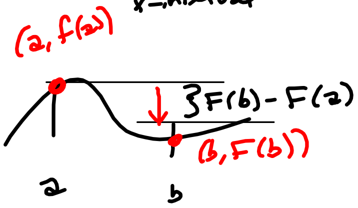
Section 2.3 - Getting Info from the Graph of a Function.

Values of a Function - Domain and Range
Comparing Function Values: Solving Equations and Inequalities Graphically
Increasing and Decreasing Functions
Local Max and Min Values of a Function

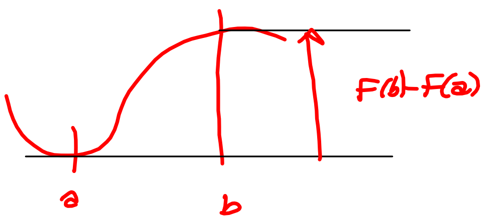
Values of a Function - Domain and Range

The value of a function is its height (or y-value).

Net change on $\underbrace{[a,b]}_{x\text{-interval}}$ is $F(b)-F(a)$

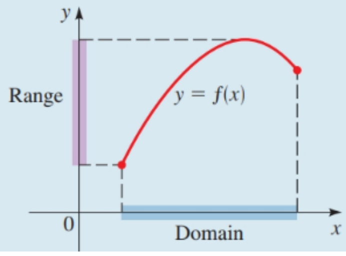


(is negative



DOMAIN AND RANGE FROM A GRAPH

The **domain** and **range** of a function $y = f(x)$ can be obtained from a graph of f as shown in the figure. The domain is the set of all x-values for which f is defined, and the range is all the corresponding y-values.



$\mathcal{D} = \{x \mid f(x) \text{ is Real}\}^*$

$\mathcal{R} = \{y \mid y = f(x) \text{ for some } x \in \mathcal{D}\}$

* Sometimes $\mathcal{D}$ is determined by the situation, not just the algebra

$f(t) = t^2$ Time ≥ 0

\# of chairs ≥ 0

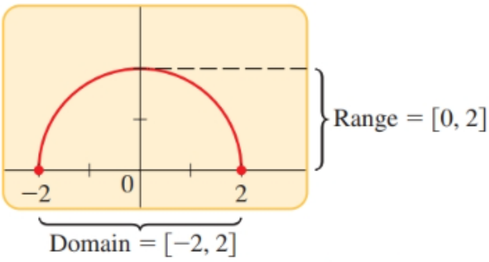


FIGURE 2 Graph of $f(x) = \sqrt{4 - x^2}$

Trick for $\mathcal{D}$ & $\mathcal{R}$ of $\sqrt{f(x)}$

Need $f(x) \geq 0$

$\mathcal{D}$: Need $4 - x^2 \geq 0$

$4 \geq x^2$

$x^2 \leq 4$

$\sqrt{x^2} \leq \sqrt{4}$

$|x| \leq 2$

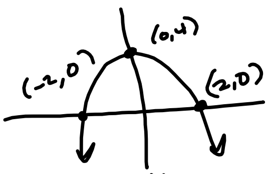
$x \leq 2$ and $x \geq -2$

$-2 \leq x \leq 2$

$\mathcal{D} = [-2, 2]$

$\sqrt{x}$ is increasing so $A \leq B \Rightarrow \sqrt{A} \leq \sqrt{B}$

$\sqrt{4-x^2}$ GRAPH $4-x^2$



N Y Y N

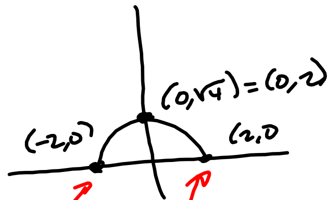
$-$ $+$ $-$ ≥ 0

-2 2

$=0$ $=0$

$= [-2, 2]$

$\sqrt{4-x^2}$



Chop it off.

Comparing Function Values: Solving Equations and Inequalities Graphically

Sol'ns of → $f(x)=g(x)$ are values of x @ which the 2 graphs intersect

$= \{x \mid f(x)=g(x)\}$

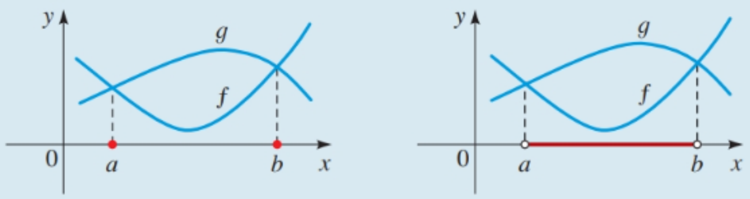
g is higher than f ?
The x-values where this happens are solutions of $f(x) < g(x)$

Solving these inequalities and equations, we're reporting the values of *x* where the inequality/equation is satisfied.

SOLVING EQUATIONS AND INEQUALITIES GRAPHICALLY

The **solution(s) of the equation** $f(x) = g(x)$ are the values of x where the graphs of f and g intersect.

The **solution(s) of the inequality** $f(x) < g(x)$ are the values of x where the graph of g is higher than the graph of f.



The solutions of $f(x) = g(x)$ are the values a and b.

The solution of $f(x) < g(x)$ is the interval (a, b).

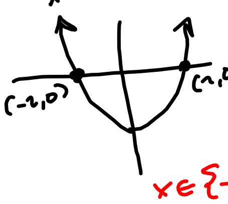
OR

where $f(x)-g(x)$ has x-intercepts!

