1340 WEEK 8 Written Solns MILLS

(1) We complete the square to write $f(x)$ in Standard Form ($a(x-h)^2+k$) and we sketch $f(x)$, showing all intercepts and its vertex (h,k). } content 1pt

(a) 5pts $f(x) = x^2-4x-21 = x^2-4x+2^2-4-21$

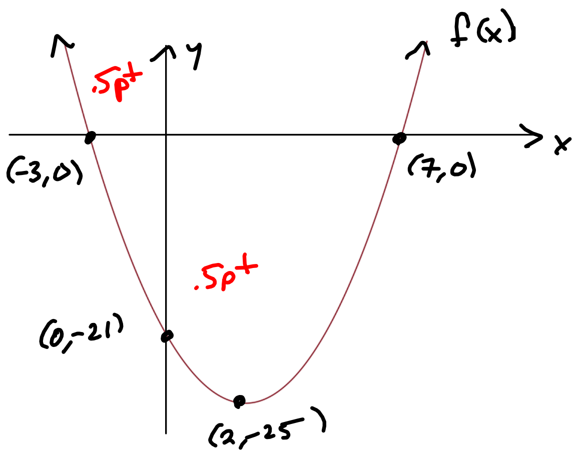
$= (x-2)^2-25 \Rightarrow \boxed{\text{vertex} = (h,k) = (2,-25)}$ and

$f(x)=0 \Rightarrow (x-2)^2 = 25$

$\Rightarrow x-2 = \pm 25 = \pm 5$

$\Rightarrow x = 2 \pm 5$ → $x=7$ or $x=-3$ → $\boxed{(7,0) \text{ x-int}}$, $\boxed{(-3,0) \text{ x-int}}$

Also, $f(0) = -21 \longrightarrow \boxed{(0,-21) \text{ is y-int}}$



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(b) 5pts $f(x) = -3x^2 - 42x - 72$

$= -3(x^2 + 14x \qquad) - 72$

$= \underbrace{-3(x^2 + 14x + 7^2) - (-3)(49)}_{\text{Subtract this off}} - 72$

Scratch: $\begin{array}{r} 49 \\ \times\ 3 \\ \hline 147 \end{array}$

$= -3(x+7)^2 + 75$

$147 - 72 = 75$

$\Longrightarrow (h,k) = \boxed{(-7, 75) = \text{vertex}}$, and

$f(x) = 0 \Longrightarrow -3(x+7)^2 + 75 = 0$

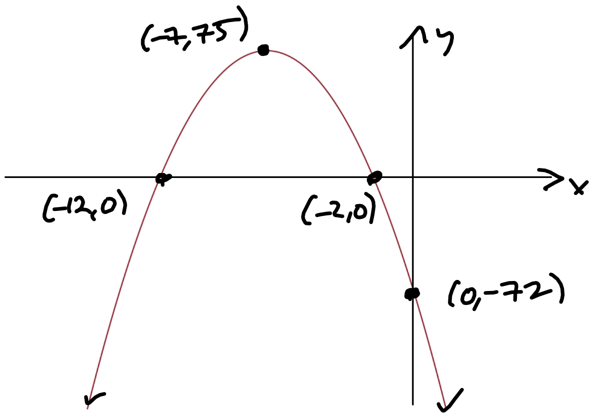
$\Longrightarrow -3(x+7)^2 = -75$

$\Longrightarrow (x+7)^2 = 25$ $\quad -2 = x \rightarrow (-2, 0)$

$\Longrightarrow x = -7 \pm 5$ or $-12 = x \rightarrow (-12, 0)$

$\boxed{(-2,0),\ (-12,0)}$ x-int's

$f(0) = -72 \Longrightarrow \boxed{(0, -72) \text{ is y-int}}$



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(b) (5pts) Alternate completing-the-square technique

$f(x) = -3x^2 - 42x - 72 = -3(x^2 + 14x + 24)$

$\Longrightarrow \dfrac{f(x)}{-3} = x^2 + 14x + 24 = x^2 - 14x + 7^2 - 49 + 24$

$= (x-7)^2 - 25 = \dfrac{f(x)}{-3} \Longrightarrow$

$\underline{f(x) = -3(x-7)^2 + 75}$, etc.

