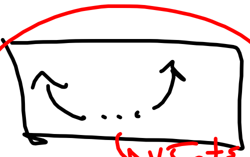
B40 WEEK 9 SOLNS MILLS

① $g(x) = 4x^4 - 16x^3 + 3x^2 + 26x + 3$

ⓐ g's end behavior is that of $4x^4$



Context .5pts

→ 4.5 pts

I'm looking for the diagram. The book teaches

$\lim_{x\to\pm\infty} g(x) = \infty$ or $\lim_{x\to-\infty} g(x) = \infty$, $\lim_{x\to\infty} g(x) = \infty$

This is OK, but it's not the simple diagram. Charge .5pt for doing it this way

ⓑ Find the possible # of positive & negative zeros.

There are 2 sign changes in the terms for $g(x)$.

2pts → 2 or 0 positive zeros. Context 1pt

$g(-x) = 4x^4 + 16x^3 + 3x^2 - 26x + 3$ has 2 sign changes

→ 2 or 0 negative zeros. → 2pts

ⓒ 5pts We list the possible rational zeros of g.

$a_4 = 4$ denom. $a_0 = 3$ numer. $\Big\}$ $\frac{p}{q} = \frac{\text{factors of } 3}{\text{factors of } 4}$

Context 1pt

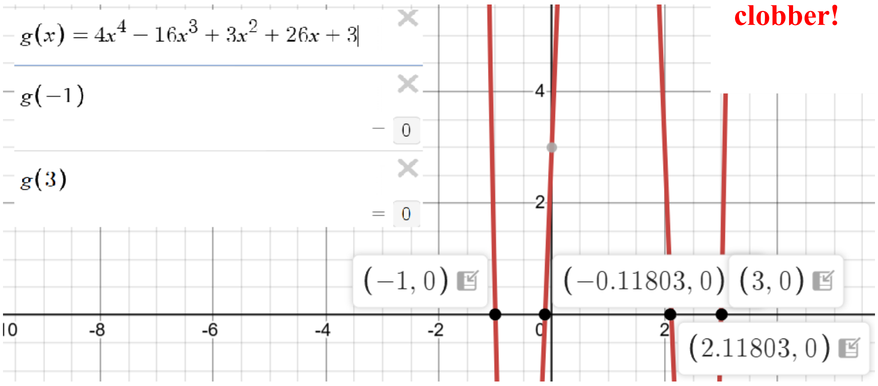
Rational zeros are from this list:

$\pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm 3, \pm\frac{3}{2}, \pm\frac{3}{4}$ → 4pts

ⓓ We find all rational zeros of *g* and factor as far as we can with those rational zeros. Here's where a grapher would save you time. Without a grapher, you have up to 12 rational candidates to test. WITH a grapher, you basically guess right, immediately.

METHOD 1 Desmos says $g(-1) = g(3) = 0$

Try those, first! If they both work, then you have a depressed polynomial that's a quadratic polynomial that you can clobber!



$g(x) = 4x^4 - 16x^3 + 3x^2 + 26x + 3$

$g(-1)$ = 0

$g(3)$ = 0

No points for part d on this page. The focus is on splitting *f* as far as you can

ⓓ Continued

$g(x) = 4x^4 - 16x^3 + 3x^2 + 26x + 3$

We divide by $x-(-1) = x+1$, and $x-3$

Context 1pt

-1	4	-16	3	26	3	
		-4	20	-23	-3	
3	4	-20	23	3	0	sweet!
		12	-24	-3		
	4	-8	-1	0		sweet!

Divide 3pts

The other 2 zeros are ugly. Let's see if they're rational by looking at the depressed polynomial

$4x^2-8x-1$

$a=4, b=-8, c=-1$

$b^2-4ac = 8^2-4(4)(-1)$

$=64+16=80$

so the other 2 zeros are real, irrational.

$8^2 < 80 < 9^2$

$64 < 80 < 81$

80 is not a perfect square

∴ the zeros of $4x^2-8x-1$ are irrational and

we factor: What we're looking for.

$\boxed{g(x) = (x+1)(x-3)(4x^2-8x-1)}$

Conclude 1pt

$\boxed{x = -1, 3 \text{ are the rational zeros}}$

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ⓔ We find the zeros of $4x^2-8x-1$.

