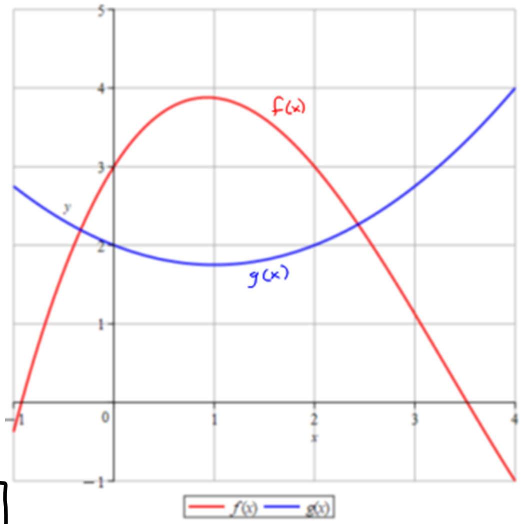
1340 WEEK 7 SOL'NS MILLS



① (5pts) Find the following, based on the figure at the right.

ⓐ $(f+g)(0) = f(0)+g(0) = 3+2 = \boxed{5}$

ⓑ $(f-g)(0) = f(0)-g(0) = 3-2 = \boxed{1}$

ⓒ $(fg)(0) = f(0)g(0) = 3(2) = \boxed{6}$

ⓓ $\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)} = \boxed{\frac{3}{2}}$

ⓔ $(f\circ g)(0) = f(g(0)) = f(2) = \boxed{3}$

② (5pts) Domain of $f\circ g = \mathcal{D}(f\circ g)$

$= \boxed{\{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}}$ or precise words to that effect.

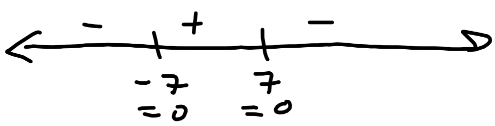
③ We find the domain of the following:

ⓐ (5pts) $f(x) = \sqrt{x} + \sqrt{49-x^2} \longrightarrow$

Need $\boxed{x \geq 0}$ and $49-x^2 \geq 0 \longrightarrow$

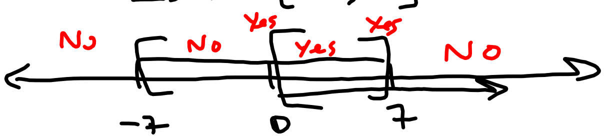
$\Rightarrow x \in [0,\infty)$

$(7-x)(7+x) \geq 0$



$\Rightarrow x \in [-7,7]$

Need



AND

$= \boxed{[0,7] = \mathcal{D}(f)}$

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3b 5pts $f(x) = \frac{\sqrt{x}}{\sqrt{49-x^2}}$ This is like 3a, except

$\sqrt{49-x^2}$ is in the denominator.

This means $49-x^2 \geq 0$ for "$\sqrt{\quad}$" as before and also

$\sqrt{49-x^2} \neq 0$ for being in the denominator.

This means we have to get rid of $x = \pm 7$ in 3a.

3a $\mathcal{D} = [0,7]$.

3b $\boxed{\mathcal{D} = [0,7)}$

4 we find $f+g$, fg, f/g, and $f \circ g$ for $f(x) = \sqrt{x}$ &

$g(x) = x^2-3x+2$. ($\mathcal{D}(f) = [0,\infty)$, $\mathcal{D}(g) = \mathbb{R}$)

a 5pts $\boxed{(f+g)(x) = \sqrt{x} + x^2-3x+2}$ $\boxed{\mathcal{D}(f+g) = [0,\infty)}$

from $x \geq 0$ in $\sqrt{x}$.

b 5pts $\boxed{(fg)(x) = \sqrt{x}(x^2-3x+2)}$ $\boxed{\mathcal{D}(fg) = [0,\infty)}$

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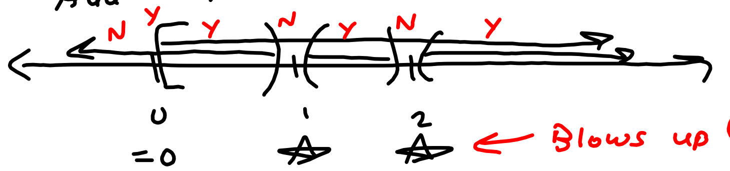
(c) (5pts) $\boxed{\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{x^2-3x+2}}$ Need $x \geq 0$ AND $x^2-3x+2 \neq 0$

$x^2-3x+2 \neq 0$

$(x-2)(x-1) \neq 0$

$x \neq 2$ AND $x \neq 1$

Add this condition to $x \geq 0$



AND

← Blows up @ $x = 1, 2,$

$\rightarrow \boxed{\mathcal{D}\left(\frac{f}{g}\right) = [0,1) \cup (1,2) \cup (2,\infty)}$

