This one is rather lengthy. Lots of bonus.

Don't be afraid to be a bit exacting and ding them for small stuff.

All done reasonably well - 70 pts plus whatever bonus earned.

Mostly done - 50 points plus whatever bonus earned

Less than half done but more than one-fourth done - 30 points plus any bonus.

There's a detailed breakdown I'd like you to follow as a formative assessment., but the summative assessment should be pretty easy to determine at a glance.

Finally, lack of context is 0.5 points off the top of each 5-pointer. After this, lacking context will be 1 full point.

1340 WEEK 7 SOL'NS MILLS

Dock 0.5 points from every 5 points' worth that doesn't have context.

① 5pts Find the following, based on the figure at the right.

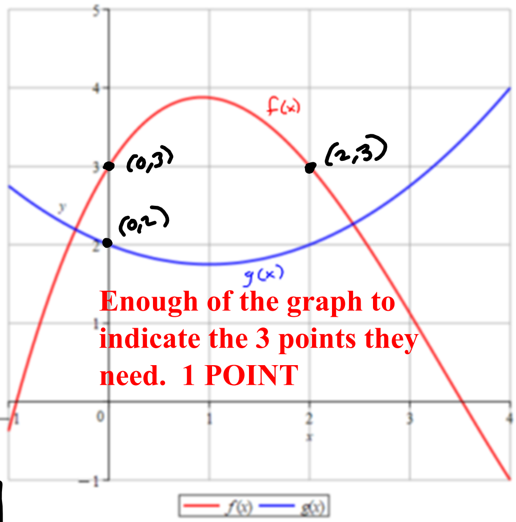
ⓐ $(f+g)(0) = f(0)+g(0) = 3+2 = \boxed{5}$ 1pt

ⓑ $(f-g)(0) = f(0)-g(0) = 3-2 = \boxed{1}$ 1pt

ⓒ $(fg)(0) = f(0)g(0) = 3(2) = \boxed{6}$ 1pt

ⓓ $\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)} = \boxed{\frac{3}{2}}$ 1pt

ⓔ $(f\circ g)(0) = f(g(0)) = f(2) = \boxed{3}$ 1pt



Enough of the graph to indicate the 3 points they need. 1 POINT

② 5pts Domain of $f\circ g = \mathcal{D}(f\circ g)$

$= \boxed{\{x \mid x\in\mathcal{D}(g) \text{ and } g(x)\in\mathcal{D}(f)\}}$ 5pts or precise words to that effect.

③ We find the domain of the following:

Context .5pts on all. Next one still be 1pt

ⓐ 5pts $f(x) = \sqrt{x} + \sqrt{49-x^2} \longrightarrow$

Need $\boxed{x\geq 0}$ 1pt and $49-x^2\geq 0$ 1pt $\longrightarrow$

$\Rightarrow x\in[0,\infty)$

$(7-x)(7+x)\geq 0$

$\xleftarrow{\quad - \quad}_{-7} \overset{+}{\quad} {}_{7} \xrightarrow{\quad - \quad}$ Sign Pattern 1pt

$-7 = 0 \qquad 7 = 0$

$\Rightarrow x\in[-7,7]$

1pt Interpret the 'AND' correctly Show the analysis

Need: NO | NO | Yes | Yes | NO — at -7, 0, 7 — AND

$= \boxed{[0,7] = \mathcal{D}(f)}$ 1pt

1340 MILLS

(3b) (5pts) $f(x) = \frac{\sqrt{x}}{\sqrt{49-x^2}}$ This is like (3a), except

$\sqrt{49-x^2}$ is in the denominator.

This means $49-x^2 \geq 0$ for "$\sqrt{\ }$" as before and also

$\sqrt{49-x^2} \neq 0$ (1pt) for being in the denominator.

This means we have to get rid of $x = \pm 7$ in (3a).

(3a) $\mathcal{D} = [0,7]$.

(3b) $\boxed{\mathcal{D} = [0,7)}$ (4pts)

(4) we find $f+g$, fg, f/g, and $f \circ g$ for $f(x) = \sqrt{x}$ & $g(x) = x^2 - 3x + 2$. $(\mathcal{D}(f) = [0,\infty), \mathcal{D}(g) = \mathbb{R})$

Content .5 each, but one statement like mine for all is OK

(a) (5pts) $\boxed{(f+g)(x) = \sqrt{x} + x^2 - 3x + 2}$ (3pts) $\boxed{\mathcal{D}(f+g) = [0,\infty)}$ (2pts) from $x \geq 0$ in $\sqrt{x}$.