$a=4, b=-8, c=-1$

By part d, $b^2-4ac=80$

Simplify $\sqrt{80}$

$2 \,|\, 80$
$2 \,|\, 40$
$2 \,|\, 20$
$2 \,|\, 10$
5

$4\sqrt{5}=\sqrt{80}$

context 1pt

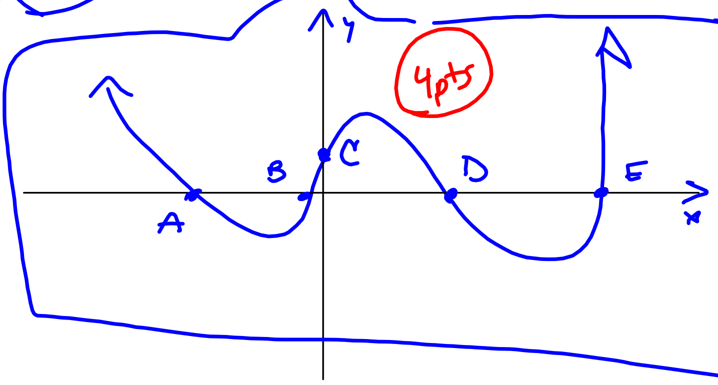
Do b^2-4ac 1st 1pt

Now, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $a=4, b=-8, c=-1$

$$=\frac{8\pm4\sqrt{5}}{2(4)}=\boxed{\frac{2\pm\sqrt{5}}{2}} \rightarrow \text{Find this } 3\text{pts}$$

ⓕ $$\boxed{g(x)=(x+1)(x-3)\left(x-\frac{2+\sqrt{5}}{2}\right)\left(x-\frac{2-\sqrt{5}}{2}\right)} \rightarrow 5\text{pts}$$

ⓖ 5pts Sketch the graph based on previous work 1pt



$A=(-1,0)$

$B=\left(\frac{2-\sqrt{5}}{2},0\right)\approx(-.12,0)$

$C=(0,3)$

$D=\left(\frac{2+\sqrt{5}}{2},0\right)\approx(2.12,0)$

$E=(3,0)$

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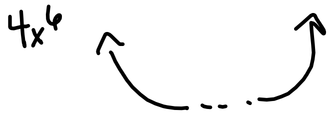
② $f(x) = 4x^6 - 36x^5 + 146x^4 - 340x^3 + 76x^2 + 776 + 174$

ⓐ (5pts) we provide an end-behavior graphic:



context - 1pt
Graphic - 4pts
If they write as limits, 3pts instead of 4pts

ⓑ We use the given fact that $x = 3-7i$ is a zero of f to split f into the product of a quadratic and a quartic

$x = 3-7i$ is a zero & all coefficients of f are real $\longrightarrow$

$x = 3+7i$ is ALSO a zero.

We split off $(x-(3-7i))(x-(3+7i))$

$= x^2 - (3+7i)x - (3-7i)x + (3-7i)(3+7i)$

$= x^2 - 3x - 7ix - 3x + 7ix + 3^2 + 7^2$

$= x^2 - 6x + 9 + 49 = x^2 - 6x + 9$. We split this factor off by dividing f by it.

This is bonus. I shouldn't have injected this, even though it's cool.

1pt - context
2pts - find $x^2 - 6x + 9$
2pts - Perform division on next page.

$$4x^4 - 16x^3 + 3x^2 + 26x + 3$$

$$x^2 - 6x + 58 \;\big)\; 4x^6 - 40x^5 + 331x^4 - 920x^3 + 21x^2 + 1490x + 174$$

$$-\left(4x^6 - 24x^5 + 232x^4\right)$$

$$-16x^5 + 99x^7 - 920x^3 + 21x^2 + 1490x + 174$$

$$-\left(-16x^5 + 96x^4 - 928x^3\right)$$

$$3x^4 + 8x^3 + 21x^2 + 1490x + 174$$

$$-\left(3x^4 - 18x^3 + 174x^3\right)$$

$$26x^3 - 153x^3 + 1490x + 174$$

$$-\left(26x^3 - 156x^2 + 1508x\right)$$

$$3x^3 - 18x + 174$$

$$-\left(3x^3 - 18x + 174\right)$$

$$0!$$

Sweet!