(d) (5pts) $(f \circ g)(x) = f(g(x)) = \sqrt{g(x)} = \boxed{\sqrt{x^2-3x+2} = f \circ g}$

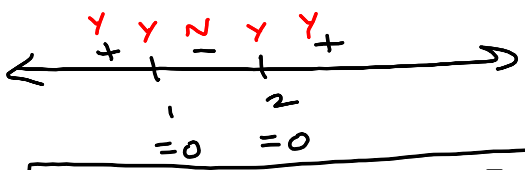
$\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$= \{x \mid x \in \mathbb{R} \text{ and } x^2-3x+2 \in \mathcal{D}(g)\}$

$= \{x \mid x \in \mathbb{R} \text{ and } x^2-3x+2 \geq 0\}$

i.e., we need $x^2-3x+2 \geq 0$

$\longrightarrow (x-1)(x-2) \geq 0$



$\rightarrow \boxed{\mathcal{D}(f \circ g) = (-\infty, 1] \cup [2,\infty)}$

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5 5pts If $f(x)=\sqrt{x}$, $g(x)=\frac{x}{x-4}$, and $h(x)=\sqrt[6]{x}$, then

$$(f\circ g\circ h)(x) = f(g(h(x))) = f(g(\sqrt[6]{x})) = f\left(\frac{\sqrt[6]{x}}{\sqrt[6]{x}-4}\right) =$$

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$$\boxed{\sqrt{\frac{\sqrt[6]{x}}{\sqrt[6]{x}-4}}}$$

Bonus 5pts

Need $\boxed{x \geq 0}$ for $\sqrt[6]{x}$. AND

Need $\sqrt[6]{x} \neq 4$ for $\frac{\sqrt[6]{x}}{\sqrt[6]{x}-4}$

$\Rightarrow \sqrt[6]{x} \neq 4 \Rightarrow$

$x \neq 4^6 = \boxed{4096 \neq x}$ AND

$\frac{\sqrt[6]{x}}{\sqrt[6]{x}-4} \geq 0^{*}$ $x=0$, $x=4096$ are key points

* to handle $\sqrt{\ }$ around everything

Test: $x=1$ $\frac{\sqrt[6]{1}}{\sqrt[6]{1}-4} = \frac{1}{-3}$ NO

$x=4097$ $\frac{\sqrt[6]{4097}}{\sqrt[6]{4097}-4} = \frac{+}{+} = +$ Yes. So

$$\boxed{\mathcal{D}(f\circ g\circ h) = (4096, \infty)}$$

($x=4096$ makes denominator zero.

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6 5pts A function is a rule that assigns to each $x \in$ a set called the "domain" to exactly one y in another set, called the "range."

A 1-to-1 function is a function such that each y in the range is assigned to exactly one x in the domain.

Simply put:

$$x_1 \neq x_2 \Longrightarrow f(x_1) \neq f(x_2) \quad \text{OR}$$

$$f(x_1) = f(x_2) \Longrightarrow x_1 = x_2.$$

7 5pts $f(x) = \frac{14}{x} \Longrightarrow f^{-1}(2)$ is found by solving $\frac{14}{x} = 2$ for x:

$$\Longrightarrow 14 = 2x$$

$$\Longrightarrow \boxed{7 = x = f^{-1}(2)}$$

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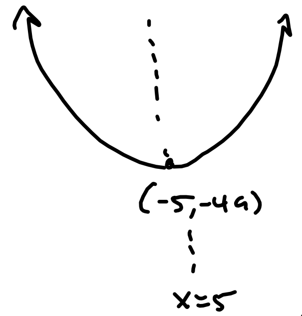
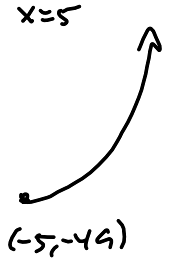
8) $f(x) = x^2+10x-24$. We restrict $\mathcal{D}(f)$ to $[-5,\infty)$.

a) 5pts The restriction to $[-5,\infty)$ makes $f(x)$ 1-to-1.

$$x^2+10x-24 = x^2+10x+5^2-25-24 = (x+5)^2-49$$



If you eliminate the left half of the graph, you eliminate the duplicate y-values to the left of the vertex @ $x=-5, y=-49$



A horizontal line crosses the graph at at most 1 point.

b) 5pts We find a "duplicate" y-value for distinct x-values.

$$f(x) = x^2+10x-24 = 0 \longrightarrow$$

$$(x+10)^2 = 49 \longrightarrow$$

$$x = -10 \pm \sqrt{49} = -10 \pm 7 \quad \begin{cases} -10+7 = -3 \\ -10-7 = -17 \end{cases}$$

Since $f(-3) = f(-7)$, f is not 1-to-1.

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c) 5pts we find $f^{-1}(x)$ & sketch it, $y=x$ & $y=f(x)$ on the same graph.

