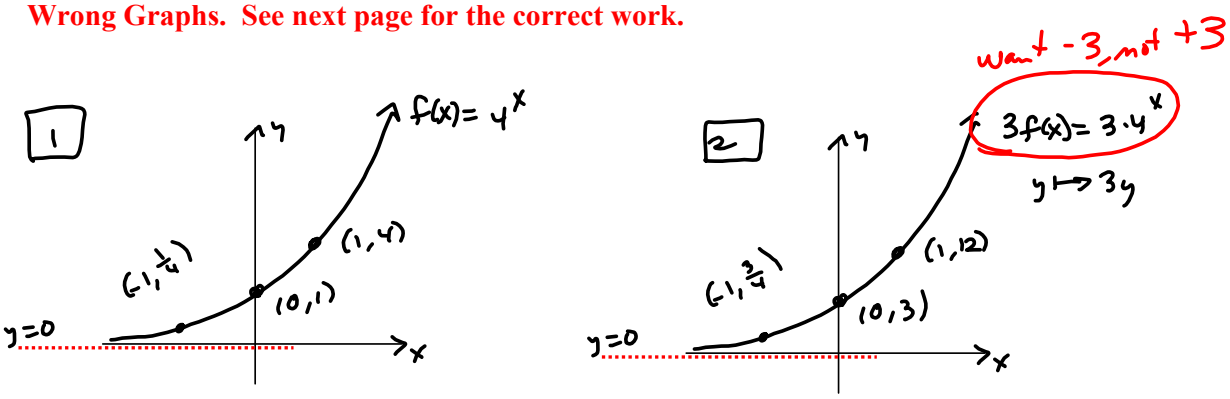
1340 Week 10 Written Mills

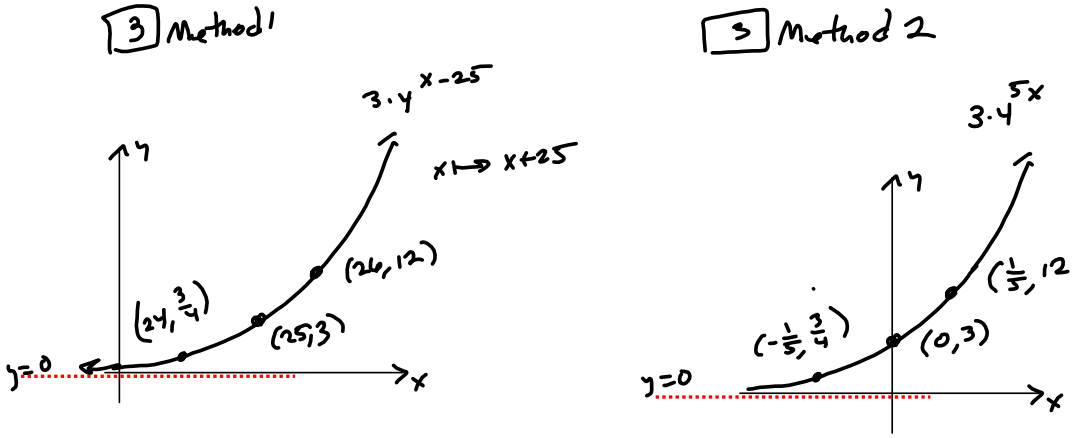
(1) we graph $g(x) = -3 \cdot 4^{5x-25}$. we track $(-1, \frac{1}{4})$, $(0,1)$, $(1,4)$

sigh I missed the "- 3" and just saw "+ 3." Nice graphs, Steve.