ADVANTAGE:
You don't have to make a tall window

$x^2 = 4$ ⟹

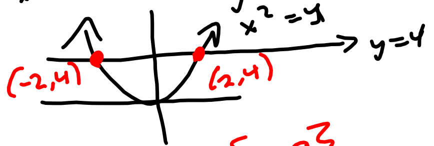
$x^2 - 4 = 0$



The x-ints are the sol'ns!

$x \in \{-2, 2\}$

$x^2=4$ their way:



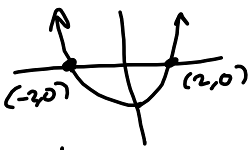
$x \in \{-2, 2\}$

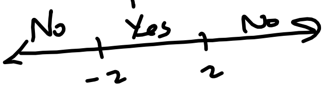
→ $f(x)-g(x) < 0$

Find all x such that $f(x)-g(x)$ graph is BELOW the x-axis

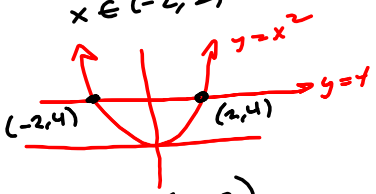
$x^2 < 4$

$x^2 - 4 < 0$





$x \in (-2, 2)$



$x \in (-2, 2)$

Increasing and Decreasing Functions

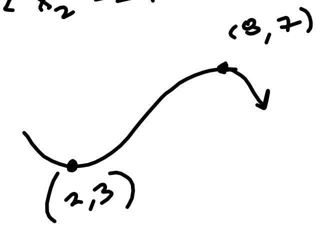
Increasing on interval I means if $x_1, x_2 \in I$. Then

$x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$

Decreasing $\Rightarrow$

$x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$

ALWAYS OPEN Intervals



Book says f is increasing on $(2,8)$

$8 \in [2,8] = I$

Any $8 \neq x \in I$ satisfies

$x < 8 \Rightarrow$

$f(x) < f(8)$

$\Rightarrow 8 \in \{$ values when y is increasing $\}$

From the definition we see that a function increases or decreases *on an interval*. It does not make sense to apply these definitions at a single point.

I guess this is their rationale for not including $x=8$

The argument would be it's not increasing @ $x=8$, b/c $f(8) < f(x)$ for x to its immediate right

LOCAL MAXIMA AND MINIMA OF A FUNCTION

1. The function value $f(a)$ is a **local maximum value** of f if

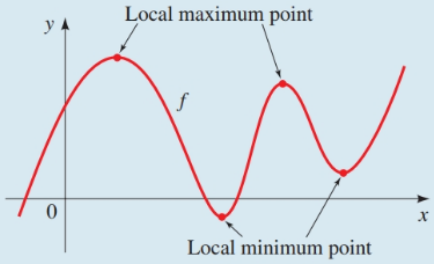
$$f(a) \geq f(x) \quad \text{when } x \text{ is near } a$$

(This means that $f(a) \geq f(x)$ for all x in some open interval containing a.) In this case we say that f has a **local maximum** at $x = a$.

2. The function value $f(a)$ is a **local minimum value** of f if

$$f(a) \leq f(x) \quad \text{when } x \text{ is near } a$$

(This means that $f(a) \leq f(x)$ for all x in some open interval containing a.) In this case we say that f has a **local minimum** at $x = a$.

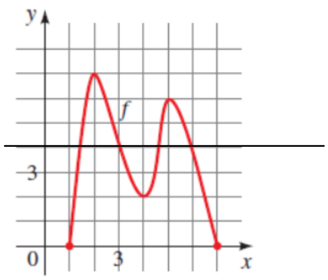


$y = 4$

$(4, 4)$

Refer to the graph of *f* shown below.

1



To find a function value $f(a)$ from the graph of f, we find the height of the graph above the x-axis at x = 3 . From the graph of f, we see that $f(3)$ = 4 and $f(6)$ = 4 . The net change in f between $x = 3$ and $x = 6$ is

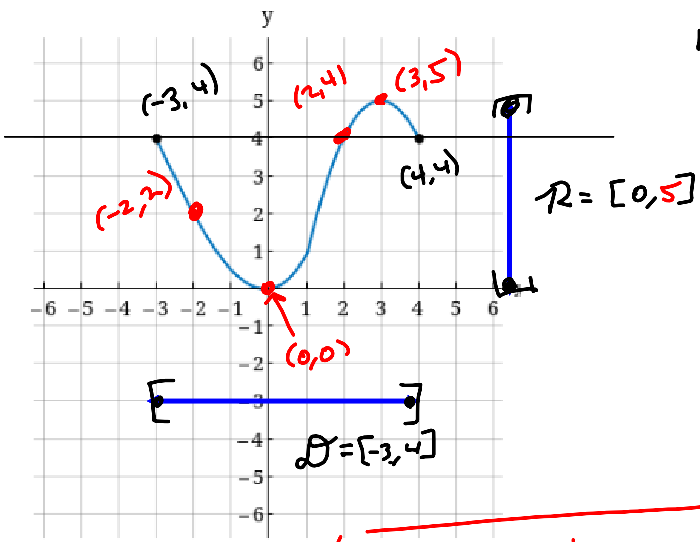
$f(6) - f(3) = 0$

For #s 2 - 6, select the clickable link of your choice:

2 See Video **3 See Video** **4 See Video** **5 See Video** **6 See Video**

The graph of a function h is given.