(b) (5pts) $\boxed{(fg)(x) = \sqrt{x}(x^2 - 3x + 2)}$ (3pts) $\boxed{\mathcal{D}(fg) = [0,\infty)}$ (2pts)

1340 MILLS

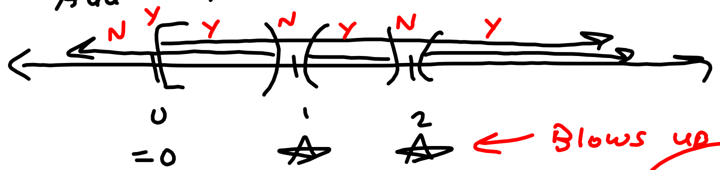
(c) (5pts) $\boxed{\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{x^2-3x+2}}$ 1pt Need $x \geq 0$ AND $x^2-3x+2 \neq 0$

$x^2-3x+2 \neq 0$

$(x-2)(x-1) \neq 0$

$x \neq 2$ AND $x \neq 1$

Add this condition to $x \geq 0$



AND

2pts Sign pattern & analysis

← Blows up @ $x = 1, 2$,

$$\boxed{\mathcal{D}\left(\frac{f}{g}\right) = [0,1) \cup (1,2) \cup (2,\infty)}$$

Final ans 2pts

(d) (5pts) $(f \circ g)(x) = f(g(x)) = \sqrt{g(x)} = \boxed{\sqrt{x^2-3x+2} = f \circ g}$

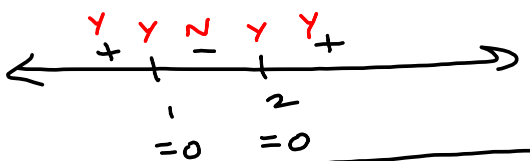
$\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$= \{x \mid x \in \mathbb{R} \text{ and } x^2-3x+2 \in \mathcal{D}(g)\}$

$= \{x \mid x \in \mathbb{R} \text{ and } x^2-3x+2 \geq 0\}$ ← $\{x \mid x^2-3x+2 \geq 0\}$ 2pts or equivalent.

i.e., we need $x^2-3x+2 \geq 0$

→ $(x-1)(x-2) \geq 0$



2pts Sign pattern

→ $\boxed{\mathcal{D}(f \circ g) = (-\infty, 1] \cup [2,\infty)}$ 1pt

1340 MILLS

5) 5pts If $f(x)=\sqrt{x}$, $g(x)=\frac{x}{x-4}$, and $h(x)=\sqrt[6]{x}$, then

$(f\circ g\circ h)(x) = f(g(h(x))) = f(g(\sqrt[6]{x})) = f\left(\frac{\sqrt[6]{x}}{\sqrt[6]{x}-4}\right) =$

$$\boxed{\sqrt{\frac{\sqrt[6]{x}}{\sqrt[6]{x}-4}}}$$ 4pts

Special: Content 1pt (full point)

Bonus 5pts

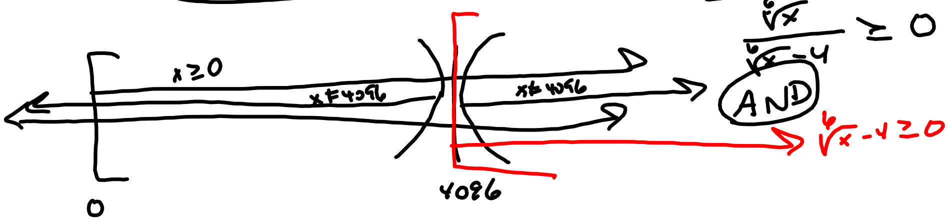
Need $\boxed{x \geq 0}$ for $\sqrt[6]{x}$. AND

Need $\sqrt[6]{x} \neq 4$ for $\frac{\sqrt[6]{x}}{\sqrt[6]{x}-4}$

$\longrightarrow \sqrt[6]{x} \neq 4 \longrightarrow$

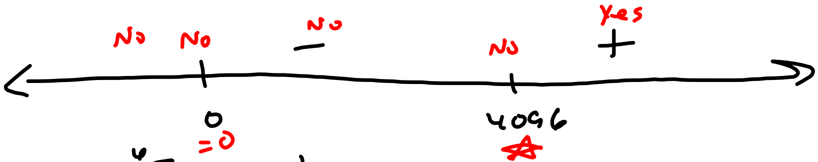
$x \neq 4^6 = \boxed{4096 \neq x}$ AND Need $\boxed{\sqrt[6]{x}-4 \geq 0 \text{ to keep}}$