So $\boxed{f(x) = (x^2 - 6x + 58)(4x^4 - 16x^3 + 3x^2 + 26x + 3)}$

Wait a minute! $4x^4 - 16x^3 + 3x^2 + 26x + 3 = g(x)$ for #1!

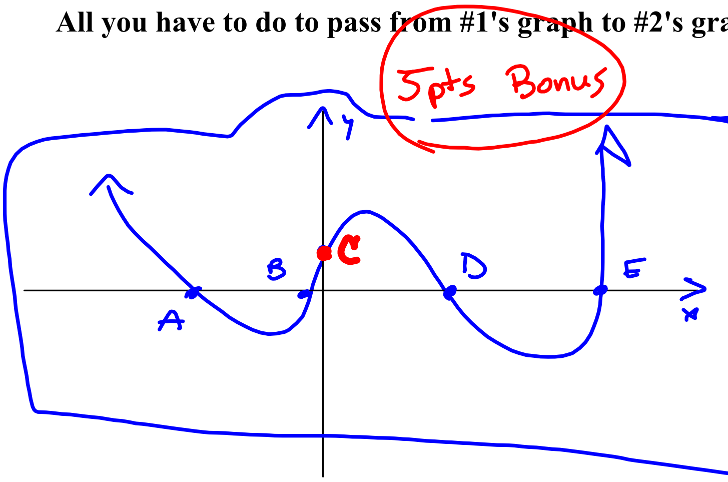
Fact: $x^2 - 6x + 58$ has non-real zeros, and therefore contributes nothing to the x-intercepts of the graph.

$x^2 - 6x + 58$ factor definitely makes an impact, but we don't see it at our level of analysis.

$x^2 - 6x + 58$ has no effect on the sign pattern for f, so its sign pattern and overall appearance will be something very much like $g(x)$ ... at THIS level of analysis.

To delve deeper, we'd have to find the highs & lows, because they won't match up. #2 has a lot more growth potential than #1.

All you have to do to pass from #1's graph to #2's graph is change the label on the y-intercept.



$A = (-1, 0)$

$B = \left(\frac{2-\sqrt{5}}{2}, 0\right) \approx (-.12, 0)$

$C = (0, 174)$

$D = \left(\frac{2+\sqrt{5}}{2}, 0\right) \approx (2.12, 0)$

$E = (3, 0)$

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③ $R(x) = \dfrac{3x^2+6x-24}{4x^2+27x+18} = \dfrac{3(x^2+2x-8)}{4x^2+24x+3x+18} = \dfrac{3(x+4)(x-2)}{4x(x+6)+3(x+6)}$