7



$h(x) \le 4$

$x \in [-3,2] \cup \{4\}$

(a) Find $h(-2)$, $h(0)$, $h(2)$, and $h(3)$.

$h(-2) = 2,\ h(0) = 0,\ h(2) = 4,\ h(3) = 5$

(b) Find the domain and range of h. (Enter your answers using interval notation.)

D = [-3, 4]

R = [0, 5]

(c) Find the values of x for which $h(x) = 4$. (Enter your answers as a comma-separated list.)

$\boxed{x = -2, 3, 4}$

(d) Find the values of x for which $h(x) \le 4$.

(e) Find the net change in h between $x = -3$ and $x = 3$.

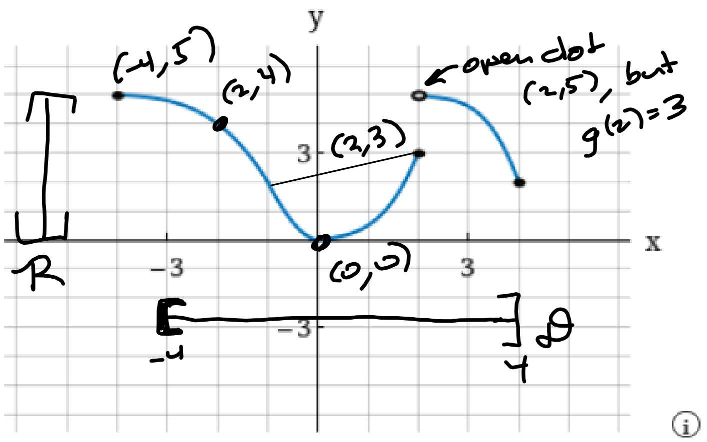
$h(3) - h(-3) = 5 - 4 = \boxed{1}$ = net vertical change.

From $(-3,4) = (-3, h(-3))$ to

$(3,5) = (3, h(3))$

The graph of a function g is given.

8



(a) Find $g(-4)$, $g(-2)$, $g(0)$, $g(2)$, and $g(4)$.

$g(-4)$ = 5

$g(-2)$ = 4

$g(0)$ = 0

$g(2)$ = 3

$g(4)$ = 2

(b) Find the domain and range of g. (Enter your answers using interval notation.)

domain $= D = [-4,4]$

range $= R = [0,5]$

(c) Find the values of x for which $g(x) = 5$. (Enter your answers as a comma-separated list.)

x = 2

(d) Find the values of x for which $g(x) \leq 0$.

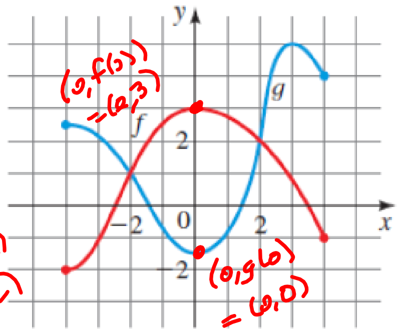
$x \in \{0\}$

(e) Net change from x = -1 to x = 2.

$g(2) - g(1) = 3 - 2 = 1$ = net change over $[-1,2]$.

Graphs of the functions f and g are given.

9



$(-4, g(-4))$
$\approx (-4, 2.5)$

$(0, f(0))$
$= (0, 3)$

$(-4, f(-4))$
$= (-4, -2)$

$(0, g(0))$
$= (0, 0)$

(a) Which is larger, $f(0)$ or $g(0)$?

- $f(0)$ is larger.
- $g(0)$ is larger.
- Neither is larger.

$f(0) = 3 > 0 = g(0)$

(b) Which is larger, $f(-4)$ or $g(-4)$?

- $f(-4)$ is larger.
- $g(-4)$ is larger.
- Neither is larger.

(c) For which values of x is $f(x) = g(x)$? (Enter your answers as a comma-separated list.)

$x =$ −2, 2

(d) For which values of x is $f(x) \le g(x)$?

- [−2, 0] and [2, 4]
- [−2, 2]
- [−2, 4]
- [−4, 2]
- [−4, −2] and [2, 4]

Union / "OR"
$\cup$ OR

$f(x) \le g(x) \quad \forall x \in [-4, -2] \cup [2, 4]$

Yes NO YES
−4 −2 2 4
$f = g$ $f = g$

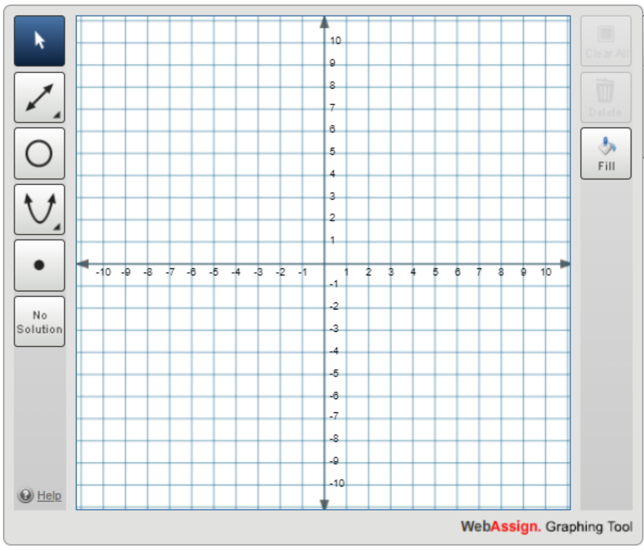
(e) For which values of x is $f(x) > g(x)$?

