1340 Week 14 Mills

1 we decompose by partial fractions

2 5pts $\frac{5x+1}{x^2-1} = \frac{5x+1}{(x-1)(x+1)} = \boxed{\frac{A}{x-1} + \frac{B}{x+1}}$

$\Rightarrow 5x+1 = A(x+1) + B(x-1)$

$x=-1$:

$5(-1)+1 = -4 = B(-1-1) = -2B$

$\Rightarrow B = \frac{-4}{-2} = \boxed{2 = B}$

$x=1$:

$5(1)+1 = 6 = A(1+1) = 2A$

$\Rightarrow A = \frac{6}{2} = \boxed{3 = A}$

We see this in Calc II

$\int \frac{5x+1}{x^2-1}\,dx = \int \left(\frac{3}{x-1} + \frac{2}{x+1}\right) dx = 3\ln|x-1| + 2\ln|x+1| + C,$

not that the above means a whole lot to you, yet.

b 5pts $\frac{x-8}{x^3+8x} = \frac{x-8}{x(x^2+8)} = \frac{A}{x} + \frac{Bx+C}{x^2+8}$

$\Rightarrow x-8 = A(x^2+8) + B(x)$

$x=0 \Rightarrow -8 = 8A \Rightarrow \boxed{A=-1}$

$x=1$: (RANDOM)

$1-8 = -7 = A(1^2+8) + B(1) = 9A + B = 9(-1) + B = B-9$

$\Rightarrow B-9 = -7$

$\Rightarrow \boxed{B=2}$

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(2) (10 pts) Feasible Region for

$2x - 7y \le 14$

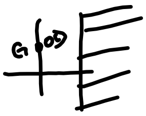
$3x + 2y > 6$

$x \le 7$

$y \le 3$

Shading:

$x \le 7$
$0 \le 7$?
$(0,0)$ Good



$x = 7$

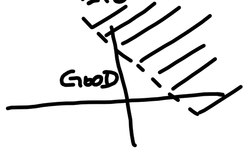
$2x - 7y \le 14$
$0 \le 14$? Yes
$(0,0)$ Good

x	y
0	-2
7	0

$3x + 2y > 6$
$0 > 6$? No
$(0,0)$ BAD

x	y
0	3
2	0

$A = (0,3)$



Corners:
$A = (0,3)$
$C = \left(\frac{14}{5}, -\frac{6}{5}\right)$

$y \le 3$
$0 \le 3$?
$(0,0)$ good

Find E:

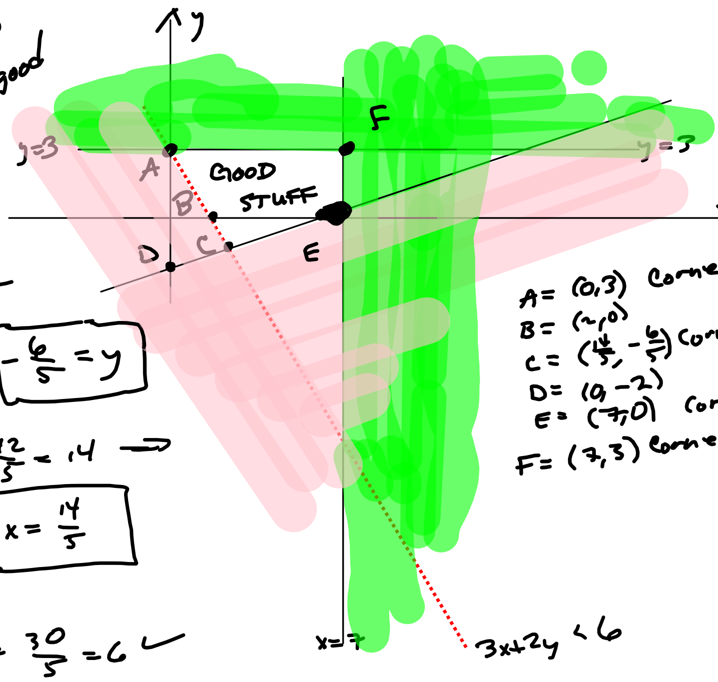
$3(2x - 7y = 14)$
$-2(3x + 2y = 6)$

$$6x - 21y = 42$$
$$-\;6x - 4y = -12$$
$$-25y = 30$$
$$y = \frac{-30}{25} = \boxed{-\frac{6}{5} = y}$$

$$2x - 7y = 2x - 7\left(-\frac{6}{5}\right) = 2x + \frac{42}{5} = 14 \rightarrow$$

$$2x = \frac{-42}{5} + \frac{70}{5} = \frac{28}{5} \rightarrow \boxed{x = \frac{14}{5}}$$

