Print Your Name_____

MAT 1340 Midterm
200 points Covers Chapters 1 - 3

Intro

Here's well over 200 points'-worth of worthy-ish questions. Even so, it's not 100% comprehensive, but these are type questions from which I will draw for the midterm. I'm shooting for 200 points

There're a few questions that were inspired by Weekly Written Assignments, but a few questions that I liked while spending far too much time building this aren't to be found in their exact format on the Weeklies.

Begin the test, proper:

Do your own work. Show all your work. Circle final answers. Organize your work for efficient processing, left to right, top to bottom.

Required Materials: Bring pencil/pen, and scientific calculator.

Recommended Materials: A cheat sheet (1 page, front and back permitted). A ruler or straight-edge for drawing lines.

Paper will be provided. Do not take anything from the test with you. Turn in your work, cheat sheet, and any scratch pages to the proctor. You should not make a separate scratch sheet. EVERYTHING you do related to a given question should be included with the work on that question, including your scratch work.

This isn't about fitting perfect answers in tiny boxes. This is about supporting your work in a readable and cogent fashion.

Do all your work on the blank paper provided. All I'm looking for on test-questions pages is your NAME. Other than your name, there are no points given or deducted for anything you write on the test-questions pages. Every bit of work should be on the blank pages provided, and will be thoroughly processed for every possible point of credit I can award.

The only difference between this test and your Written Assignments is that you don't have to write out the question on a test. No annoying "Context" deductions.

1. We sketch the feasible region of a system of linear inequalities in stages.

    a. (5 pts) Sketch the graphs of $y = 10$ and $x = 10$ on the same set of coordinate axes.
    b. (5 pts) Sketch the graph of $5x + 2y \geq 10$ on its own set of coordinate axes. Label the x- and y-intercepts.
    c. (5 pts) Find the intersection of the lines $y = 10$ and $5x + 2y = 10$.
    d. (5 pts) Find the intersection of the lines $x = 10$ and $5x + 2y = 10$.
    e. (5 pts) Sketch the feasible region for the following system of inequalities. Go Big!

    $$\begin{aligned} y &< 10 \\ x &< 10 \\ 5x + 2y &\geq 10 \end{aligned}$$

    Label all corner points.

2. (5 pts) Use the converse of the Pythagorean Theorem to show that the triangle with vertices $A=(6,1), B=(-1,2),$ and $C=(2,5)$.

3. Solve the absolute-value inequalities.

   a. (5 pts) $|4x-9| \le 7$

   b. (5 pts) $|4x-9| > 7$

   c. (5 pts) $|4x-9| \le 0$

   d. $|4x-9| > -7$

4. The equation $x^2+y^2+2x-10y=-1$ represents a circle.

   a. (5 pts) Write the equation of the circle in standard form. What is the center? What is the radius?

   b. (5 pts) Sketch the graph of the circle. Label the North, South, East and West Poles.

   c. (**Bonus 5 pts**) Find the *x*- and *y*-intercepts of the circle.

5. Let $f(x)=\sqrt{x-18}$, $g(x)=\dfrac{1}{x}$, and $h(x)=x^2-3x$

   a. (5 pts) Find the domain of *f*, *g*, and *h*.

   b. (5 pts) Find $f+g$ and state its domain.

   c. (5 pts) Find $f \circ g$ and state its domain.

   d. (5 pts) Find $f \circ h$ and state its domain.

6. Let A = (2, -3) and B = (7, 8).

   a. (5 pts) Find an equation $y=m(x-x_1)+y_1$ of the line passing through A and B.

   b. (5 pts) Find an equation of the line $y=m(x-x_1)+y_1$ passing through (11, 7) that is parallel to the line from part a.

   c. (5 pts) Find an equation of the line $y=m(x-x_1)+y_1$ passing through (11, 7) that is perpendicular to the line from part a.

7. Consider the quadratic function $f(x)=3x^2-5x-1$.

a. (5 pts) Solve the equation $f(x)=0$ using the Quadratic Formula. For full credit, evaluate the discriminant and its square root, first. I'm looking for exact answers, not decimal approximations.

b. (5 pts) Solve the equation $f(x)=0$ by completing the square. Find the exact solution. Do not use a calculator.

c. (5 pts) Write $f(x)$ in standard form $f(x)=a(x-h)^2+k$ and sketch its graph. Include the x- and y-interepts, as well as the vertex in your graph.

d. (5 pts) What is the domain of f? What is the range of f?

