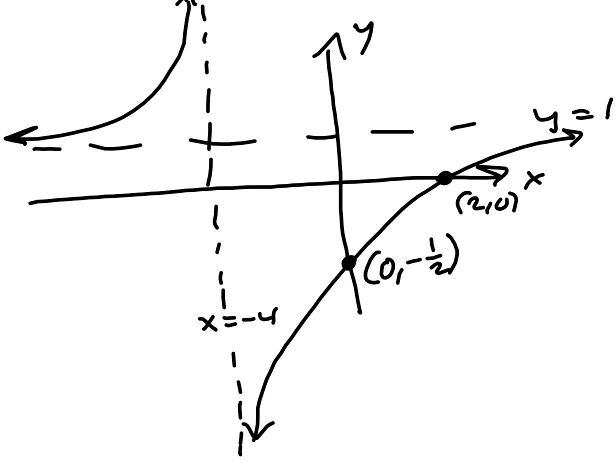
$x+4=0$

$\rightarrow \boxed{x=-4 \quad V.A.}$

H.A.: $\frac{x}{x} = \boxed{1 = y \text{ is H.A.}}$ (End Behavior (+)



y-int: $(0, -\frac{1}{2})$



(b) $T(x) = \dfrac{x^2-4x-32}{x-5}$ sketch

$= \dfrac{(x-8)(x+4)}{x-5} = 0 \Rightarrow x = 8, x = -4$

$$\begin{array}{c|ccc} 5 & 1 & -4 & -32 \\ & & 5 & \\ \hline & 1 & 1 & \end{array}$$

$\boxed{y = x+1 \text{ is O.A.}}$

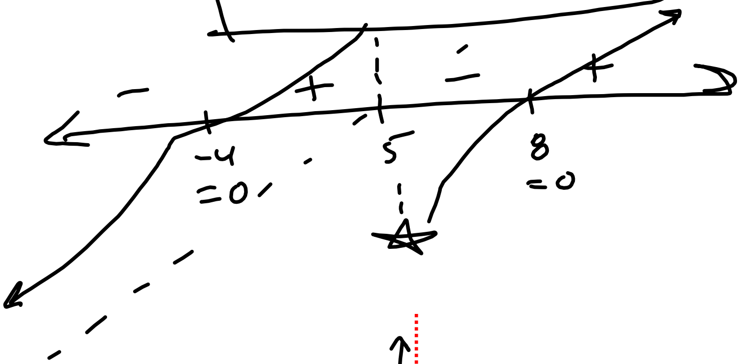
$\boxed{x = 5 \text{ V.A.}}$

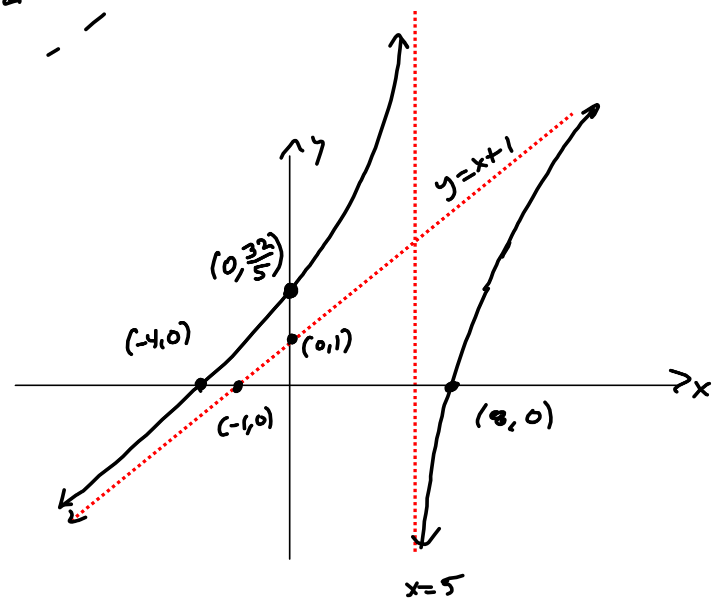
$\boxed{\text{H.A.: No joke}}$

$T(x) = 0 \Rightarrow (x+4)(x-8) = 0$

$\Rightarrow x = -4, 8 \Rightarrow$

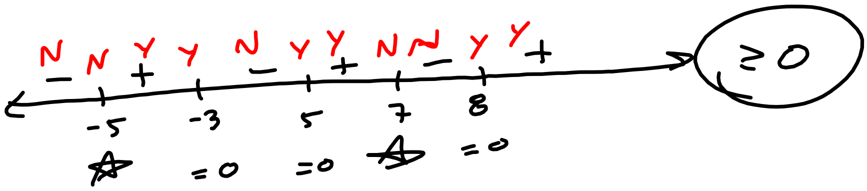
$\boxed{\text{x-ints: } (-4,0) \quad (8,0)}$





14 $u(x) = \dfrac{(x+3)(x-5)(x-8)}{(x+5)(x-7)}$ what's domain of $v(x) = \sqrt{u(x)}$