$f(x) > g(x)$
$(-2, 2)$

A function f is given.

11 $f(x) = 2x - 4$

(a) Sketch a graph of f.



Graph Layers

After you add an object to the graph you can use Graph Layers to view and edit its properties.

(b) Use the graph to find the domain and range of f. (Enter your answers using interval notation.)

domain

range

A function f is given.

12

$$f(x) = x - 3, \quad -3 \le x \le 7$$

Handle that, too, man.

(a) Sketch a graph of f.

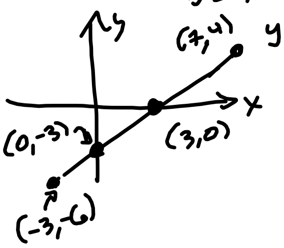
Proper

x	y
0	-3
3	0

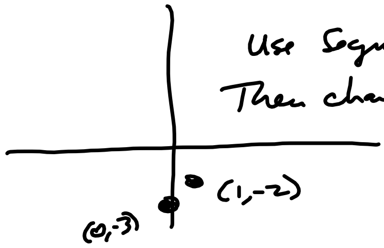
$0 = x - 3$

$3 = x$

$y = f(x) = x - 3$



Practical (WebAssign)



Use Segment tool.
Then change • to o
by clicking
on "LINE1"

To handle the restricted domain

x	y
-3	-6
7	4

Those'd be the points to use for WebAssign.

(b) Use the graph to find the domain and range of f. (Enter your answers using interval notation.)

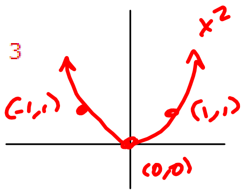
$\mathcal{D} = [-3, 7]$ $\left(= \{x \mid -3 \le x \le 7\} \right.$

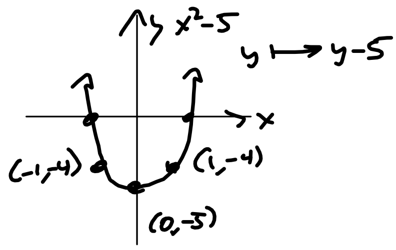
$\left. = \{x \mid -3 \le x \text{ and } x \le 7\} \right)$

A function f is given.

13 $f(x) = x^2 - 5, \quad -3 \le x \le 3$

(a) Sketch a graph of f.



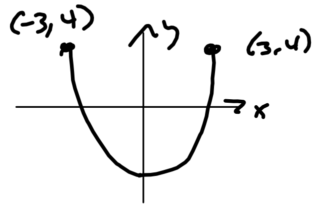


$y \mapsto y-5$

This restriction on x is totally artificial. As a function in the abstract, its domain is all real numbers and its range is all real numbers greater than or equal to $y = -5$

Restrict to $-3 \le x \le 3$:

x	y
-3	4
3	4



Restricted Domain:

$\{x \mid -3 \le x \le 3\} = \boxed{[-3,3] = D}$

$R = \{y \mid y \ge -5\}$

oops! restricted!

$R = \{y \mid -5 \le y \le 4\} = \boxed{[-5,4] = R}$

A function f is given.

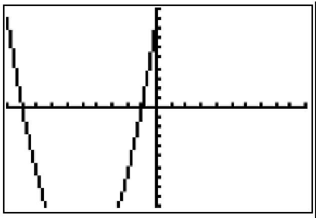
14 $f(x) = x^2 + 10x + 9$

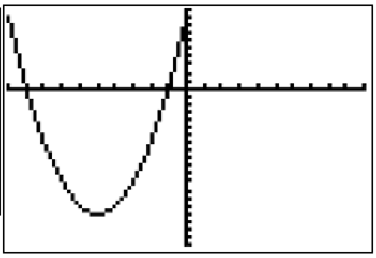
Use a graphing device to draw the graph of f. Find the domain and range of f from the graph. (Enter your answers using interval notation.)

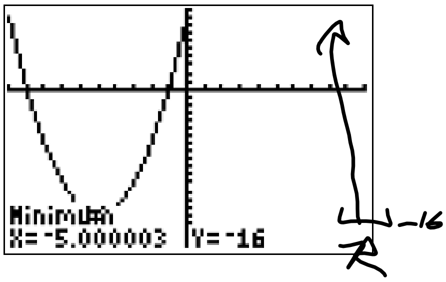
domain $= (-\infty, \infty)$

range $= [-16, \infty)$

These can and will be hand-sketched in Writing Project #2.







16

A function f is given.

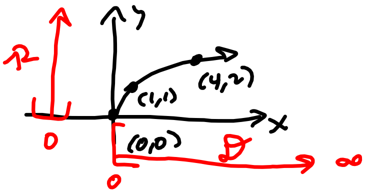
$f(x) = \sqrt{x - 6}$

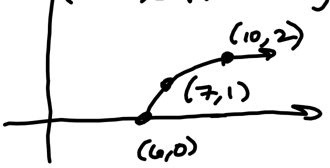
Use a graphing device to draw the graph of f. Find the domain and range of f from the graph. (Enter your answers using interval notation.)

$g(x) = \sqrt{x}$ BASIC FUNC.

$x \mapsto x + 6$

$f(x) = \sqrt{x-6} = g(x-6)$





$x - 6 \geq 0$

$x \geq 6$

$\boxed{\mathcal{D} = [6, \infty) \quad \mathcal{R} = [0, \infty)}$ as before.