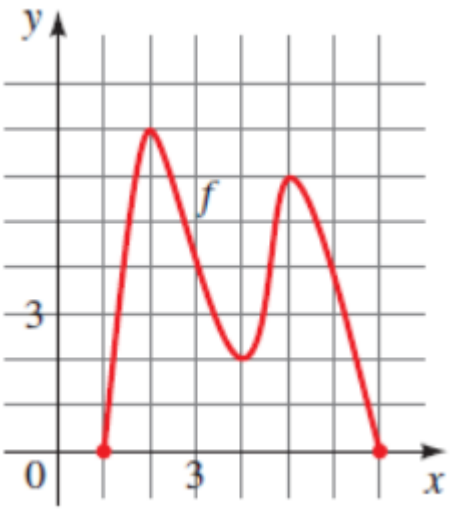
8. (**Bonus 5 pts**) Factor $140x^2-111x-198$.

9. (5 pts) Solve the equation $\dfrac{x-9}{x-4}=\dfrac{-12}{x-5}$ for x.

10. (5 pts) Solve the inequality $\dfrac{x-9}{x-4}\geq\dfrac{-12}{x-5}$ for x.

11. Refer to the figure on the right for a graph of a function $f(x)$.

It's tempting to write the answers on this sheet. You certainly may, but if you want points for it, use the blank paper provided. Just write
10a – your answer to a
bi – your answer to bi
ii – your answer to bii
c – your answer to c.
d – your answer to d.
You don't have to write anything else or reproduce the figure on your blank sheet. This is one place where just the answers will suffice, unlike on the Weeklies, where I want the work to stand alone as a report.



a. (5 pts) State the domain D and Range R of f.

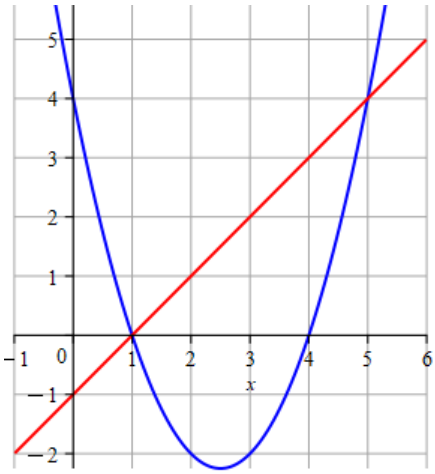
b. (5 pts) Complete the statemen:
   i. f is increasing on...
   ii. f is decreasing on...

c. (5 pts) State the local maximum and minimum values. Report these values as ordered pairs, i.e., as points in the plane, like $(x, y) = (4, f(4)) = (4, 2)$, where $x = 2$ is where we obtain a y-value or $f(x)$-value of $4 = f(2)$. I'm pretty sure I said this wrong on Week 5 Written Assignment. Yes. I

did. Editing... Fixed.

d. (5 pt) State the solutions of the equation $f(x)=0$. Give your solutions as a set.

12. The graphs of $f(x)=x^2-5x+4$ and $g(x)=x-1$ are shown on the same set of coordinate axes on the right.



a. (5 pts) Solve the equation $f(x)=g(x)$

b. (5 pts) Complete the statement:

i. $f(2)=\ldots$

ii. $g(3)=\ldots$

c. (5 pts) $(f\circ g)(5)=$

d. (5 pts) The set of all x such that $f(x)=-2$ is...

13. (10 pts) Sketch the graph of $g(x)=-5\sqrt{-4x-20}+10$ by transforming the graph of $f(x)=\sqrt{x}$ with 5 graphs, counting the graph of $f(x)$ as the 1st, before you make any moves (transformations). Track the original (0, 0) and (1, 1) through all the transformations, like in the Weeklies.

14. Let $f(x)=x^2-2x-3$

a. (5 pts) If (h, k) is the vertex of f, restrict the domain to $x\geq h$, to keep it 1-to-1, and then find the inverse $f^{-1}(x)$.

b. (5 pts) Sketch the graph of the restricted $f(x)$ and the graph of $f^{-1}(x)$ on the same set of coordinate axes. You may use $y=x$ as a guide.

c. (5 pts) State the domain and range of the restricted f and the inverse f^{-1}.