Wrong Graphs. See next page for the correct work.

want -3, not +3





4 $3f(5x-25) = 3f(5(x-5)) = g(x)$

m1: $= 3 \cdot 4^{5x-25}$, $x \mapsto \frac{1}{5}x$

$x = \frac{24}{5}$

$x = \frac{25}{5} = 5$

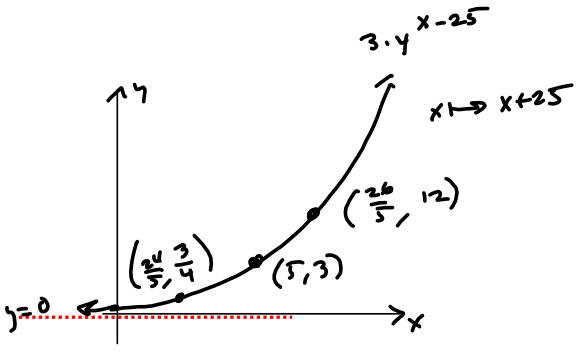
$x = \frac{26}{5}$

m2: $= 3 \cdot 4^{5(x-5)}$, $x \mapsto x+5$

$x = -\frac{1}{5} + 5 = \frac{24}{5}$

$x = 0 + 5 = 5$

$x = \frac{1}{5} + 5 = \frac{26}{5}$



The instructions say do it in 5 steps, counting the graph of *f* but I really only gave you 4 moves that need to be made, because there's no constant tacked on. I *thought* I was assigning something like

$g(x) = -3 \cdot 4^{5x-25} + 13$, but I see no "+13"!

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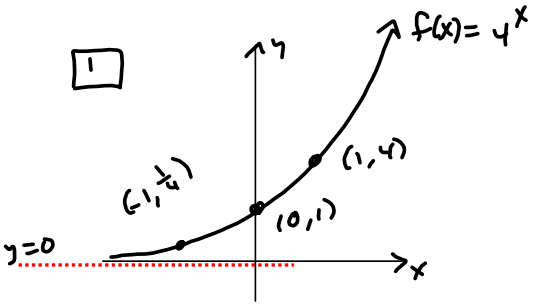
2 points each sketch. Looking for the 3 points in each graph. 2 points for breathing.

1



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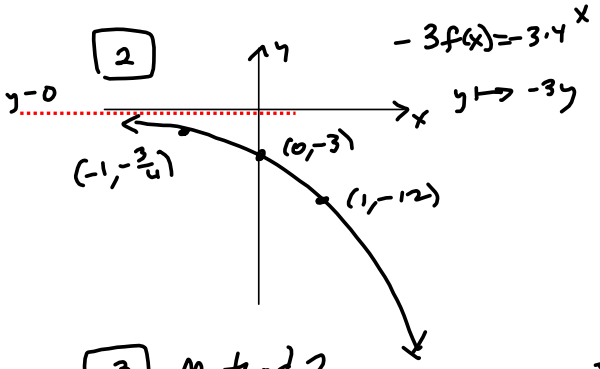
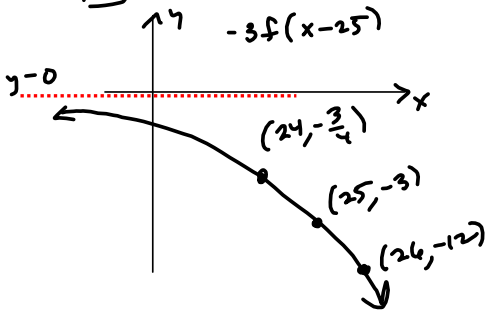
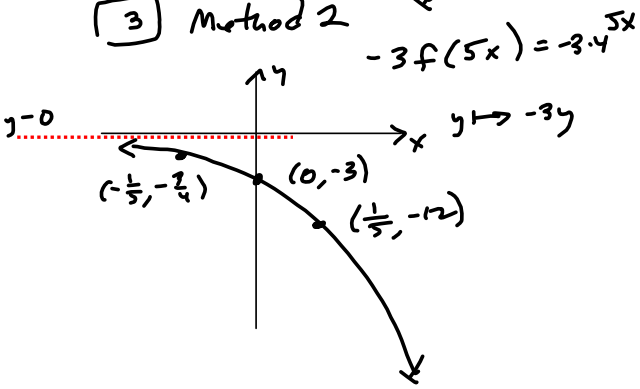