$\mathcal{D}$: Need $x \geq 0$

i.e. $\mathcal{D} = [0, \infty)$

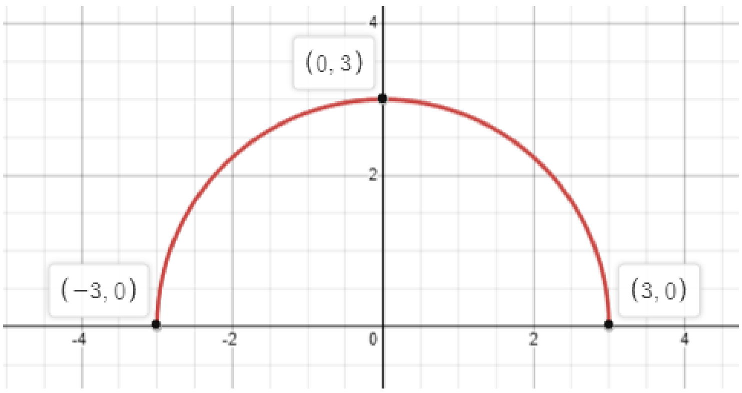
$\mathcal{R} = [0, \infty)$

17

A function f is given.

$f(x) = -\sqrt{9 - x^2}$

(a) Use a graphing calculator to draw the graph of f.



$x^2 + y^2 = 9$. circle, $r = 3$, $(h,k) = (0,0)$

$y^2 = 9 - x^2$

$y = \pm\sqrt{9 - x^2}$ ⟨ $+\sqrt{9-x^2}$ Top half; $-\sqrt{9-x^2}$ Bottom half

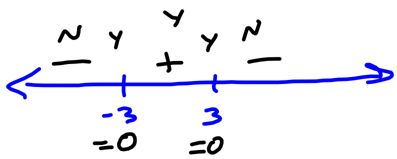
$\mathcal{D} = [-3, 3]$

$\mathcal{R} = [0, 3]$

Hand Analysis:

Need $9 - x^2 \geq 0$

$(3-x)(3+x) \geq 0$



≥ 0

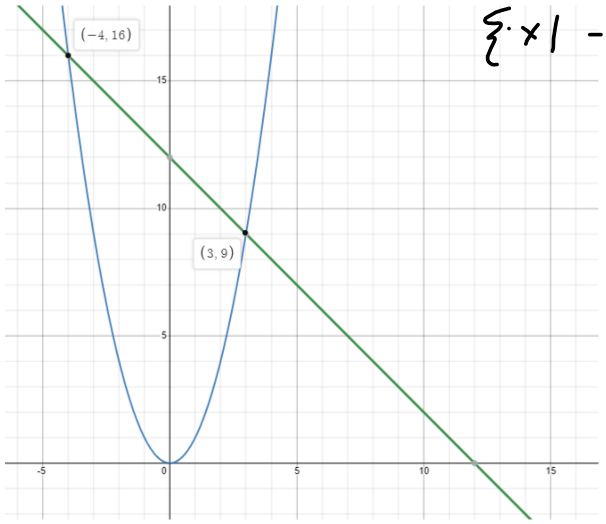
$= [-3, 3]$

Solve the given equation or inequality graphically.

18 (a) $x^2 = 12 - x$ (Enter your answers as a comma-separated list.)

$x =$ $\boxed{-4, 3}$

(b) $x^2 \le 12 - x$ (Enter your answer using interval notation.)



$\{x \mid -4 \le x \le 3\} = [-4, +3]$

asks where is x^2 below or equal to $12-x$?

$x^2 \le 12 - x$

$x^2 + x - 12$

$= (x+4)(x-3) = 0$

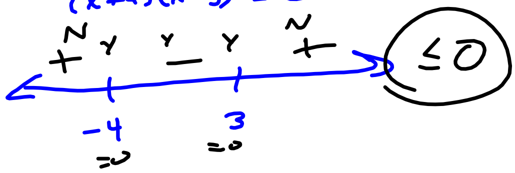
$x \in \{-4, 3\}$

Now $x^2 \le 12 - x$

$x^2 + x - 12 \le 0$

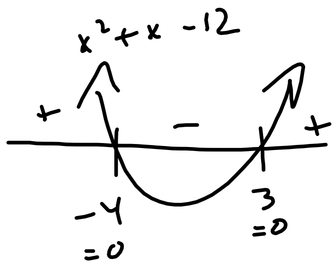
where's x^2+x-12 below the x-axis?

$(x+4)(x-3) \le 0$



$x \in$

$= [-4, 3]$



A graphing device is recommended.

Solve the given equation or inequality graphically. State your answers rounded to two decimals.

19 (a) $8x^2 - x^3 = -x^2 + 4x + 5$ (Enter your answers as a comma-separated list.)

$x =$

$-x^3 + 8x^2 = -x^2 + 4x + 5$

$-x^3 + 9x^2 - 4x - 5 = 0$ so much better.