$2 \cdot 2 \cdot 3 \cdot 3 \cdot 2$

$= \dfrac{3(x+4)(x-2)}{(x+6)(4x+3)}$ → 2pts

V.A.: $x = -6,\ x = -\frac{3}{4}$

x-int: $(-4, 0),\ (2, 0)$

y-int: $(0, -\frac{24}{18}) = (0, -\frac{4}{3})$

HA: $y = \frac{3}{4}$ → 5pts

ⓐ $\mathcal{D} = \mathbb{R} \setminus \{-6, -\frac{3}{4}\} =$

$= (-\infty, -6) \cup (-6, -\frac{3}{4}) \cup (-\frac{3}{4}, \infty)$

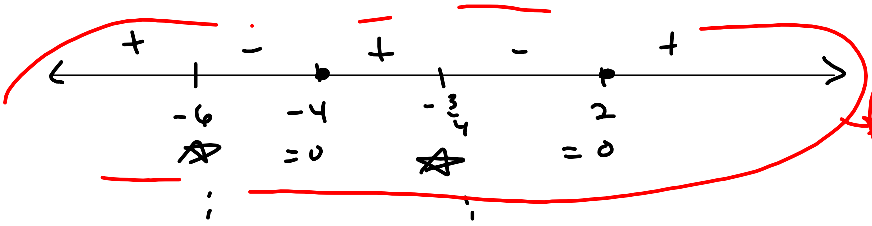
Either is OK 5pts

ⓑ zeros: $x = -4, 2$ 5pts

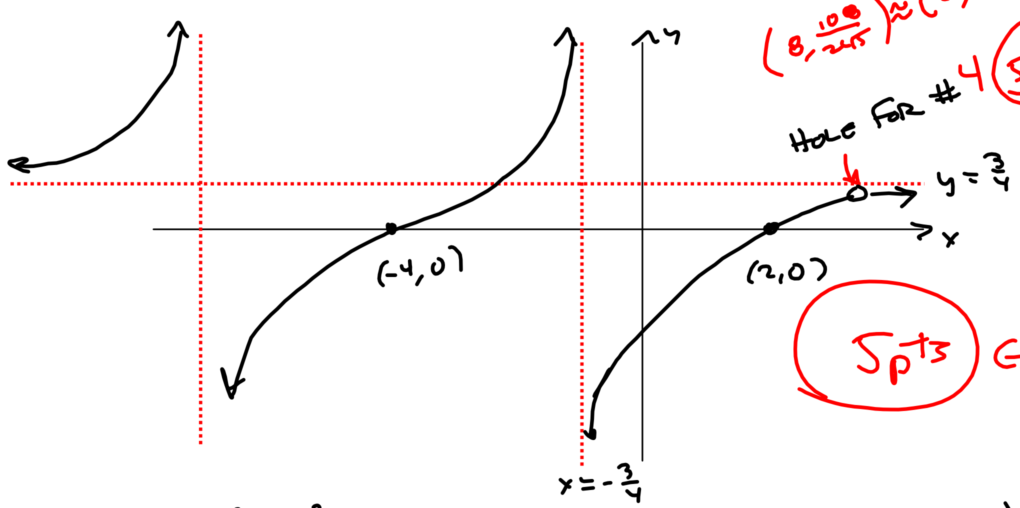
ⓒ Hor. Asymp.: $\dfrac{3x^2}{4x^2} = \dfrac{3}{4} \longrightarrow$ $y = \frac{3}{4}$ is H.A. → 5pts

Hard way: $\dfrac{3x^2+6x-24}{4x^2+27x+18} = \dfrac{x^2(3 + \frac{6}{x} - \frac{24}{x^2})}{x^2(4 + \frac{27}{x} + \frac{18}{x^2})} \xrightarrow{x \to \infty} \dfrac{3}{4}$

#3d Factor $R(x) = \frac{3(x+4)(x-2)}{4x(x+6)+3(x+6)}$ → 2pts



→ 3pts



$(8, \frac{108}{245}) \approx (8, .4408) \approx$ HOLE

Hole for #4 (5pts) Support for hole, below.

5pts Graph.

$$\hat{R}(x) = \frac{3x^3\ 18x^2 - 72x + 192}{4x^3 - 5x^2 - 198x - 144} = \frac{3(x^3 - 6x^2 - 24x + 64)}{\sim}$$

We know it has same zeros as R, so: $x=-4$ & $x=2$:

$$\begin{array}{r|rrrr} -4 & 1 & -6 & -24 & 64 \\ & & -4 & 40 & -64 \\ \hline 2 & 1 & -10 & 16 & 0 \\ & & 2 & -16 & \\ \hline & 1 & -8 & 0 & \end{array}$$

→ $x-8=0$ → $x=8$ is zero. →

Support for #4 hole (Finding the hole (a) $x=8$)

SEE GRAPH for #3

$(8, R(8))$ is the hole.

$$R(8) = \frac{3(8+4)(8-2)}{(8+6)(4(8)+3)} = \frac{3\overset{6}{\cancel{(12)}}(6)}{\underset{7}{\cancel{14}}(35)} = \frac{3(36)}{7(35)} = \frac{108}{245} \approx$$

$\approx$ 0.4408163265

⑤ $T(x)=\frac{3x^3-18x^2-72x+192}{4x^2+27x+18}=\frac{3(x+4)(x-2)(x-8)}{(x+6)(4x+3)}$