$\frac{\sqrt[6]{x}}{\sqrt[6]{x}-4} \geq 0$



$\frac{\sqrt[6]{x}}{\sqrt[6]{x}-4} \geq 0^{*}$ $x=0$, $x=4096$ are key points

* to handle $\sqrt{\ }$ around everything



Sign Pattern – 2pts
Analysis – 2pts
write answer – 1pt

Test: $x=1$ $\frac{\sqrt[6]{1}}{\sqrt[6]{1}-4} = \frac{1}{-3}$ NO

$x=4097$ $\frac{\sqrt[6]{4097}}{\sqrt[6]{4097}-4} = \frac{+}{+} = +$ Yes. So

$$\boxed{\mathcal{D}(f\circ g\circ h) = (4096, \infty)}$$

($x=4096$ makes denominator zero.)

1340 MILLS

6 5pts A function is a rule that assigns to each $x \in$ a set called the "domain" to exactly one y in another set, called the "range." 2pts

A 1-to-1 function is a function such that each y in the range is assigned to exactly one x in the domain. 3pts

Simply put:

$x_1, x_2 \in D \Rightarrow x_1 \neq x_2 \longrightarrow f(x_1) \neq f(x_2)$ OR

Equivalent to $f(x_1) = f(x_2) \longrightarrow x_1 = x_2$. Alternate way of saying it.

7 5pts $f(x) = \frac{14}{x} \Rightarrow f^{-1}(2)$ is found by solving $\frac{14}{x} = 2$ for x:

Context - 2pts

$\longrightarrow 14 = 2x$

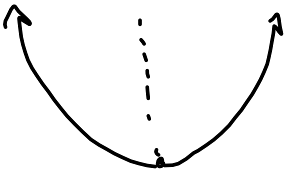
$\longrightarrow \boxed{7 = x = f^{-1}(2)}$ 3pts

1340 MILLS

8 $\boxed{f(x) = x^2+10x-24}$ 1pt. We restrict $\mathcal{D}(f)$ to $[5,\infty)$.

a) 5pts The restriction to $[5,\infty)$ makes $f(x)$ 1-to-1.

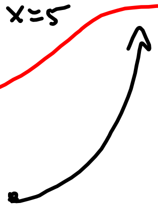
$$x^2+10x-24 = x^2+10x+5^2-25-24 = (x+5)^2-49$$ 1pt



(-5,-49)

x=5

If you eliminate the left half of the graph, you eliminate the duplicate y-values to the left of the vertex @ $x=-5, y=-49$



(-5,-49)

A horizontal line crosses the graph at at most 1 point.

3pts

b) 5pts We find a "duplicate" y-value for distinct x-values. FREE 5pts

$$f(x) = x^2+10x-24 = 0 \Longrightarrow$$ OR any number $y > -49$.

$$(x+10)^2 = 49 \Longrightarrow$$

$$x = -10 \pm \sqrt{49} = -10 \pm 7 \quad \begin{matrix} -10+7 = -3 \\ -10-7 = -17 \end{matrix}$$

Many ways to do this.

Since $f(-3) = f(-7)$, f is not 1-to-1.

As long as they show 2 y-values for one x-value, they're good.

They could also try to find *f*-inverse and show that it has two solutions, and isn't a function. If their argument is sound, full credit.

$$x^2+10x-24 = y$$

$$= x^2+10x+5^2-25-24 = y$$

$$= (x+5)^2 = y+49$$

$$x = -5 \pm \sqrt{y+49}$$

Any $y > -49$ corresponds to two x-values in the domain

Everybody gets 5pts for part b

1340 MILLS

(c) 5pts We find $f^{-1}(x)$ & sketch it, $y=x$ & $y=f(x)$ on the same graph.