(2) Let $f(x) = x^6 + 11x^5 + 19x^4 - 115x^3 - 200x^2 + 500x$ & $p(x) = x - 2$.

(a) (5pts) Find quotient $g(x)$ and remainder $r(x)$ for $f(x) \div p(x)$. Then write $f(x) = p(x)\,g(x) + r(x)$. USE SYNTHETIC DIVISION

	6	5	4	3	2	1	0
2	1	11	19	-115	-200	500	0
		2	26	90	-50	-500	0
	1	13	45	-25	-250	0	$\boxed{0 = r(x)}$

This says $f(x) = (x-2)(x^5 + 13x^4 + 45x^3 - 25x^2 - 250x) + 0$, $r(x) = 0$ & $p(x) = x - 2$ is a factor of $f(x)$.

(b) (5pts) Crap. I wanted a $p(x)$ that wasn't a factor.

In any case, $R(x) = \dfrac{f(x)}{p(x)} = x^5 + 13x^4 + 45x^3 - 25x^2 - 250x = g(x)$ and there's no $\dfrac{r(x)}{p(x)}$ at all. So $\dfrac{f(x)}{p(x)} = g(x)$ is a polynomial everywhere.

I was aiming for something more like $p(x) = x - 3$

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We use synthetic division to divide *f* by *x* - 3:

3	1	11	19	-115	-200	500	0
		3	42	183	204	12	1536
	1	14	61	68	4	512	$\boxed{1536 \cdot \text{remainder}}$

This says $f(x) = (x-3)(x^5 + 14x^4 + 61x^3 + 68x^2 + 4x + 512) + 1536$

This is why the Remainder Theorem, which says

$$f(3) = (3-3)(3^5 + 4(3)^4 + 61(3)^3 + 68(3)^2 + 4(3) + 512) + 1536$$

$= \boxed{f(3) = 1536}$ = remainder, when $f(x)$ is divided by $x-3$.

Furthermore $\dfrac{f(x)}{p(x)} = x^5 + 14x^4 + 61x^3 + 68x^2 + 4x + 512 + \dfrac{1536}{x-3}$

So this rational function is basically a polynomial plus a $\dfrac{1536}{x-3}$ term that vanishes as $x \to \pm\infty$.

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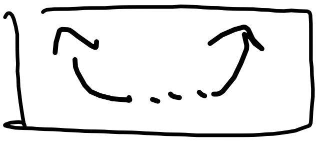
(c) (5pts) Based on 2a, $\boxed{f(2)=0}$

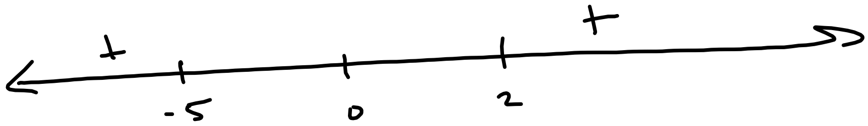
(d) (5pts) End behavior of f is controlled by the leading term x^6, whose end behavior is



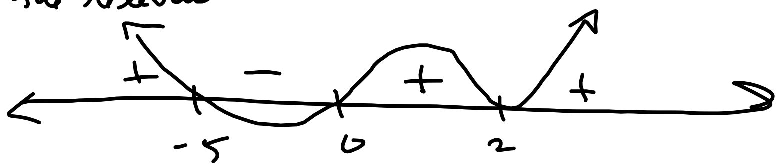
(e) (5pts) $f(x) = (x)(x-2)^2(x+5)^3$ is factored form of f.

we sketch $f(x)$ using its x- & y-in~~tercepts~~, y-intercept, and end behavior

x^6 gives this much of the sign pattern

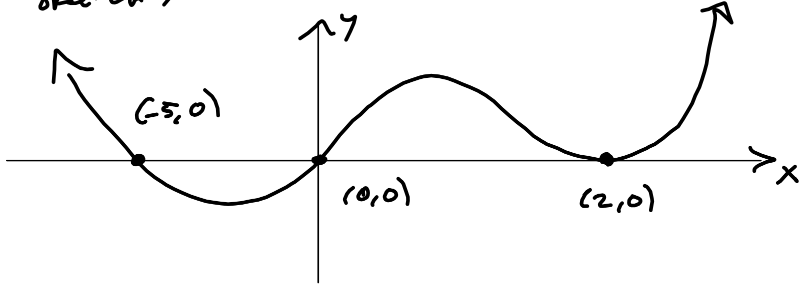


You get the rest by looking for odd/even powers of the relevant factors.