3 Method 1



3 Method 2



4 $-3f(5x-25) = -3f(5(x-5)) = g(x)$

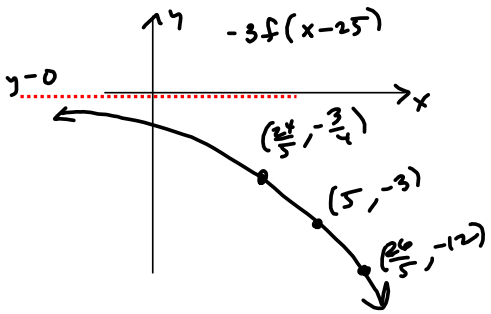
m1 $5x-25$ $= -3\cdot 4$ m2 $5(x-5)$ $= -3\cdot 4$

$x \mapsto \frac{1}{5}x$ $x \mapsto x+5$

$x = \frac{24}{5}$ $x = -\frac{1}{5} + 5 = \frac{24}{5}$

$x = \frac{25}{5} = 5$ $x = 0+5 = 5$

$x = \frac{26}{5}$ $x = \frac{1}{5} + 5 = \frac{26}{5}$



(2) (10 pts) Find EXACT x- & y-ints for #1

y-int: $g(0) = -3 \cdot 4^{5(0)-25} = -3 \cdot 4^{-25}$

y-int: $\left(0, -\frac{3}{4^{25}}\right)$ 5 pts

x-int: $-3 \cdot 4^{5x-25} = 0 \Rightarrow$ DNE! 5 pts

No x-int.!

(3) $g(x) = -3 \cdot 4^{5x-25}$ We find $g^{-1}(x)$:

$-3 \cdot 4^{5y-25} = x \Rightarrow$ 2 pts context

$4^{5y-25} = -\frac{x}{3} \Rightarrow$

$\log_4(\quad) = \log_4\left(-\frac{x}{3}\right)$

$5y - 25 = \log_4\left(-\frac{x}{3}\right)$

$5y = \log_4\left(-\frac{x}{3}\right) + 25$

8 pts Show Support

$g^{-1}(x) = y = \frac{1}{5}\left[\log_4\left(-\frac{x}{3}\right) + 25\right]$ OR $\frac{1}{5}\log_4\left(-\frac{x}{3}\right) + 5$

OR $g^{-1}(x) = \frac{1}{5\ln(5)}\ln\left(-\frac{x}{3}\right) + 5$

Any of the above versions will suffice

(4) We solve $\ln(x-5)+\ln(x+2)=\ln(18)$: → 2pts

$\ln((x-5)(x+2))=\ln(18)$ → 2pts

$x^2-3x-10=18$ → 2pts

$x^2-3x-28=0$ → 2pts

$(x-7)(x+4)=0$ →

$x=-4,7.$

$x=-4 \notin D$ → $x=7$ 2pts

check

$\ln(2)+\ln(9)$

$=\ln(2\cdot 9)=\ln(18)$ ✓

(5) The $\frac{1}{2}$-life of C-14 is 5400 yrs.

(2) We derive exponential decay model $A(t)=A_0e^{kt}$

$\frac{1}{2}$-life is 5400 yrs →

$A_0e^{5400k}=\frac{1}{2}A_0$ → Not preferred But OK. SOME WILL START HERE

$e^{5400k}=\frac{1}{2}$ →

$5400k=\ln(1/2)=-\ln(2)$ →

$k=\frac{-\ln(2)}{5400}\approx -0.0001283605890$

5pts

Find k & write the model.

(5) How old is a sample w/ 28% of C-14 remaining? Round to nearest year.

$A_0e^{kt}=.28A_0$

$e^{kt}=.28$

$kt=\ln(.28)$

$t=\frac{\ln(.28)}{k}=\frac{\ln(.28)}{-\frac{\ln(2)}{5400}}=\frac{5400\ln(.28)}{-\ln(2)}$

Use judgement on supporting work.