Check $3\left(\frac{14}{5}\right) + 2\left(-\frac{6}{5}\right) = \frac{42 - 12}{5} = \frac{30}{5} = 6$ ✓

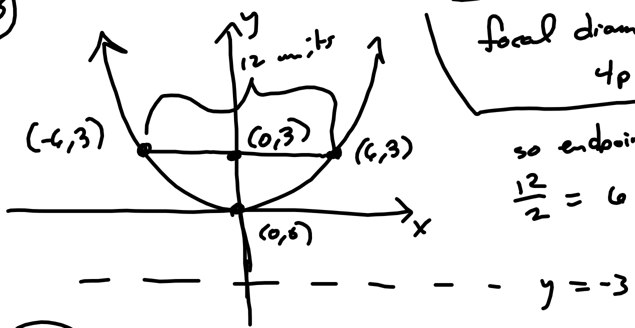


$A = (0,3)$ Corner
$B = (2,0)$
$C = \left(\frac{14}{5}, -\frac{6}{5}\right)$ Corner
$D = (0,-2)$
$E = (7,0)$ Corner
$F = (7,3)$ Corner

1340 Week 14 Written Mills

3. We find the focal length p, the equation of the directrix, and the focal diameter $4p$, and we label the focus, vertex, directrix, and the latus rectum, including its endpoints.

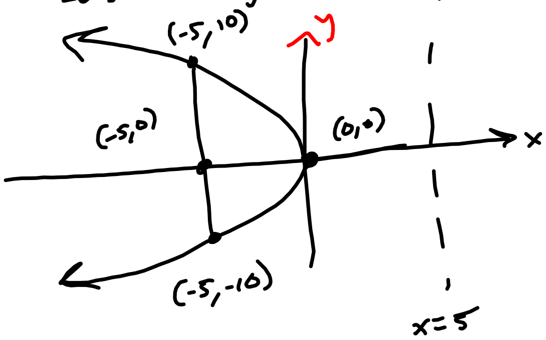
a) 5pts $y = \frac{1}{12}x^2 \Rightarrow x^2 = 12y = 4py \Rightarrow p = 3$



$\boxed{\text{focal diameter: } 4p = 12}$

so endpoints of latus rectum are $\frac{12}{2} = 6$ units left & right of the focus.

b) 5pts $x = -\frac{1}{20}y^2 \Rightarrow y^2 = 20x = 4px \Rightarrow p = 5$



$4p = 20$

$\frac{20}{2} = 10$

$-\frac{1}{20}y^2$ opens left.

1340 Week 14 Written Mills

(c) (5 pts) $y = \frac{1}{8}x^2 - \frac{5}{4}x - \frac{71}{8}$

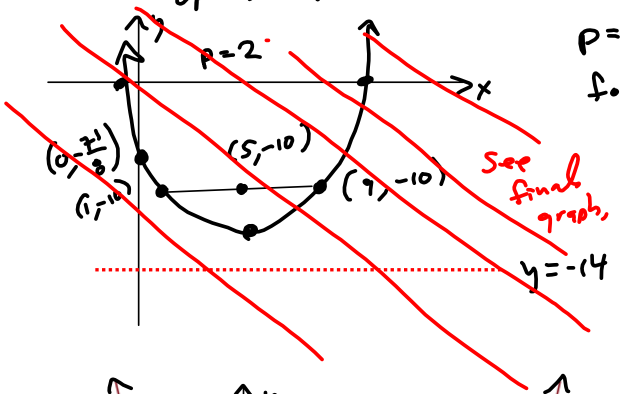
$\Rightarrow 8y = x^2 - 10x - 71$

$= x^2 - 10x + 5^2 - 25 - 71$

$= (x-5)^2 - 96 \Rightarrow y = \frac{1}{8}(x-5)^2 - \frac{96}{8}$

$\Rightarrow 8y = 4py \Rightarrow p = 2$ $\qquad = \frac{1}{8}(x-5)^2 - 12 \Rightarrow$ $\boxed{(h,k) = (5,-12)}$

opens up.