$f(x) = g(x) \Rightarrow$

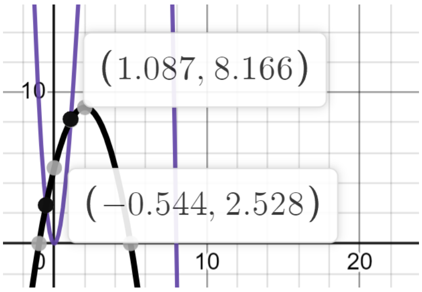
$f(x) - g(x) = 0$

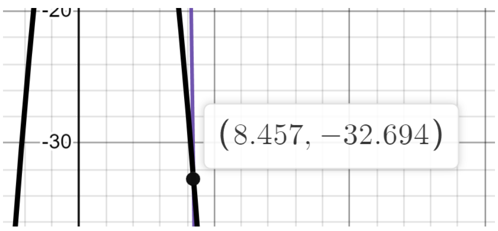
(b) $8x^2 - x^3 \le -x^2 + 4x + 5$ (Enter your answer using interval notation.)

$x =$

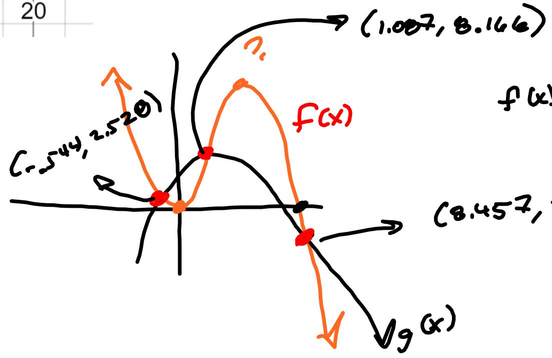
$f(x) \le g(x) \Rightarrow$

$f(x) - g(x) \le 0$





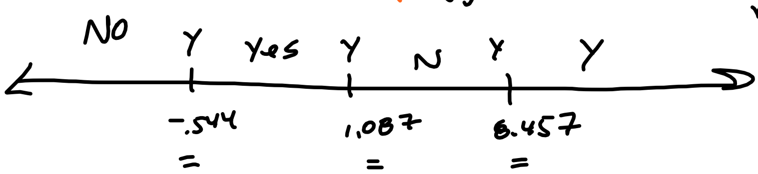
$8x^2 - x^3 = x^2(8 - x)$



$f(x) = g(x)$

red ≤ black

$f \le g$



$x \in [-.544, 1.087] \cup [8.457, \infty)$

A graphing device is recommended.

Solve the given equation or inequality graphically. State your answers rounded to two decimals.

20

(a) $25x^3 + 25x^2 = x + 1$ (Enter your answers as a comma-separated list.)

we graph $25x^3 + 25x^2 - x - 1$ & find x-ints.

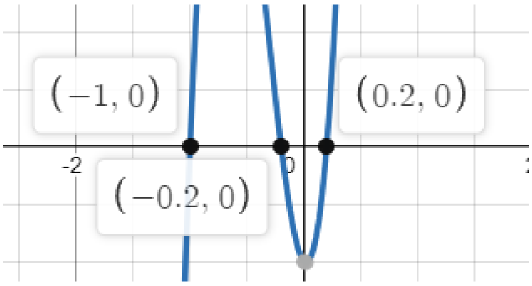
$$25x^3 + 25x^2 = x + 1$$
$$-x-1 \quad = -x-1$$
$$25x^3 + 25x^2 - x - 1 = 0$$

(b) $25x^3 + 25x^2 \ge x + 1$ (Enter your answer using interval notation.)

we graph $25x^3 + 25x^2 - x - 1$ & find where it's ≥ 0

$f(x) = g(x) \implies f(x) - g(x) = 0$

$f(x) \ge g(x) \implies f(x) - g(x) \ge 0$

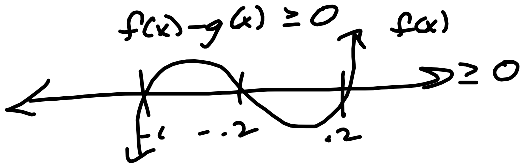


$f(x) = g(x) \implies$

$x \in \{-1, -0.2, 0.2\}$

$f(x) \ge g(x)$

$f(x) - g(x) \ge 0$



N Y Y Y N Y Y

− + − +

−1 −.2 .2

=0 =0 =0

$x \in [-1, -.2] \cup [.2, \infty)$

Not $x = [-1, -.2] \cup [.2, \infty)$

x is NOT equal to the set it's in!

$x \in$, not $x =$!

Not when you're presenting a collection (set) containing x.

A graphing device is recommended.

21 Solve the given equation or inequality graphically. State your answers rounded to two decimals.

(a) $1 + \sqrt{x} = \sqrt{x^2 + 1}$ (Enter your answers as a comma-separated list.)

we solve

$1+\sqrt{x} - \sqrt{x^2+1} = 0$

x $=$ $\approx$

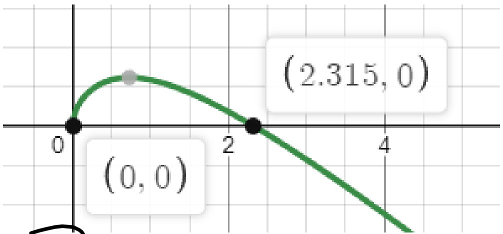
$\mathcal{D}$: $\sqrt{x}$ Need $x \geq 0$

$\sqrt{x^2+1}$ Need $x^2+1 \geq 0$, which it always is!

$\mathcal{D} = [0, \infty)$

(b) $1 + \sqrt{x} > \sqrt{x^2 + 1}$ (Enter your answer using interval notation.)

x $>$ No! $x \in$



(a) $x \approx 0, 2.32$
$x \in \{0, 2.32\}$ either is ok.