$(x+5)^3$	x^1	$(x-2)^2$
3 is odd	1 is odd	2 is even
sign changes	sign changes	sign doesn't change.
cross	cross	kiss

Sketch:



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2 f 5pts $D(x) = x^2 - 3x - 15$. We find the quotient $Q(x)$ and remainder $R(x)$, when $f(x)$ is divided by $D(x)$.

QUOTIENT $Q(x)$: $\boxed{x^4 + 14x^3 + 76x^2 + 323x + 1909}$

$$x^2 - 3x - 15 \;\big)\; x^6 + 11x^5 + 19x^4 - 115x^3 - 200x^2 + 500x$$

$$-(x^6 - 3x^5 - 15x^4)$$

$$14x^5 + 34x^4 - 115x^3 - 200x^2 + 500x$$

$$-(14x^5 - 42x^4 - 210x^3)$$

$$76x^4 + 95x^3 - 200x^2 + 500x$$

$$-(76x^4 - 228x^3 - 1140x^2)$$

$$323x^3 + 940x^2 + 500x$$

$$-(323x^3 - 969x^2 - 4845x)$$

$$1909x^2 + 5345x$$

$$-(1909x^2 - 5727x - 28635)$$

$$11072x + 28635$$

REMAINDER $\boxed{R(x) = 11072x + 28635}$

$76 \times 3 = 228$

$(76)(15) = 760 + 380 = 1140$

$(15)(323) = 3230 + 1615 = 4845$

$\frac{3230}{2} = 1615$, $1615 + 3230 = 4845$

$1909 \times 3 = 5727$

$(15)(1909) = 19090 + 9545 = 28635$

$\frac{19090}{2} = 9545$

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The above work shows that

$$f(x) = D(x)\,Q(x) + R(x)$$
$$= (x^2-3x-15)(x^4+14x^3+76x^2+323x+1909) + 11072x + 20635$$

3.6 Preview!

$$\frac{f(x)}{D(x)} = \frac{x^4+11x^5+19x^4-115x^3-200x^2+500x}{x^2-3x-15}$$

$$= x^4+14x^3+76x^2+323x+1909 + \frac{11072x+20635}{x^2-3x-15}$$

Now, as $x \longrightarrow \pm\infty$, $\frac{f(x)}{D(x)} \longrightarrow x^4+14x^3+76x^2+323x+1909$

because $\frac{11072x+20635}{x^2-3x-15} \longrightarrow 0$ as $x \longrightarrow \pm\infty$

(x^2 out-grows $11072x$)

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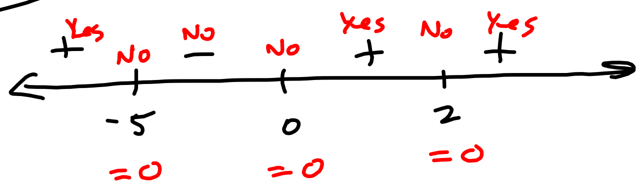
(g) 5pts $f(x)=x(x-2)^2(x+5)^3 \longrightarrow$

$x=0$ is a zero of multiplicity $m=1$,
$x=2$ " " " " " $m=2$, and
$x=-5$ " " " " " $m=3$.

3pts

(h) Bonus 5pts Solve $f(x)>0$

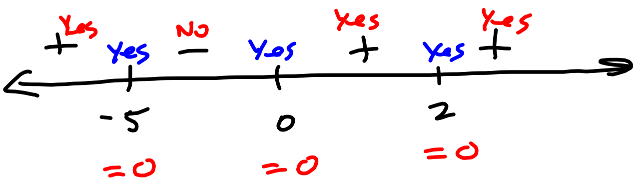


>0

$\longrightarrow x \in (-\infty,-5)\cup(0,2)\cup(2,\infty)$

(i) Bonus 5pts We find the domain $\mathcal{D}(g)$ for $g(x)=\sqrt{f(x)}$

$\longrightarrow$ we need $f(x)\geq 0$



≥ 0

$\longrightarrow \boxed{\mathcal{D}(g)=(-\infty,-5]\cup[0,\infty)}$