1340 WEEK 6 WRITTEN SOLUTIONS H. MILLS

(1) Let $s(x) = x^2 - 5x$. → write 1pt

(a) 5pts The net change in s from $x=1$ to $x=5$ is — Context 1pt

$$s(5)-s(1) = 5^2-5(5)-(1^2-5(1)) = -1+5 = \boxed{4 = s(5)-s(1)}$$

→ 1pt → 2pts

(b) 5pts The average rate of change from $x=1$ to $x=5$ is — Context 1pt

$$\boxed{\frac{s(5)-s(1)}{5-1} = \frac{4}{4} = 1}$$

→ 1pt → 3pts

(c) 5pts We simplify the difference quotient — Context - 1pt

$$\frac{s(a+h)-s(a)}{h} = \frac{(a+h)^2-5(a+h)-(a^2-5a)}{h}$$

1pt 1pt

$$= \frac{a^2+2ah+h^2-5a-5h-a^2+5a}{h}$$

$$= \frac{2ah+h^2-5h}{h} = \frac{h(2a+h-5)}{h} = \boxed{2a+h-5} \quad (h \neq 0)$$

2pts → 1 Bonus

= Difference quotient.

(2) 5pts $f(x) = \dfrac{1}{x-3}$ → 1pt

$$\frac{f(a+h)-f(a)}{h} = \frac{\frac{1}{a+h-3} - \frac{1}{a-3}}{h} = \frac{\left(\frac{1}{a+h-3}\right)\left(\frac{a-3}{a-3}\right) - \left(\frac{1}{a-3}\right)\left(\frac{a+h-3}{a+h-3}\right)}{h}$$

→ 1pt, 1pt

$$= \frac{\frac{a-3-(a+h-3)}{(a+h-3)(a-3)}}{h} = \frac{1}{h}\left[\frac{a-3-a-h+3}{(a+h-3)(a-3)}\right]$$

1pt

$$= \frac{1}{h}\left[\frac{-h}{(a+h-3)(a-3)}\right] = \boxed{\frac{-1}{(a+h-3)(a-3)}} \quad (h \neq 0)$$

1 Bonus

→ 1pt

1340 MILLS

(3) (5pts) The table gives the population P in a small town from 2002-2020. Figures are for January 1st each year

Year	2002	2004	2006	2008	2010	2012	2014	2016	2018	2020
Pop	3220	3645	4357	4869	5871	6375	6288	5318	4921	4636

At least enough of the table to cover 2010 & 2014

Yr	2010	2014
Pop	5871	6288

2pts context

The average rate of change from 2010-2014 is

$$\frac{P(2014)-P(2010)}{2014-2010} = \frac{6288-5871}{4} = \boxed{\frac{417}{4}}$$

3pts as long as it's supported

(4) (5pts) The avg. rate of change for $f(x)=2\sqrt{x}$ between $x=a+h$ & $x=a$ is

Context - 1pt

$$\frac{f(a+h)-f(a)}{h} = \frac{2\sqrt{a+h}-2\sqrt{a}}{h}$$ 1pt

$$= \boxed{\left(\frac{2\sqrt{a+h}-2\sqrt{a}}{h}\right)\left(\frac{2\sqrt{a+h}+2\sqrt{a}}{2\sqrt{a+h}-2\sqrt{a}}\right)} = \frac{4(a+h)-4a}{h(2\sqrt{a+h}+2\sqrt{a})}$$ 1pt

$$= \frac{4a+4h-4a}{h(2\sqrt{a+h}+2\sqrt{a})} = \frac{4h}{h(2\sqrt{a+h}+2\sqrt{a})} = \frac{4}{2\sqrt{a+h}+2\sqrt{a}}$$ 1pt

$$= \frac{4}{2(\sqrt{a+h}+\sqrt{a})} = \boxed{\frac{2}{\sqrt{a+h}+\sqrt{a}}}$$ 1pt

1340 CONTEXT - .5 MILLS

(5) A pond is filling with H_2O at a rate of $10 \frac{\text{gallons}}{\text{min}}$. or words to this effect.
Initially, there are 500 gl of H_2O in the pond.

(a) 5pts We model V = volume of H_2O in the pond (in gallons) as a function of t = time (in minutes)

Lexicon: words & units for variables 1pt

$V = \text{initial} + \begin{pmatrix}\text{rate of}\\ \text{filling}\end{pmatrix}(\text{time})$

$= \boxed{500 + 10t}$ 3.5pts

(b) 5pts If the pond holds 2000 gallons, how long will it take to fill it? Context .5pts

We solve $V(t) = 2000$

$500 + 10t = 2000$ 2pts

$10t = 1500$

$\boxed{t = 150 \text{ minutes}}$ 2.5pts

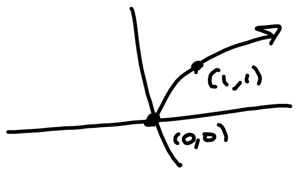
(d) 5pts Assume you stop when the pond is full. Context 1pt
what's the domain of $V(t)$?