15. Let $f(x)=x^4-5x^3+7x^2-x-2$.

a. (5 pts) List all possible rational zeros of f.

b. (5 pts) Use synthetic division and the rational zeros of f to split f into the product of 2 linear factors and one irreducible quadratic factor.

c. (5 pts) Find the nonreal zeros of f, using your work from part b.

d. (5 pts) Split f into the product of 4 linear factors.

16. Sketch the graph of the following. Show all intercepts and asymptotes.

a. (10 pts) $R(x)=\dfrac{x-2}{x+4}$.

b. (10 pts) $S(x)=\dfrac{(x-3)(x+7)}{(x+3)(x-8)}=\dfrac{x^2+4x-21}{x^2-5x-24}$

c. (10 pts) $T(x)=\dfrac{x^2+4x-21}{x-5}$

(20 pts) $U(x)=\dfrac{(x-3)(x+7)(x-6)}{(x+3)(x-8)}=\dfrac{x^3-2x^2-45x+126}{x^2-5x-24}$

17. (10 pts) What is the domain of $V(x)=\sqrt{\dfrac{(x-3)(x+7)^2(x-6)}{(x+3)(x-8)}}$?

18. (10 pts) Sketch the graph of $g(x)=\dfrac{-5}{(4x-20)^2}+10$ by transforming the graph of $f(x)=\dfrac{1}{x^2}$. Track where the original (- 1 , 1) and (1, 1) from the graph of f move, at each step.

19. Let $f(x)=4x^6-40x^5+331x^4-920x^3+21x^2+1490x+174$.

a. (5 pts) Use a simple graphic to describe the end behavior of *f*. If you don't know what I mean, they you may benefit from the See Video for #1a.

a. (5 pts) Suppose I told you that $f(x)=4x^6-36x^5+146x^4-340x^3+76x^2+776x+174$ has a complex zero $3-7i$. Use this information and long division of polynomials to factor f into the product of a quadratic polynomial and a quartic polynomial. See Video for #2b and #2c

b. (5 pts) Based on your work in part b, you have a quadratic polynomial whose zeros are $3\pm 7i$. You also have a quartic (4^{th}-degree) polynomial, whose zeros are yet to be determined. This is called the "depressed polynomial."

If all has gone well, the depressed polynomial is the polynomial from #1! So f has the same real zeros as g from #1! Sketch its graph, using only the information from its intercepts. It's identical to #1's graph, with one exception. What's the only difference?

20. Let $R(x)=\dfrac{3x^2+6x-24}{4x^2+27x+18}$. See Video for #3.

c. (5 pts) What is the domain of *R*?

d. (5 pts) Find the zeros of *R*. Also find the *y*-intercept of *R*. These will be labeled points on the graph.

e. (5 pts) Find any horizontal asymptotes of *R*.

f. (5 pts) Re-write *R* with its numerator and denominator factored (See parts a and b.). Then provide a sign pattern for *R*. Take care to distinguish between zeros of *R* and vertical asymptotes of *R*, both of which control any sign changes of *R*. Use the parity (sign) of the horizontal

asymptote and the *y*-intercept to kick-start your sign pattern.

g. (5 pts) Render the graph of *R*, showing all intercepts and asymptotes. This is what "Graph *R*" means.

21. Let $\hat{R}(x)=\frac{3x^3-18x^2-72x+192}{4x^3-5x^2-198x-144}$. $\hat{R}$ has the same graph as *R*, with one exception: $\hat{R}$ has a hole. See Video for #4.

h. (5 pts) Where is the hole? Give your answer as an ordered pair (x,y).

i. (5 pts) Go back to your graph of R in #3. Add the hole you found in part a, above to its graph. That will suffice in earning credit for the graphs of both *R* and $\hat{R}$. If you wish, you may do a separate graph for $\hat{R}$, showing all intercepts, asymptotes, and the hole.

22. Let $T(x)=\frac{3x^3-18x^2-72x+192}{4x^2+27x+18}$. T has a pair of vertical asymptotes and a slant (oblique) asymptote. See Video for #5.

j. (5 pts) Use long division to determine the slant asymptote. Call it *s*(*x*).

k. (5 pts) Sketch the graph of *T*, showing all intercepts and asymptotes. Most of the work has already been done, as T has the same denominator as R, and the same numerator as $\hat{R}$.