(b) + −

0.00 $= 0$ 2.32 $= 0$

want > 0

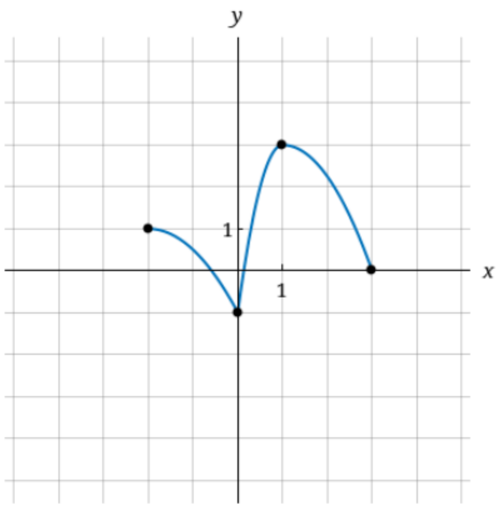
we want $(0, 2.32)$

Desmos really led me astray, because it rounds to 3 decimal places. Taking the rounded figure and rounding it to 2 places, results in being wrong in the second digit. Wolfram Alpha can clobber this to however many digits, and that would've saved me.

Click here for Wolfram Alpha!

The graph of a function *f* is given. Use the graph to estimate the following. (Enter your answers using interval notation.)

22



$\mathcal{D} = [-2, 3]$
$\mathcal{R} = [-1, 3]$

Increasing $\forall x \in (0, 1)$
Decreasing $\forall x \in (-2, 0) \cup (1, 3)$

(a) The domain and range of *f*.

(b) The intervals on which *f* is increasing and on which *f* is decreasing.

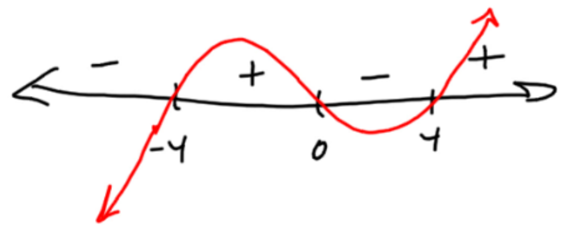
23 See Video

A function f is given.

25 $f(x) = x^3 - 16x = x(x^2-16) = x^1(x-4)^1(x+4)^1 \overset{\text{set}}{=} 0 \Rightarrow x \in \{-4, 0,$

(a) Use a graphing device to draw the graph of f. Find the domain and range of f. (Enter your answers usi
notation.)

(b) State approximately the intervals on which f is increasing and on which f is decreasing. (Enter your an
interval notation. Round your answers to two decimal places.)



$\mathcal{D} = \mathbb{R}$ (Poly)

$\mathcal{R} = (-\infty, \infty)$

Does require a grapher.

Desmos is dangerous about round-off

Show how to check w/ wolfram?

Graphing CALCULATOR will do more digits

Need the max and min values (or rather, the x-values giving the max and min y-values. That's the key to finding intervals of increase and decrease, because these intervals have boundaries at the x-values corresponding to the local maximum and minimum y-values.

WolframAlpha computational intelligence.

Enter this

Local Minimum of x^3-16x

NATURAL LANGUAGE MATH INPUT EXTENDED KEYBOARD EXAMPLES UPLOAD

Input interpretation

local minima $x^3 - 16x$

Result Approximate form Step-by-step solution

click here

$\min\{x^3 - 16x\} = -\frac{128}{3\sqrt{3}}$ at $x = \frac{4}{\sqrt{3}}$

local minima $x^3 - 16x$

Result More digits Exact form Step-by-step solution

click here

$\min\{x^3 - 16x\} \approx -24.634$ at $x \approx 2.3094$

local minima $x^3 - 16x$

Result Fewer digits More digits Exact form Step-by-step solution

$\min\{x^3 - 16x\} \approx -24.6336114854240$ at $x \approx 2.30940107675850$

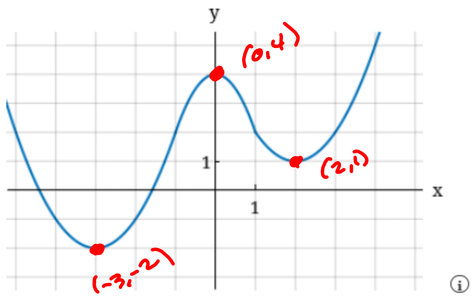
plenty of digits!

using Wolfram Alpha

click here!

The graph of a function f is given. Use the graph to estimate the following.

27



(a) All the local maximum and minimum values of the function and the value of x at which each occurs.

local maximum $(x, y) =$ (0,4)

local minimum (smaller x-value) $(x, y) =$ (-3,-2)

local minimum (larger x-value) $(x, y) =$ (2,1)

(b) The intervals on which the function is increasing and on which the function is decreasing. (Enter your answers using interval notation.)

increasing $(-3,0) \cup (2,\infty)$

decreasing $(-\infty,-3) \cup (0,2)$

WebAssign doesn't appear too picky about whether you use [ ] or ( ). But I AM.

A graphing device is recommended.

A function is given.

28

$g(x) = x^4 - 3x^3 - 19x^2 = x^2(x^2 - 3x - 19)$

$\frac{-9-76}{4} = -\frac{85}{4}$

(a) Find all the local maximum and minimum values of the function and the value of x at which each occurs. State each answer rounded to two decimal places.