METHOD 1 complete the square

$y^2+10y-24=x$

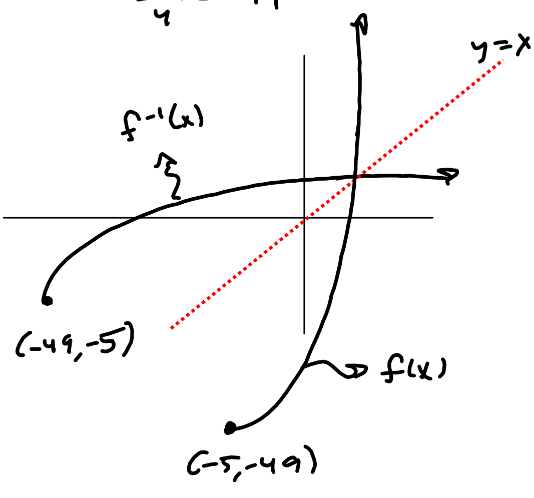
$y^2+10y+5^2-25-24=x$

$(y+5)^2=x+49$

$y=-5\pm\sqrt{x+49}$

$\Rightarrow y=\boxed{-5+\sqrt{x+49}=f^{-1}(x)}$

$\frac{196}{4}=49$



METHOD 2 Quadratic formula

$y^2+10y-24=x$

$y^2+10y-24-x=0$

$a=1,\ b=10,\ c=-24-x$

$b^2-4ac=10^2-4(1)(-24-x)$

$=100+4(x+24)=100+4x+96$

$=4x+196$

$y=\frac{-10\pm\sqrt{4x+196}}{2(1)}$

$=\frac{-10\pm\sqrt{4(x+49)}}{2}$

$=\frac{-10\pm2\sqrt{x+49}}{2}$

$=-5\pm\sqrt{x+49}$

$\Rightarrow \boxed{f^{-1}(x)=-5+\sqrt{x+49}}$

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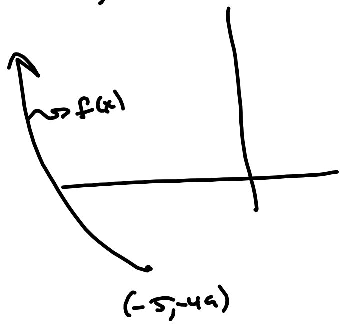
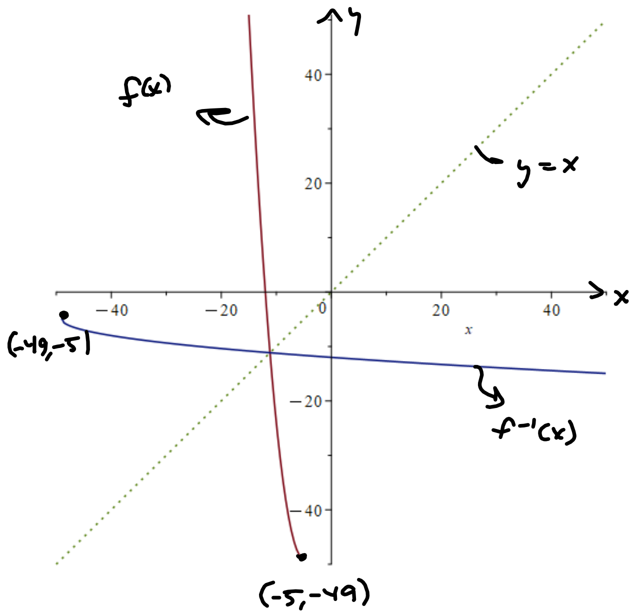
Bonus Sol's If you restrict to the domain $(-\infty, -5]$ you get this



f^{-1} for this restriction would be

$f^{-1}(x) = -5 - \sqrt{x+49}$

It's the "2nd solution."



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9) $f(x) = \dfrac{x-1}{x-4}$

a) 5pts We find $f^{-1}(x)$:

$\dfrac{y-1}{y-4} = x \longrightarrow$

$y-1 = x(y-4) = xy - 4x \longrightarrow$

$y - xy = -4x+1 \longrightarrow$

$y(1-x) = -4x+1 \longrightarrow$

$y = \underset{\text{ok}}{\dfrac{-4x+1}{1-x}} = \boxed{\underset{\text{pretty}}{\dfrac{4x-1}{x-1}} = f^{-1}(x)}$

b) 5pts $\mathcal{D}(f) = \{x \mid x \neq 4\} = \boxed{\mathbb{R} \setminus \{4\} = \mathcal{D}(f)}$

$\longrightarrow \boxed{\mathcal{R}(f^{-1}) = \mathbb{R} \setminus \{4\}}$

$\mathcal{D}(f^{-1}) = \{x \mid x \neq 1\} = \boxed{\mathcal{D}(f^{-1}) = \mathbb{R} \setminus \{1\}}$

$\longrightarrow \boxed{\mathcal{R}(f) = \mathbb{R} \setminus \{1\}}$