Section 5.3 - Partial Fractions

1 Write the form of the partial fraction decomposition of the function (as in Example 4). Do not determine the numerical values of the coefficients.

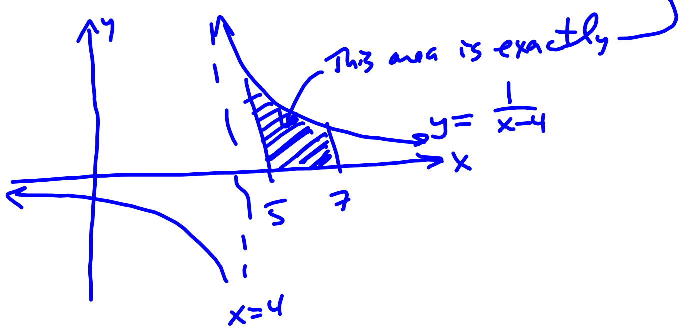
$$\frac{1}{(x-4)(x+3)} = \frac{A}{x-4} + \frac{B}{x+3}$$, where A & B are real #s

We find A & B by reverse-engineering the $\frac{1}{(x-4)(x+3)}$.

$\frac{A}{x-4}$ is nice for Calculus.

$$\int \frac{dx}{x-4} = \ln|x-4| + C$$

$$\int_5^7 \frac{dx}{x-4} = \ln|7-4| - \ln|5-4|$$
$$= \ln(3).$$



Write the form of the partial fraction decomposition of the function (as in Example 4). Do not determine the numerical values of the coefficients.

2 $\frac{x}{x^2+6x-7} = \frac{x}{(x+7)(x-1)} = \frac{A}{x+7} + \frac{B}{x-1}$

3 Write the form of the partial fraction decomposition of the function (as in Example 4). Do not determine the numerical values of the coefficients.

$$\frac{x^2-3x+5}{(x-3)^2(x+1)} = \frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{C}{x+1}$$

Linear factor of $x-3$ raised to a power of 2.

4 Write the form of the partial fraction decomposition of the function (as in Example 4). Do not determine the numerical values of the coefficients.

$$\frac{7}{x^4-x^3} = \frac{7}{x^3(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-1}$$

$x^4-x^3 = x^3(x-1)$

5 Write the form of the partial fraction decomposition of the function (as in Example 4). Do not determine the numerical values of the coefficients.

$$\frac{x^2}{(x-8)(x^2+4)} = \frac{A}{x-8} + \frac{Bx+C}{x^2+4}$$

Keep it real!

x^2+4 is irreducible over the reals.

6 Write the form of the partial fraction decomposition of the function (as in Example 4). Do not determine the numerical values of the coefficients.

$$\frac{1}{x^4-625} = \frac{A}{x-5} + \frac{B}{x+5} + \frac{Cx+D}{x^2+25}$$

$$x^4-625 = x^4-25^2 = (x^2)^2-25^2 = (x^2-25)(x^2+25)$$

$$= (x-5)(x+5)(x^2+25)$$

7 Write the form of the partial fraction decomposition of the function (as in Example 4). Do not determine the numerical values of the coefficients.

$$\frac{x^3 - 4x^2 + 2}{(x^2 + 25)(x^2 + 8)} = \frac{Ax+B}{x^2+25} + \frac{Cx+D}{x^2+8}$$

Write the form of the partial fraction decomposition of the function. Do not determine the numerical values of the coefficients.

8 $$\frac{x^4 + x^2 + 1}{x^2(6x^2 + 7)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{6x^2+7} + \frac{Ex+F}{(6x^2+7)^2}$$

Write the form of the partial fraction decomposition of the function (as in Example 4). Do not determine the numerical values of the coefficients.

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$$\frac{1}{(x^3 - 125)(x^2 - 25)} = \frac{1}{(x-5)(x^2+5x+25)(x-5)(x+5)}$$

$$x^3 - 5^3 = (x-5)\underbrace{(x^2+5x+25)}_{\text{irreducible}}$$

$$= \frac{1}{(x-5)^2(x+5)(x^2+5x+25)} = \frac{A}{x-5} + \frac{B}{(x-5)^2} + \frac{C}{x+5} + \frac{Dx+E}{x^2+5x+25}$$

Find the partial fraction decomposition of the rational function.