$\approx 9917.106847 \approx$ 9917 yrs, rounded to nearest year

5pts

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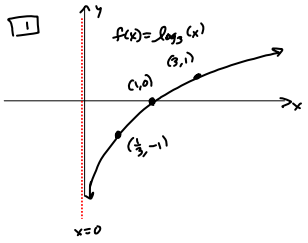
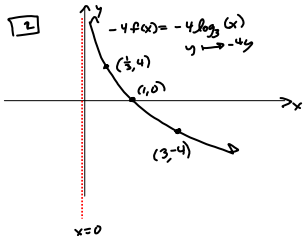
6

We sketch the graph of $-4\log_3(3x+21)+2$ by transforming the graph of $f(x)=\log_3(x)$

We track where (1/3, -1), (1, 0), and (3, 1) go at each step.

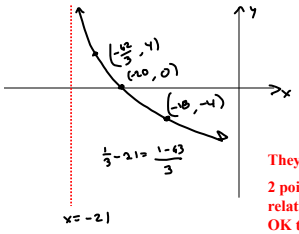




3 Method 1

$-4f(x+21)=-4\log_3(x+21)$

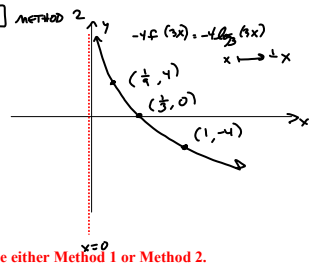
$x \to x-21$



3 METHOD 2

$-4f(3x)=-4\log_3(3x)$

$x \to \frac{1}{3}x$



They may use either Method 1 or Method 2.

2 points for each sketch. Looking for shape, key labels and relative positions of points. If they use BOTH M1 and M2, it's OK to award an extra 2 points bonus (if they do both correctly). For a possible 12 points before the 10 bonus, below

4

$-4f(3x+21) = -4f(3(x+7))$

$x \mapsto \frac{1}{3}x$ $\quad x \mapsto x-7$

$\frac{-62}{3} \div 3 = \frac{-62}{9}$

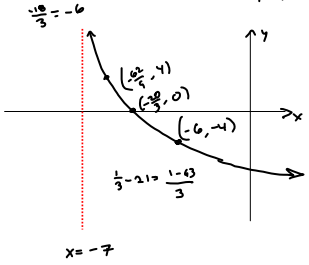
$\frac{-20}{3}$

$\frac{-18}{3} = -6$

$\frac{1}{9}-7 = \frac{1-63}{9} = -\frac{62}{9}$ ✓

$\frac{1}{3}-7 = \frac{1-21}{3} = \frac{-20}{3}$ ✓

$1-7=-6$ ✓



g(x)

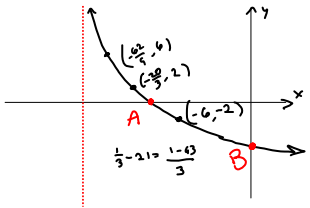
5 points bonus for x-intercept and supporting work.

5 points bonus for y-intercept and supporting work.

5

$g(x)=-4\log_3(3x+21)+2$

$y \mapsto y+2$ $\quad 4+2=6$ $\quad 0+2=2$ $\quad -4+2=-2$



g(x)

Bonus 10pts Find x- & y-ints w/o being asked.

5pts $A=(\frac{\sqrt{3}}{3}-7,0) \approx (-6.422649731,0)$

x-int: $g(x)=0$

$-4\log_3(3x+21)+2=0$

$-4\log_3(3x+21)=-2$

$\log_3(3x+21)=\frac{-2}{-4}=\frac{1}{2}$

$3x+21=3^{\frac{1}{2}}$

$3x=3^{\frac{1}{2}}-21$

$x=\frac{3^{\frac{1}{2}}-21}{3}$

or $=\frac{\sqrt{3}}{3}-7$

$\approx$ -6.422649731

y-int:

$g(0)=-4\log_3(0+21)+2$

$=-4\log_3(21)+2 \approx$ -9.08497500

or $-4\log_3(3\cdot7)+2$

$=-4[\log_3(3)+\log_3(7)]+2$

$=-4[1+\log_3(7)]+2$

$=-4-4\log_3(7)+2$

$=-4\log_3(7)-2$!

5pts $B=(0,-4\log_3(7)+2) \approx (0,-9.08497500)$