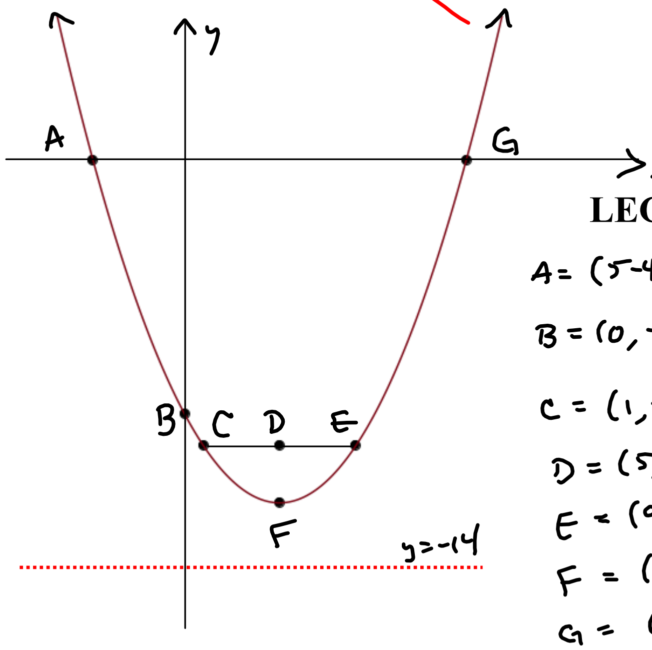


$p = 2 \Rightarrow$

focal diameter is $4p = 8$

$\frac{8}{2} = 4$

$(5,-12)$



LEGEND

$A = (5 - 4\sqrt{6}, 0)$ x-int

$B = (0, -\frac{71}{8})$ y-int.

$C = (1, -10)$ Endpt. of latus rectum.

$D = (5, -10)$ Focus

$E = (9, -10)$ Endpt. of latus rectum

$F = (5, -12)$ Vertex

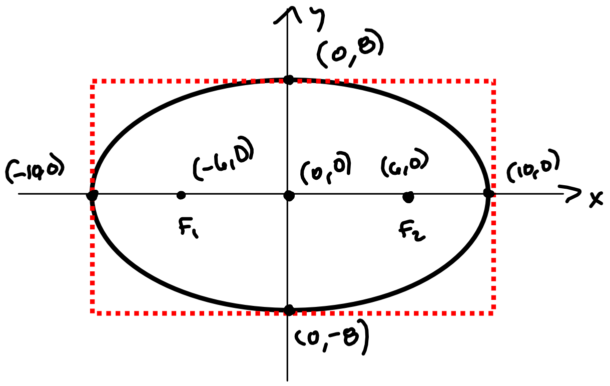
$G = (5 + 4\sqrt{6}, 0)$ x-int

1340 Week 14 Written Mills

(4) We graph the ellipse given by the equation, including the center, foci, and endpoints of the major & minor axes.

(a) (5 pts) $\frac{x^2}{100} + \frac{y^2}{64} = 1$

$a^2 = 100$
$\Rightarrow a = 10$
$b^2 = 64$
$\Rightarrow b = 8$
$a^2 - b^2 = 100 - 64 = 36 = 6^2$
$\Rightarrow c = 6 =$ focal length



(b) (5 pts) $49x^2 + 25y^2 + 196x - 150y - 804 = 0$

$49x^2 + 196x \quad + 25y^2 - 150y \quad = 804$

$49(x^2 + 4x \quad) + 25(y^2 - 6y \quad) = 804$

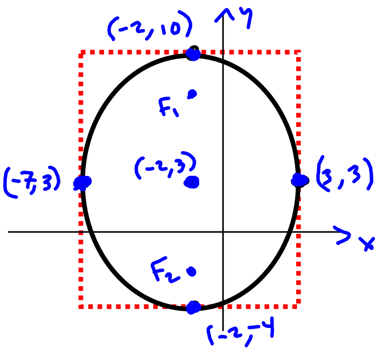
$49(x^2 + 4x + 2^2) + 25(y^2 - 6y + 3^2) = 804 + 49(4) + 25(9)$

$\frac{49(x+2)^2}{(49)(25)} + \frac{25(y-3)^2}{(49)(25)} = 804 + 196 + 225 = \frac{1225}{(49)(25)}$