11 $\frac{4}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} \Rightarrow$

$4 = A(x+1) + B(x-1) \Rightarrow$

$4 = Ax + A + Bx - B \Rightarrow$

$Ax + Bx = 0$ $\qquad A - B = 4$ +

$x(A+B) = 0 \Rightarrow$ $\qquad A = B - 4$

$x = 0$ or $A + B = 0$

$\Rightarrow A = -B$

$\Rightarrow -B = B + 4$

$-2B = +4$

$\boxed{B = -2}$ &

$A = -B \Rightarrow \boxed{A = +2}$

-1

$\frac{-2}{x-1} + \frac{2}{x+1}$

$= \frac{-2(x+1) + 2(x-1)}{(x-1)(x+1)} = \frac{-2x - 2 + 2x - 2}{(x-1)(x+1)} = \frac{-4}{(x-1)(x+1)}$

Find the partial fraction decomposition of the rational function.

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$$\frac{9x^2 - 14x + 16}{2x^3 - x^2 - 8x + 4} = \frac{A}{2x-1} + \frac{B}{x-2} + \frac{C}{x+2}$$

$x^2(2x-1) - 4(2x-1)$

$= (2x-1)(x^2-4) = (2x-1)(x-2)(x+2)$

$9x^2 - 14x + 16 = A(x-2)(x+2) + B(2x-1)(x+2) + C(2x-1)(x-2)$

$= A(x^2-4) + B(2x^2+3x-2) + C(2x^2-5x+2)$

$\Rightarrow 9x^2 = Ax^2 + 2Bx^2 + 2Cx^2$ $\qquad -14x = 3Bx - 5Cx$

$9 = A + 2B + 2C$ $\qquad -14 = 3B - 5C$

$16 = -4A - 2B + 2C$

$A + 2B + 2C = 9$ E1
$3B - 5C = -14$ E2
$-4A - 2B + 2C = 16$ E3

E1 $\quad A + 2B + 2C = 9$ E1
E2 $\quad 3B - 5C = -14$ E2
4E1 + E3 $\quad 6B + 10C = 52$ E3

E1 $\quad A + 2B + 2C = 9$
E2 $\quad 3B - 5C = -14$
-2E2 + E3 $\quad 20C = 80$

$\Rightarrow \boxed{C = 4}$

$\Rightarrow$ E2 $\Rightarrow 3B - 5(4) = 3B - 20 = -14$

$\Rightarrow 3B = 6$

$\Rightarrow \boxed{B = 2}$

$\Rightarrow$ E1 $\Rightarrow A + 2(2) + 2(4) = A + 4 + 8 = A + 12 = 9$

$A = -3$

$$= \frac{A}{2x-1} + \frac{B}{x-2} + \frac{C}{x+2} = \frac{-3}{2x-1} + \frac{2}{x-2} + \frac{4}{x+2}$$

16 Find the partial fraction decomposition of the rational function.

$$\frac{x^3 - 4x^2 - 3x + 5}{x^4} = \frac{x^3}{x^4} - \frac{4x^2}{x^4} - \frac{3x}{x^4} + \frac{5}{x^4}$$

$$= \frac{1}{x} - \frac{4}{x^2} - \frac{3}{x^3} + \frac{5}{x^4}$$

17 Find the partial fraction decomposition of the rational function.

$$\frac{3x - 15}{x^3 + 5x} = \frac{A}{x} + \frac{Bx+C}{x^2+5} \qquad x(x^2+5)$$

$3x-15 = A(x^2+5) + (Bx+C)x$

$3x-15 = Ax^2 + 5A + Bx^2 + Cx$

$5A = -15$

$\boxed{A = -3}$

$\boxed{C = 3}$

$Ax^2 + Bx^2 = 0$

$A + B = 0$

$B = -A = +\boxed{3 = B}$

18 Find the partial fraction decomposition of the rational function.

$$\frac{4x^4 + 4x^3 + 3x^2 - 2x + 4}{x(x^2+1)^2}$$

$$= \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2} \Longrightarrow$$

$$4x^4+4x^3+3x^2-2x+4 = A(x^2+1)^2 + (Bx+C)(x(x^2+1)) + (Dx+E)(x)$$

$$= A(x^2+1)^2 + Bx(x^3+x) + C(x^3+x) + Dx^2+Ex$$

$$= A(x^4+2x^2+1) + B(x^4+x^2) + C(x^3+x) + Dx^2+Ex$$

$4x^4 = (A+B)x^4$ $\quad A+B = 4+B = 4 \Rightarrow \boxed{B=0}$

$4x^3 = Cx^3 \Rightarrow \boxed{C=4}$

$3x^2 = (2A+B+D)$ $\quad 2A+B+D=3$

$2(4)+0+D=3$

$\boxed{D=-5}$

$-2x = (C+E)x$ $\quad -2 = C+E = 4+E$

$\boxed{4=A}$ $\quad \boxed{-6=E}$

$$\frac{4}{x} + \frac{4}{x^2+1} - \frac{+5x+6}{(x^2+1)^2}$$