

5.1 – Systems of Linear Equations in Two Variables

5.2 – Systems of Linear Equations in Many (Three) Variables

This is drawn from an old Writing Project #4, for which there are [Notes and Videos](#).

1 Solve the system of linear equations  $\begin{matrix} 2x - 5y = 10 \\ 7x + 2y = 28 \end{matrix}$  in 3 ways:

(a) (10 pts) Find the general vicinity of the solution by graphing the system. This should give you a general idea. Don't worry about it being super-accurate, although the more care you take, the better the estimate will be. Just graph the two lines by the intercept method. Supply the exact answer after you work parts b and c, below. I care much more about OPLs than tickmarks. OPLs are required. Tickmarks are not. Do not use graph paper!

(b) (10 pts) Use the Substitution Method

(c) (10 pts) Use the Elimination Method.

2. (10 pts) Use Elimination to solve the independent system of linear equations:  $\begin{matrix} x - 2y + z = 18 \\ 2x - 3y + 3z = 38 \\ 2x - 6y + z = 37 \end{matrix}$

3. Consider the dependent system of linear equations:  $\begin{matrix} 3x + 12y - 8z = 3 \\ 2x + 7y - 5z = 4 \\ -x - 5y + 3z = 1 \end{matrix}$

(a) (10 pts) Give the general solution. Be kind to your teacher and let  $z$  be free! That means, find an expression for  $x$  and  $y$  in terms of the variable  $z$ .

(b) (10 pts) Give the particular solutions corresponding to  $z = 0$ ,  $z = 1$  and  $z = -1$ .

4. **The Underlying Assumption:** All of the techniques we learn for solving systems of linear equations are based on the *assumption* that the systems *have* solutions. So when we arrive at a false (*absurd!*) statement after a few elimination steps, the only explanation is that there was no solution in the first place\*. Our incorrect assumption\* led to something absurd, like  $0 = 10$  or  $0 = -5$ .

\*... or you made a mechanical error and should check your work, just to make sure. Stay organized and always check your work.

**Higher Learning:** In higher mathematics, this is the most basic method of proving something is false: "Assume it's true and conclude something absurd (like ' $0 = 1$ ')." It's important that you realize what's happening when you arrive at those absurdities at the end of a perfectly logical and legal sequence of moves. That said, let me *finally* get to the question:

$$3x + 12y - 8z = 3$$

(10 pts) **Your Task:** Show that the dependent system of linear equations  $2x + 7y - 5z = 4$

$$-x - 5y + 3z = 3$$

has no solution. I expect to see the word "absurd" in your discussion.