$\frac{(x+2)^2}{25} + \frac{(y-3)^2}{49} = 1$

$\frac{\cancel{1225}}{\cancel{(49)(25)}}$ $\frac{1}{1}$ ✓

$(h,k) = (-2,3)$
$a = 5,\ b = 7$
$b^2 - a^2 = 49 - 25 = 24 = c^2$
$\Rightarrow c = \sqrt{24} = 2\sqrt{6} \approx 4.898979486 \approx$ FOCAL LENGTH



Foci:

$\boxed{F_1 = (-2, 3+2\sqrt{6}) \approx (-2, 7.899) \\ F_2 = (-2, 3-2\sqrt{6}) \approx (-2, -1.899)}$

1340 **Week 14 Written** **Mills**

(5) we graph the hyperbola given by the eqn, labeling center, foci, and vertices

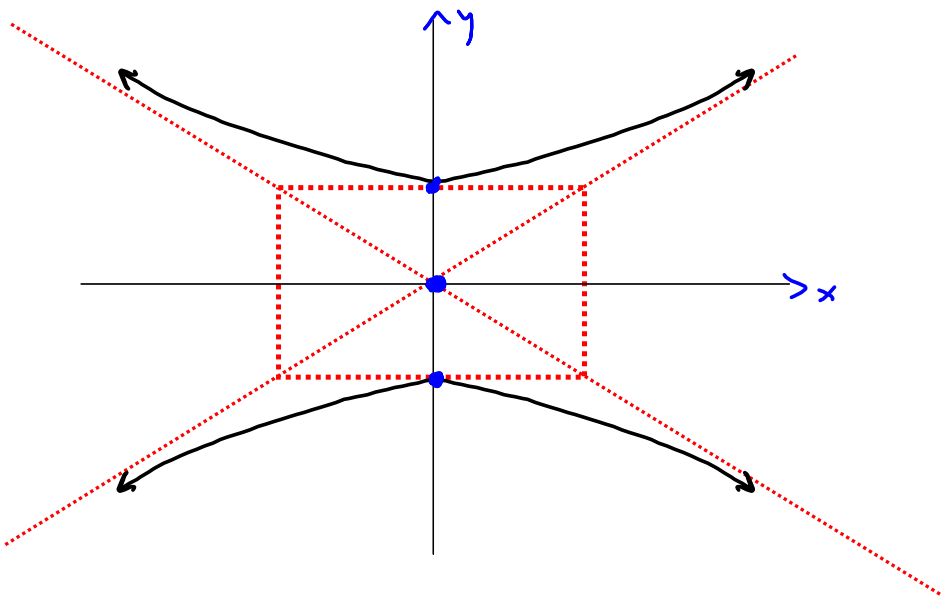
(2) 5pts $\frac{y^2}{64} - \frac{x^2}{100} = 1$

$a = \sqrt{64} = 8$, $b = \sqrt{100}$.

$c^2 = a^2 + b^2 = 64 + 100 = 164$

$c = \sqrt{164} = 2\sqrt{41}$

$2 \underline{| 164}$
$2 \underline{| 82}$
41



1340 **Week 14 Written** Mills

(b) (5pts) $25x^2-36y^2-150x-144y-819=0 \Longrightarrow$

$25x^2-150x \quad -36y^2-144y \quad =819$

$\Rightarrow 25(x^2-6x \quad)-36(y^2+4y \quad)=819$

$\Rightarrow 25(x^2-6x+3^2)-36(y^2+4y+2^2)=819+25(9)-36(4)$

$\Rightarrow 25(x-3)^2-36(y+2)^2=819+225-144=1044-144=900$

$\Rightarrow \dfrac{25(x-3)^2}{25(36)}-\dfrac{36(y+2)^2}{25(36)}=819+225-144=1044-144=\dfrac{900}{25(36)}$