ⓐ Long division for slant asymptote:

$$\frac{3}{4}x-\frac{153}{16}$$

$$4x^2+27x+18\,\overline{)\,3x^3-18x^2-72x+192}$$

$$-\left(3x^3+\frac{81}{4}x^2\right)\sim$$

$$-\frac{153}{4}x^2+\sim$$

$\frac{3x^3}{4x^2}=\frac{3}{4}x$

$-18-\frac{81}{4}$

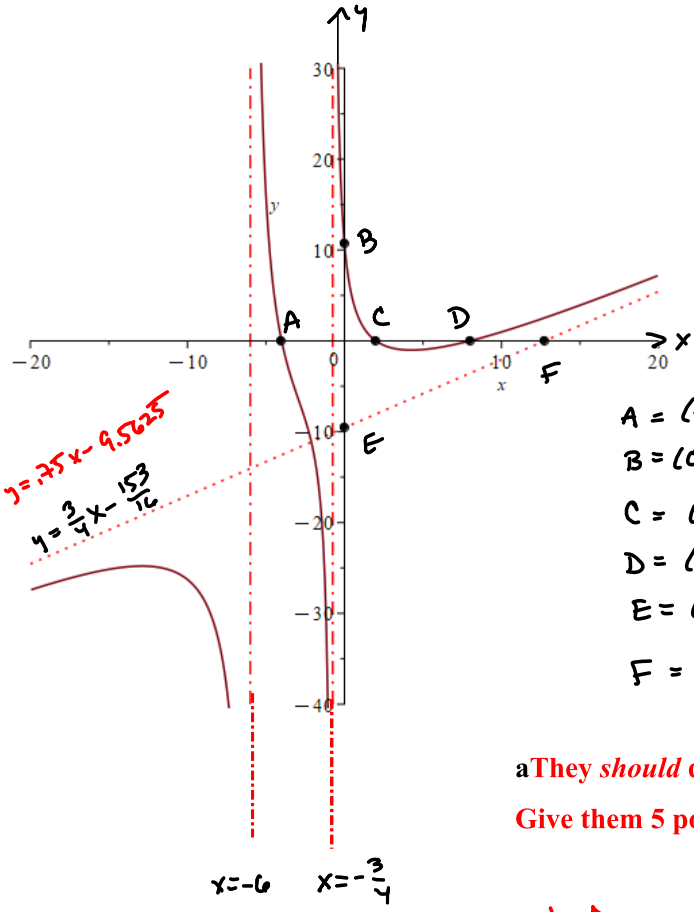
$= -\frac{72-81}{4}$

$=\frac{-153}{4}$

$\frac{-\frac{153}{4}x^2}{4x^2}=\frac{-153}{16}$

$\boxed{y=\frac{3}{4}x-\frac{153}{16} \text{ is S.A.}}$ 5pts

Generous w/ partial credit



A = (-4,0)

B = $(0,\frac{32}{3})=(0,10.\overline{6})$

C = (2,0)

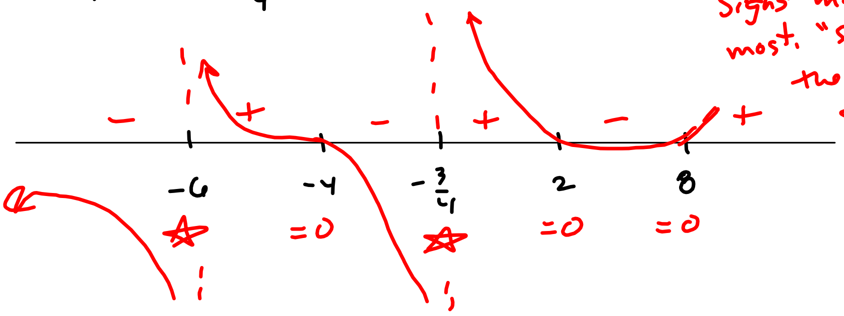
D = (8,0)

E = $(0,-\frac{153}{16})=(0,-9.5625)$

F = $(\frac{51}{4},0)=(12.75,0)$

a**They *should* do a sign pattern to get this right.**

Give them 5 points bonus for the following:



Signs matter on most. "sketch" on the pattern is suggested, not required.