Domain: Need $u(x) \geq 0$

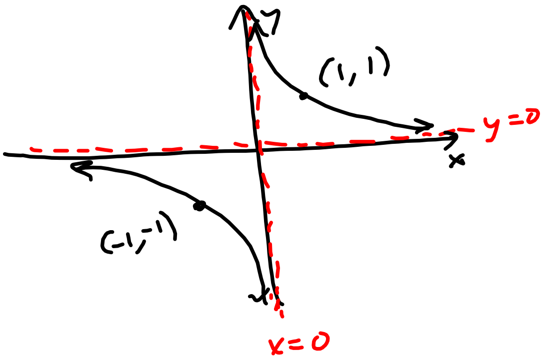


$= (-5, -3] \cup [5, 7) \cup [8, \infty)$

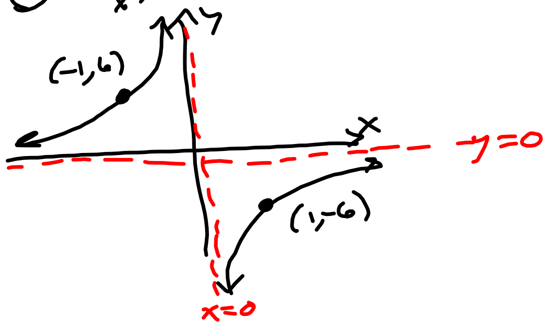
$$\frac{(100+3)(100-5)(100-8)}{(100+5)(100-7)}$$

(15) Sketch $g(x) = \frac{-6}{(3x+21)^3} + 10$. $f(x) = \frac{1}{x^3}$ (Like $\frac{1}{x}$)

(1) $f(x) = \frac{1}{x^3}$



(2) $\frac{-6}{x^3}$ $y \mapsto -6y$

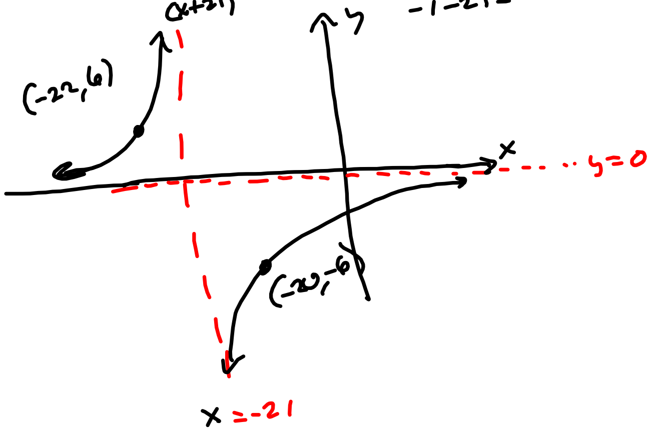


(3) METHOD 1

$-6f(x+21)$ $x \mapsto x-21$

$= \frac{6}{(x+21)^3}$ $1-21=-20$

$-1-21=-22$



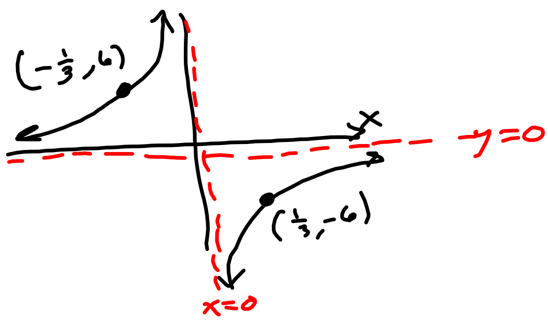
(3) METHOD 2

$-6f(3x) = \frac{-6}{(3x)^3}$

$x \mapsto \frac{1}{3}x$

$(-1, 6) \mapsto (-\frac{1}{3}, 6)$

$(1, -6) \mapsto (\frac{1}{3}, -6)$



METHOD 1

$-6f(3x+21) = \frac{-6}{(3x+21)^3} =$

$x \mapsto \frac{1}{3}x$

METHOD 2

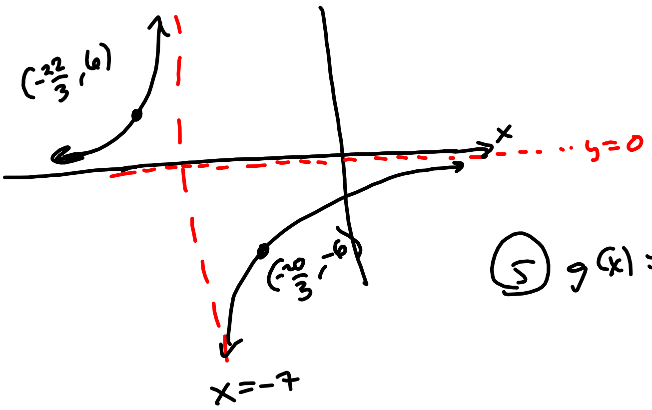
$3x+21 = 3(x+7)$

$x \mapsto x-7$

$-\frac{1}{3}-7 = \frac{-1-21}{3} = -\frac{22}{3}$

$\frac{1}{3}-7 = \frac{1-21}{3} = -\frac{20}{3}$

4



5 $g(x) = -6f(3x+21)+10$

$y \mapsto y+10$