MAT 1340 Final, Fall, 2025
140 points Comprehensive

(5 pts) Print Your Name_____

NOTE TO PROCTORS: The time control is 2 hours. Please encourage students to write on only one side of each sheet of paper. This way they can refer to previous work for current work.

Materials Permitted: Pencil/Pen, Scientific Calculator (not graphing), Straight Edge, 2-page cheat sheet (either one sheet, 2 sided or 2-page, 1-sided). Include student cheat sheets in scans. Place them at the end.

TO STUDENTS: Show all work. That means scratch work goes with the problem, and not on a separate sheet. **Do your own work**. Leave at least ½-inch margins around each sheet. If you're taking the test in Horizon Hall, this has already been done for you, with a border.

You may work up to 4 bonus problems, at your discretion. There's something of everything on this test.

Remember #1 is above #2 is above #3a is above #3b is above #3c, ... If your work is in two columns, with #5 to the right of #3, you won't receive any points for #5. Writing to communicate is a requirement. If I can't read your work, I can't give you a passing grade.

Leave plenty of space. If it's too cramped to efficiently award partial credit, there will be no partial credit. If you get the answer right, but I can't understand what you did, you will get ½-credit, at most. I need to see the support. I need to see all the scratch work for each problem WITH that problem, not on a separate sheet.

Turn in your test sheets, your work, and your cheat sheet. Test sheets on top. Your work, next, and Cheat Sheet at the bottom.

Draw Pictures! Fare well!

1. (**Bonus** 10 pts) Consider the system of linear inequalities: $\begin{aligned} 2x-7y &\le 70 \\ 2x+3y &\le 24 \\ x &\ge 0 \\ y &\le 0 \end{aligned}$.

   Sketch the system. Find and label all x- and y-intercepts with ordered-pair labels, such as $(-2, 0)$. Don't waste time on tick marks. Use the intercept method.

   Find the corner points of the feasible region. For full credit, give exact answers as fractions.

2. Solve the absolute-value inequalities.

   a. (5 pts) $|-5x-9| \le 6$

   b. (5 pts) $|-5x-9| > 6$

   c. (5 pts) $|-5x-9| < -6$

3. Let $f(x)=\sqrt{x+10}$, $g(x)=\dfrac{1}{x+10}$, and $h(x)=x^2-7x$

   a. (5 pts) Find the domain of *f*, *g*, and *h*.

   b. (5 pts) Find $\dfrac{f}{g}$ and state its domain.

   c. (5 pts) Find $g \circ h$ and state its domain.

   d. (**Bonus** 10 pts) Find $f \circ h$ and state its domain.

4. Let $A = (-3, 4)$ and $B = (5, 11)$.

   a. (5 pts) Find an equation $y = m(x - x_1) + y_1$ of the line passing through *A* and *B*.

   b. (5 pts) Find an equation of the line $y = m(x - x_1) + y_1$ passing through $\left(-\dfrac{3}{17}, \dfrac{53}{29}\right)$ that is parallel to the line from part a.

   c. (5 pts) Find an equation of the line $y = m(x - x_1) + y_1$ passing through $\left(-\dfrac{3}{17}, \dfrac{53}{29}\right)$ that is perpendicular to the line from part a.

5. Let $f(x)=x^2-5x-5$.

   a. (5 pts) Solve the equation $f(x)=0$ using the Quadratic Formula. For full credit, evaluate the discriminant and its square root, first. Simplify. THEN plug into the quadratic formula (for literally the last time!) I'm looking for exact answers, not decimal approximations.

   b. (5 pts) Solve the equation $f(x)=0$ by completing the square. Find the exact solution. Do not use a calculator.

   c. (5 pts) Write $f(x)$ in standard form $f(x)=a(x-h)^2+k$ and sketch its graph. Include the *x*- and *y*-interepts, as well as the vertex in your graph.

   d. (5 pts) What is the domain of f? What is the range of f?

6. (5 pts) Solve the equation $\dfrac{3x+11}{x+2} = -\dfrac{16}{x-7}$ for *x*.

7. (**Bonus** 10 pts) Solve the inequality $\frac{3x+11}{x+2} \geq -\frac{16}{x-7}$ for x.

8. Let $f(x) = 2x^4 + 3x^3 - 7x^2 + 8x + 6$.

   a. (5 pts) List all possible rational zeros of f.

   b. (5 pts) Use synthetic division and the rational zeros of f to split f into the product of 2 linear factors and one irreducible quadratic factor.

   c. (5 pts) Find the nonreal zeros of f, using your work from part b.

   d. (5 pts) Sketch a graph of f, showing all intercepts.

   e. (**Bonus** 10 pts) Split f into the product of 4 linear factors.

9. (5 pts) Sketch the graph of $R(x) = \frac{x-3}{(x+1)(x-5)}$. Show all intercepts and asymptotes.

10. (**Bonus** 10 pts) Sketch the graph of $T(x) = \frac{x^2+2x-15}{x-7}$. Show all intercepts and asymptotes.

11. (5 pts) Let $V(x) = \log_9\left(\frac{(x+3)^2(x-5)^3(x-8)}{(x+5)^2(x-7)}\right)$ What is the domain of $V(x)$?

12. (5 pts) Sketch the graph of $g(x) = 3 \cdot 5^{4x+8} - 11$ by transforming the basic function $f(x) = 5^x$ in 5 steps, counting $f(x) = 5^x$ as the first step.

13. (**Bonus** 10 pts) Find the x- and y-intercepts of $g(x)$ in #12. Give exact answers, with no calculator.

14. (**Bonus** 10 pts) Find the inverse of $g(x)$ from #12.

15. (**Bonus** 10 pts) Sketch the graph of $h(x) = 5\log_4(3x-9)$ by transforming the basic function $f(x) = \log_4(x)$ in 4 steps, counting the graph of $f(x) = \log_4(x)$ as the first.

16. (5 pts) Sketch the graph of the parabola $y + 7 = -\frac{1}{16}(x-5)^2$. Label the center, focus, and endpoints of the *latus rectum*. Also label the x- and y-intercepts.

17. (5 pts) Graph the hyperbola given by the equation $\frac{x^2}{25} - \frac{y^2}{81} = 1$. Label the center, foci, vertices and asymptotes. I don't need to see the equations of the asymptotes, if you make "the box" and show its x- and y-intercepts,

that should suffice.

18. (5 pts) Show that the equation $36x^2 + 81y^2 - 360x - 1134y = -1953$ represents an ellipse, by completing the square, and write the ellipse's equation in standard form.

19. (**Bonus** 10 pts) Sketch the graph of the ellipse in #18, showing the center, the endpoints of its major and minor axes, and both of its foci. If you can find its *y*-intercepts, that's worth an extra 5 points.

20. (5 pts) Use Pascal's triangle to expand $(2x - 3y)^6$. You needn't simplify, but if you do, there's 5 pts bonus in it for you.

21. (5 pts) What is the coefficient of x^4y^6 in the expansion of $(2x - 3y)^{10}$?

22. (5 pts) How many ways can you pick 4 volunteers from a group of 15 volunteers?

23. (5 pts) How many ways can a president, vice-president, treasurer and secretary be chosen from a club with 30 members?

24. (**Bonus** 10 pts) Solve the following system of equations by elimination method. If no solution, state why. If infinitely many solutions, state the general solution.

$$\begin{aligned} x - 5y - 7z &= 15 \\ 3x - 14y - 19z &= 34 \\ 2x - 12y - 17z &= 35 \end{aligned}$$