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WEEK 6 WRITTEN
SOLUTIONS

H. MILLS

(1) Let $s(x) = x^2 - 5x$.

(a) (5 pts) The net change in s from $x=1$ to $x=5$ is

$$s(5) - s(1) = 5^2 - 5(5) - (1^2 - 5(1)) = -1 + 5 = \boxed{4 = s(5) - s(1)}$$

(b) (5 pts) The average rate of change from $x=1$ to $x=5$ is

$$\boxed{\frac{s(5) - s(1)}{5 - 1} = \frac{4}{4} = 1}$$

(c) (5 pts) We simplify the difference quotient

$$\frac{s(a+h) - s(a)}{h} = \frac{(a+h)^2 - 5(a+h) - (a^2 - 5a)}{h}$$

$$= \frac{a^2 + 2ah + h^2 - 5a - 5h - a^2 + 5a}{h}$$

$$= \frac{2ah + h^2 - 5h}{h} = \frac{h(2a + h - 5)}{h} = \boxed{2a + h - 5} \quad (h \neq 0)$$

= Difference quotient.

(2) (5 pts) $f(x) = \frac{1}{x-3} \rightarrow$

$$\frac{f(a+h) - f(a)}{h} = \frac{\frac{1}{a+h-3} - \frac{1}{a-3}}{h} = \frac{\left(\frac{1}{a+h-3}\right)\left(\frac{a-3}{a-3}\right) - \left(\frac{1}{a-3}\right)\left(\frac{a+h-3}{a+h-3}\right)}{h}$$

$$= \frac{a-3 - (a+h-3)}{(a+h-3)(a-3)} = \frac{1}{h} \left[\frac{a-3 - a - h + 3}{(a+h-3)(a-3)} \right]$$

$$= \frac{1}{h} \left[\frac{-h}{(a+h-3)(a-3)} \right] = \boxed{\frac{-1}{(a+h-3)(a-3)}} \quad (h \neq 0)$$

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MILLS

3) (5 pts) The table gives the population P in a small town from 2002-2020. Figures are for January 1st each year

Year	2002	2004	2006	2008	2010	2012	2014	2016
Pop	320	365	435	486	597	637	688	531

Year	2018	2020
Pop	492	463

The average rate of change from 2010-2014 is

$$\frac{P(2014) - P(2010)}{2014 - 2010} = \frac{6288 - 5971}{4} = \boxed{\frac{417}{4}}$$

4) (5 pts) The avg. rate of change for $f(x) = 2\sqrt{x}$ between

$x = a+h$ & $x = a$ is

$$\frac{f(a+h) - f(a)}{h} = \frac{2\sqrt{a+h} - 2\sqrt{a}}{h}$$

$$= \left(\frac{2\sqrt{a+h} - 2\sqrt{a}}{h} \right) \left(\frac{2\sqrt{a+h} + 2\sqrt{a}}{2\sqrt{a+h} + 2\sqrt{a}} \right) = \frac{4(a+h) - 4a}{h(2\sqrt{a+h} + 2\sqrt{a})}$$

$$= \frac{4a + 4h - 4a}{h(2\sqrt{a+h} + 2\sqrt{a})} = \frac{4h}{h(2\sqrt{a+h} + 2\sqrt{a})} = \frac{4}{2\sqrt{a+h} + 2\sqrt{a}}$$

$$= \frac{4}{2(\sqrt{a+h} + \sqrt{a})} = \boxed{\frac{2}{\sqrt{a+h} + \sqrt{a}}}$$

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CONTEXT - .5

(5) A pond is filling with H_2O at a rate of $10 \frac{\text{gallons}}{\text{min}}$.
Initially, there are 500 gal of H_2O in the pond, MILLS or words to this effect.