$\boxed{\begin{array}{l} D(V) = [0, 150]. \\ R(V) = [500, 2000] \end{array}}$ 4pts

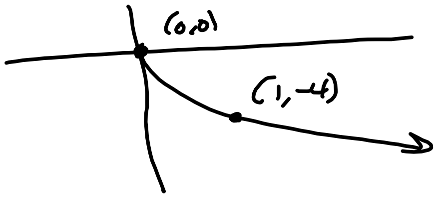
1340 MILLS

(6) 10 pts BONUS We graph $g(x) = -4\sqrt{-5x-20} + 64$ by transforming the graph of $f(x) = \sqrt{x}$:

(0) $f(x) = \sqrt{x}$



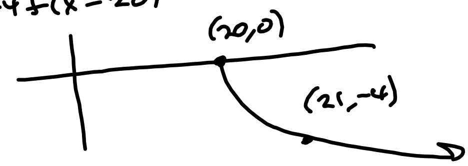
(1) $-4f(x) = -4\sqrt{x}$ $y \longmapsto -4y$



Scratch: $g(x) = -4\sqrt{-5(x+4)} + 64$

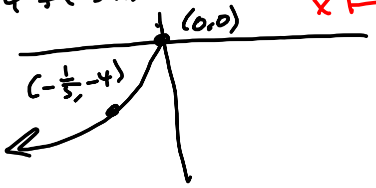
(2) METHOD 1:

$-4f(x-20)$ $x \longmapsto x+20$



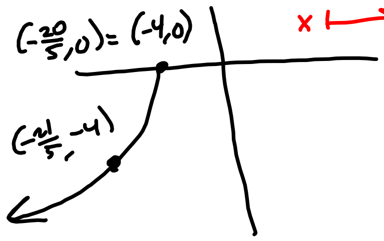
(2) METHOD 2:

$-4f(-5x) = -4\sqrt{-5x}$ $x \longmapsto -\frac{1}{5}x$



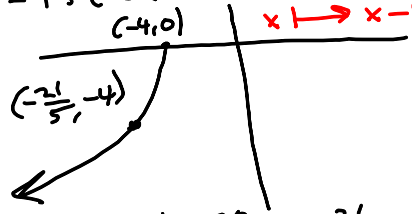
(3) METHOD 1

$-4f(-5x-20)$ $x \longmapsto -\frac{1}{5}x$



(3) METHOD 2

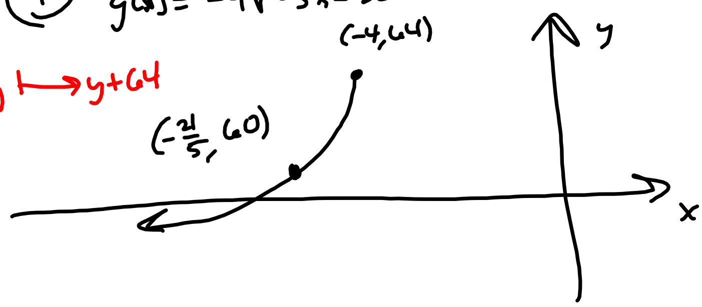
$-4f(-5(x+4)) = -4\sqrt{-5(x+4)}$ $x \longmapsto x-4$



$-\frac{1}{5} - 4 = -\frac{1}{5} - \frac{20}{5} = -\frac{21}{5}$

(4) $g(x) = -4\sqrt{-5x-20} + 64 = -4\sqrt{-5(x+4)} + 64$

$y \longmapsto y+64$



In general 2 pts per sketch. METHOD 1 OR METHOD 2 are fine. $\frac{1}{2}$-credit (5 pts) if they get the last graph and show the endpoint. Try to give credit for correct moves. This makes it tough to grade if they make early mistakes, but do other moves correctly.

7 5pts Bonus

$g(x)$ has no y-int.

$g(x) = 0 \Rightarrow$

$-4\sqrt{-5x-20} + 64 = 0$

$\Rightarrow -4\sqrt{-5x-20} = -64$

$\Rightarrow \sqrt{-5x-20} = \frac{-64}{-4} = 16$

$\Rightarrow -5x-20 = 16^2 = 256$

$\Rightarrow -5x = 276$

$\Rightarrow \boxed{x = \frac{276}{-5}} \Rightarrow \boxed{\left(-\frac{276}{5}, 0\right) \text{ is x-int}}$

Also an ok answer, but prefer the ordered pair.

The $x \mapsto x+20$, etc. are instructive notes for students looking at the work, later.

Not required on student papers.

TAKE OFF 3points if they call everything "$f(x)$," willy-nilly.

$f(x) = \sqrt{x}$. Everything else is a transformed $f(x)$, like $-4f(x)$, $-4f(-5x)$, $-4f(x-20)$, $f(-5(x+4))$, $-4f(-5x-20)$, $-4f(-5x-20)+64 = g(x)$

They can just write "$g(x)$" on last one

may.