$\Rightarrow \boxed{\dfrac{(x-3)^2}{36}-\dfrac{(y+2)^2}{25}=1}$

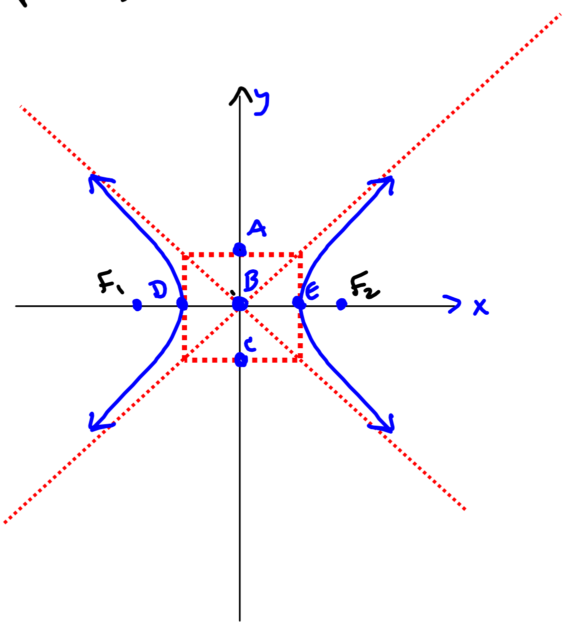
check:
$25 \times 36 = 150 + 750 = 900$

$(h,k)=(3,-2)$

$a=6, b=5$

$c=\sqrt{a^2+b^2}$ = focal length $=\sqrt{6^2+5^2}$

$=\sqrt{36+25}=\sqrt{61}\approx$ 7.810249676



$A=(3,3)$

$B=(3,-2)$ Center

$C=(3,-7)$

$D=(-3,-2)$

$E=(9,-2)$

$F_1=(3-\sqrt{61},-2)$ vertex

$F_2=(3+\sqrt{61},-2)$

1340 Week 14 Written Mills

(6) Use Pascal's Triangle or the Binomial Theorem for the following

(a) (5pts) Use Pascal's Triangle to expand $(3x-2y)^4$

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$

$(3x)^4 + 4(3x)^3(-2y) + 6(3x)^2(-2y)^2 + 4(3x)(-2y)^3 + (-2y)^4$

$= 3^4x^4 + 4(3^3)x^3(-2)y + 6(3^2x^2)((-2)^2y^2) + 4(3x)((-2)^3y^3) + ((-2)^4y^4$

$= 81x^4 + 4(-2)(27)x^3y + 6(9)(4)x^2y^2 + 12(-8)xy^3 + 16y^4$

$= 81x^4 - 216x^3y + 216x^2y^2 - 96xy^3 + 16y^4$

$$
\begin{array}{r}
{}^{5}27 \\
8 \\
\hline
216
\end{array}
$$

(b) (5pts) We find the coefficient of x^9 in the expansion of $(3x-2y)^{16}$

9th term is $\binom{16}{9}(2x)^9(-2y)^7 = 11440(3)^9x^9(-2)^7y^7$

$= -11{,}440(3)^9(2)^7x^9y^7 = -28{,}822{,}210{,}560\,x^9y^7$

Don't want to do this w/o a calculator.

$\boxed{\Longrightarrow \text{coefficient of } x^9 \text{ is } -28{,}822{,}210{,}560\,y^7}$

(Easy to miss the y^7.)

$\binom{16}{9} = \dfrac{16!}{9!(16-9)!} = \dfrac{16!}{9!7!} = \dfrac{16\cdot 15\cdot 14\cdot 13\cdot 12\cdot 11\cdot 10}{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2} = 80(26)(11)(5)$

Can be done by hand.

$= (80)(26)(55) = 208(55) = 11440$

$$
\begin{array}{r}
208 \\
55 \\
\hline
1040 \\
10400 \\
\hline
11440
\end{array}
$$

1340 **Week 14 Written** **Mills**

360360

(7) (5pts) Choose 5 from 15 and arrange.

$$P(10,5) = \frac{15!}{5!\,10!} = \frac{\overset{3}{\cancel{15}}\cdot\overset{7}{\cancel{14}}\cdot 13\cdot 12\cdot 11}{\cancel{5}\cdot 4\cdot 3\cdot\cancel{2}} = 21\cdot 13\cdot 11 = \boxed{360{,}360 = P(15,5)}$$

(8) (5pts) Choose 5 from 10:

$$C(10,5) = \binom{10}{5} = \frac{10!}{5!\,5!} = \frac{\overset{2}{\cancel{10}}\cdot 9\cdot\overset{2}{\cancel{8}}\cdot 7\cdot\cancel{6}}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}} = \boxed{252 = \binom{10}{5}}$$