(2) (5pts) We may $V =$ volume of H_2O in the pond
(in gallons) as a function of
 $t =$ time (in minutes)

$$V = \text{initial} + (\text{rate of filling})(\text{time})$$

$$= 500 + 10t$$

(b) (5pts) If the pond holds 2000 gallons, how long will it take to fill it?

$$\text{We solve } V(t) = 2000 \rightarrow$$

$$500 + 10t = 2000 \rightarrow$$

$$10t = 1500 \rightarrow$$

$$t = 150 \text{ minutes}$$

(d) (5pts) Assume you stop when the pond is full.
What's the domain of $V(t)$?

$$D(V) = [0, 150]$$

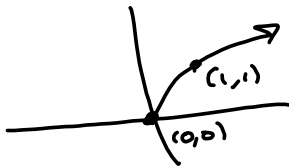
$$R(V) = [500, 2000]$$

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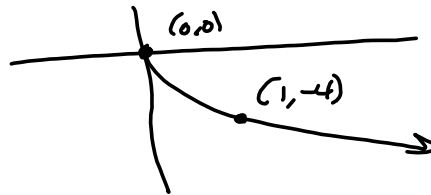
MILLS

We graph $g(x) = -4\sqrt{-5x-20} + 64$
 by transforming the graph of $f(x) = \sqrt{x}$:

① $f(x) = \sqrt{x}$

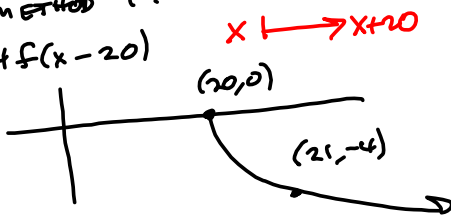


① $-4f(x) = -4\sqrt{x}$ $y \mapsto -4y$

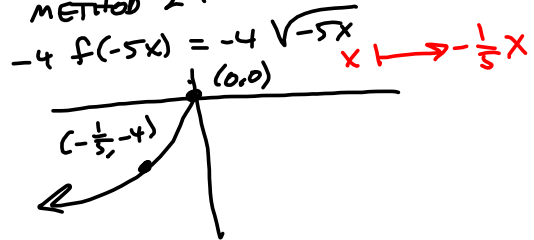


Scratch: $g(x) = -4\sqrt{-5(x+4)} + 64$

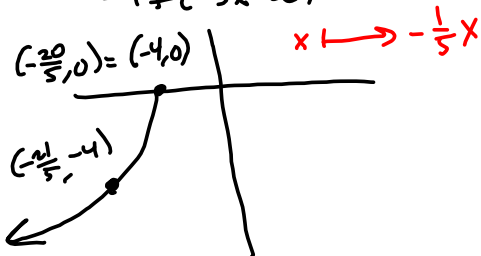
② METHOD 1:
 $-4f(x-20)$



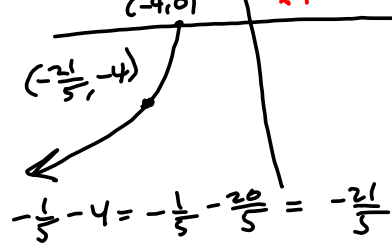
② METHOD 2:
 $-4f(-5x) = -4\sqrt{-5x}$



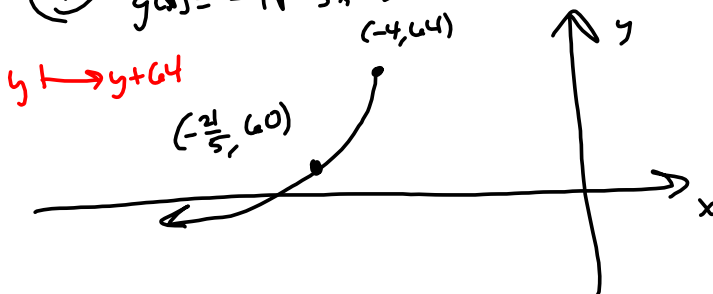
③ METHOD 1
 $-4f(-5x-20)$



③ METHOD 2
 $-4f(-5(x+4)) = -4\sqrt{-5(x+4)}$



④ $g(x) = -4\sqrt{-5x-20} + 64 = -4\sqrt{-5(x+4)} + 64$



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mills

7) Spts Bonus $g(x)$ has no y -int.

$$g(x) = 0 \Rightarrow$$

$$-4\sqrt{-5x-20} + 64 = 0$$

$$\Rightarrow -4\sqrt{-5x-20} = -64$$

$$\Rightarrow \sqrt{-5x-20} = \frac{-64}{-4} = 16$$

$$\Rightarrow -5x-20 = 16^2 = 256$$

$$\Rightarrow -5x = 276$$

$$\Rightarrow x = \frac{276}{-5} \Rightarrow \left(-\frac{276}{5}, 0 \right) \text{ is } x\text{-int}$$