$x^2 - 3x - 19 = x^2 - 3x + \left(\frac{3}{2}\right)^2 - \frac{9}{4} - \frac{19}{1}\cdot\frac{4}{4}$

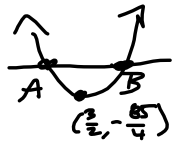
local maximum $(x, y) =$ (0,0)

$= \left(x - \frac{3}{2}\right)^2 - \frac{85}{4} \overset{\text{SET}}{=} 0$

See below

local minimum (smaller x-value) $(x, y) =$

$\Rightarrow x - \frac{3}{2} = \pm\frac{\sqrt{85}}{2}$



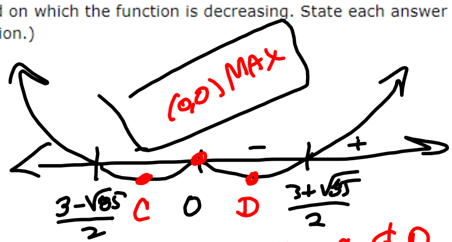
local minimum (larger x-value) $(x, y) =$

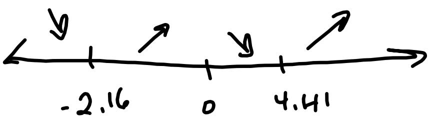
$x = \frac{3 \pm \sqrt{85}}{2}$

(b) Find the intervals on which the function is increasing and on which the function is decreasing. State each answer rounded to two decimal places. (Enter your answers using interval notation.)

increasing on $(-2.16, 0) \cup (4.41, \infty)$

decreasing on $(-\infty, -2.16) \cup (0, 4.41)$





We need C & D

we already have (0,0) as local max.

$\boxed{\begin{array}{l} C \approx (-2.16, -36.65) \text{ LOCAL MIN} \\ D \approx (4.41, -248.58) \text{ LOCAL MIN} \end{array}}$

A graphing device is recommended.

A function is given.

29 $U(x) = x\sqrt{2 - x}$

$\mathcal{D}$: Need $2 - x \geq 0$

$-x \geq -2$

$x \leq 2$

$\mathcal{D} = (-\infty, 2]$

(a) Find the local maximum value of the function and the value of x at which this occurs. State the answer rounded to two decimal places.

$(x, y) = \approx (1.33, 1.09$

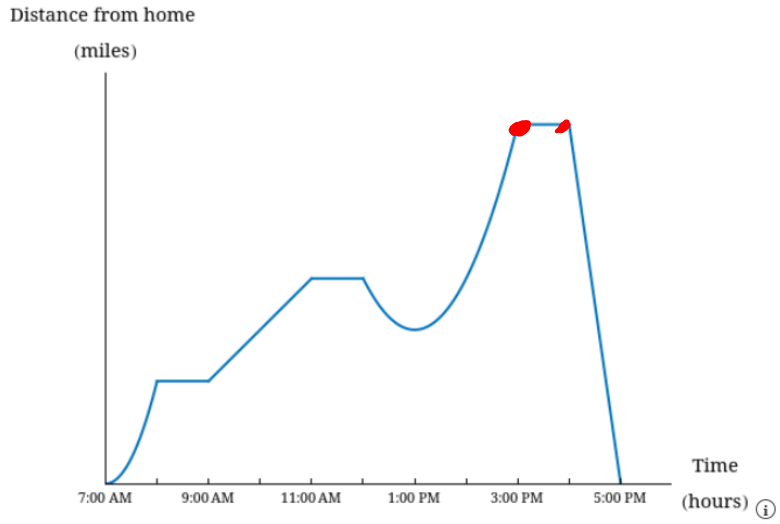
(b) Find the intervals on which the function is increasing and on which the function is decreasing. State each answer rounded to two decimal places. (Enter your answers using interval notation.)

increasing

decreasing

The graph gives a sales representative's distance from home as a function of time on a certain day.

30



(a) Determine the time intervals on which the distance from home was increasing. (Select all that apply.)

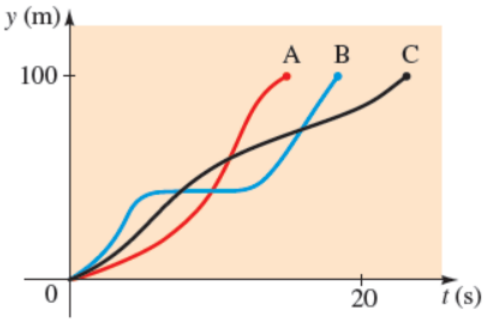
Distance Increasing 7am-8am, 9am-11am, 1pm-3pm

Distance Decreasing 12pm-1pm, 4pm-5pm

Net change between 3 pm and 4 pm None!

Three runners compete in a 100-meter hurdle race. The following graph depicts the distance run as a function of time for each runner. Describe in words what the graph tells you about this race.

32



Who won the race?

- ○ Runner A
- ○ Runner B
- ○ Runner C

Did each runner finish the race?

- ○ Yes
- ○ No

What do you think happened to Runner B?

- ○ Runner B stumbled at the start of the race and began after the other two runners.
- ○ Runner B stopped running and walked the rest of the race.
- ○ Runner B increased speed the first part of the race, ran at a constant speed during the middle of the race, then increased speed the last part of the race.
- ○ Runner B fell, but got up and finished the race.
- ○ Runner B ran the first part of the race, walked during the middle, then ran the last part of the race.