METHOD 1 complete the square

$y^2+10y-24 = x$

$y^2+10y+5^2-25-24=x$

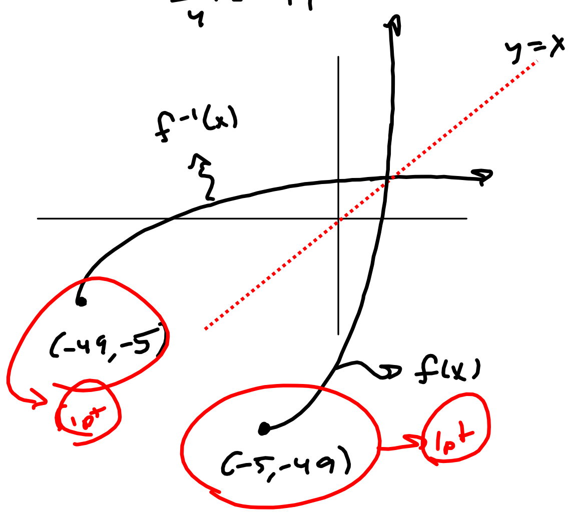
$(y+5)^2 = x+49$

$y=-5\pm\sqrt{x+49}$

2pts

$\rightarrow y=\boxed{-5+\sqrt{x+49}=f^{-1}(x)}$

$\frac{196}{4}=49$



METHOD 2 Quadratic formula

$y^2+10y-24=x$

$y^2+10y-24-x=0$

$a=1,\ b=10,\ c=-24-x$

$b^2-4ac=10^2-4(1)(-24-x)$

$=100+4(x+24)=100+4x+96$

$=4x+196$

$y=\frac{-10\pm\sqrt{4x+196}}{2(1)}$

$=\frac{-10\pm\sqrt{4(x+49)}}{2}$

$=\frac{-10\pm2\sqrt{x+49}}{2}$

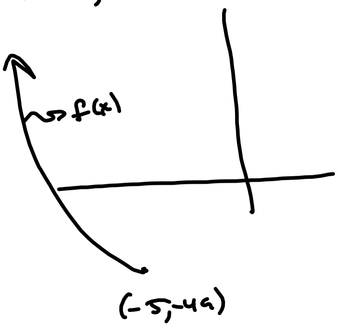
$=-5\pm\sqrt{x+49}$

$\rightarrow \boxed{f^{-1}(x)=-5+\sqrt{x+49}}$

Shape — 1pt

1340 MILLS

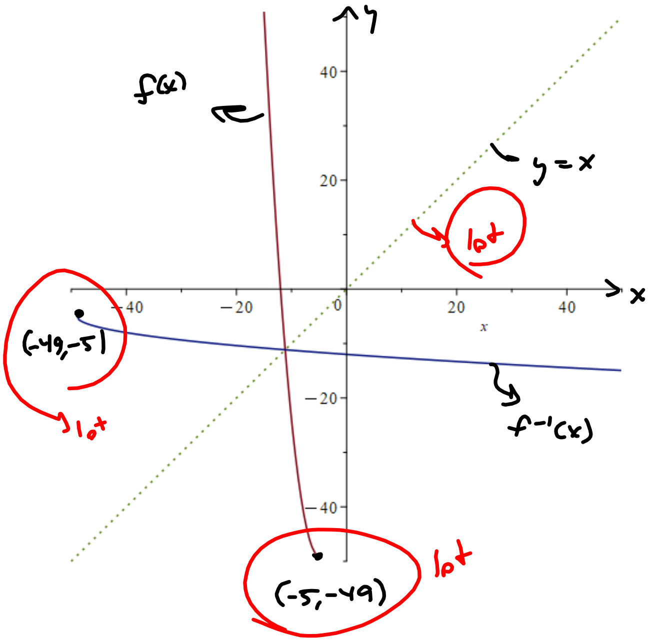
Bonus 5pts If you restrict to the domain $(-\infty, -5]$ you get this. We graph this $f(x)$, $f^{-1}(x)$ and $y=x$ for this one.



f^{-1} for this restriction would be

$f^{-1}(x) = -5 - \sqrt{x+49}$

It's the "2nd solution."



2pts - shape

1340 MILLS

9) $f(x) = \frac{x-1}{x-4}$

a) 5pts We find $f^{-1}(x)$:

$\frac{y-1}{y-4} = x \longrightarrow$

$y-1 = x(y-4) = xy - 4x \longrightarrow$

$y - xy = -4x+1 \longrightarrow$

$y(1-x) = -4x+1 \longrightarrow$

This or equivalent work 3pts

$y = \frac{-4x+1}{1-x} = \boxed{\frac{4x-1}{x-1} = f^{-1}(x)}$ 2pts

ok pretty

b) 5pts $\mathcal{D}(f) = \{x \mid x \neq 4\} = \boxed{\mathbb{R} \setminus \{4\} = \mathcal{D}(f)}$ 2pts

$\longrightarrow \boxed{\mathcal{R}(f^{-1}) = \mathbb{R} \setminus \{4\}}$ 1pt

$\mathcal{D}(f^{-1}) = \{x \mid x \neq 1\} = \boxed{\mathcal{D}(f^{-1}) = \mathbb{R} \setminus \{1\}}$ 1pt

$\longrightarrow \boxed{\mathcal{R}(f) = \mathbb{R} \setminus \{1\}}$ 1pt