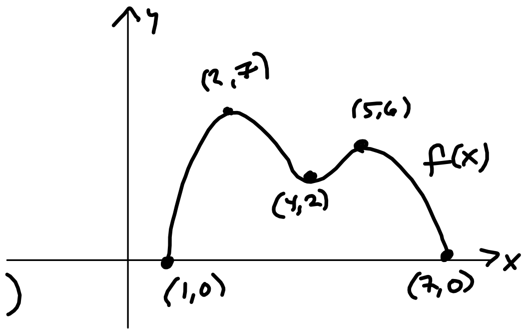
1340 WEEK 5 SOLUTIONS H. MILLS

1 (5pts) we refer to the figure on the right to answer the following:



a) $\mathcal{D}(f) = [1,7]$

b) $\mathcal{R}(f) = [0,7]$

c) f is increasing on $(1,2) \cup (4,5)$

d) f is decreasing on $(2,4) \cup (5,7)$

e) Local max @ $(2,7)$, $(5,6)$
" min " $(4,2)$.

Your eBook says:

"By convention we write the intervals on which a function is increasing or decreasing as open intervals. (It would also be true to say that the function is increasing or decreasing on the corresponding closed interval."

I say that if you follow the strict definition, the intervals should be closed intervals, and there's an overlap of one point between intervals of increase at the local max and local min points.

In Calculus I, they use the same definition, and the WebAssign is looking for closed intervals. In general, you're thinking of places where the slope is positive or negative, and you're really interested in the places where the slope is zero (at local max/min points).

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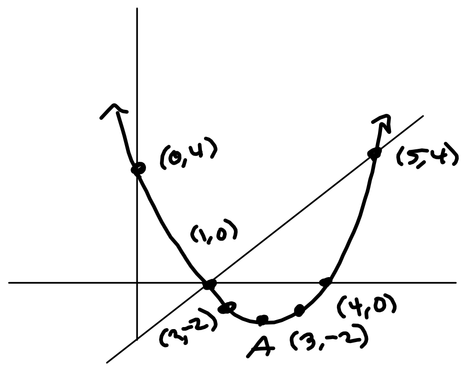
(2) Graphs of $f(x) = x^2 - 5x + 4$ and $g(x) = x - 1$ are shown. Solve $f(x) = g(x)$ in two ways



(a) (5 pts) Using the graph

$f(x) = g(x) \Longrightarrow x \in \{1, 5\}$

(b) (5pts) Algebraically:

$x^2 - 5x + 4 = x - 1$

$\Longrightarrow x^2 - 6x + 5 = 0$

$\Longrightarrow (x-5)(x-1) = 0$

$\Longrightarrow \boxed{x \in \{1, 5\}}$

EXTRA: A: $x^2 - 5x + 4$

$= x^2 - 5x + \left(\frac{5}{2}\right)^2 - \frac{25}{4} - \frac{16}{4}$

$= \left(x - \frac{5}{2}\right)^2 - \frac{41}{4}$

$A = (h, k) = \left(\frac{5}{2}, -\frac{41}{4}\right)$

(3) (5pts) Still using same figure,

(a) $f(2) = -2$ (I added this point, later, b/c #3 asked)

(b) $g(3) = 2$ (" " " " " " " ")

(c) The set of all x such that $f(x) = 4$ is $\{x \mid f(x) = 4\} = \{0, 5\}$

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(4) (5 pts) We sketch $f(x) = x^2 - 5x - 24$, $-4 \le x \le 9$

A couple ways to do this:

(1) Preferred: Complete the square for the vertex.

$$x^2-5x-24 = x^2-5x+\left(\frac{5}{2}\right)^2 - \frac{25}{4} - \frac{24}{1}\cdot\frac{4}{4}$$

$$= \left(x-\frac{5}{2}\right)^2 - \frac{25}{4} - \frac{96}{4}$$

$$= \left(x-\frac{5}{2}\right)^2 - \frac{121}{4} \longrightarrow (h,k) = \left(\frac{5}{2}, -\frac{121}{4}\right)$$

$f(x)=0 \longrightarrow$

$$\left(x-\frac{5}{2}\right)^2 = \frac{121}{4}$$

$$\Longrightarrow x - \frac{5}{2} = \pm\sqrt{\frac{121}{4}} = \pm\frac{11}{2}$$

$$\Longrightarrow x = \frac{5\pm 11}{2}$$

x-ints: $\frac{5+11}{2} = \frac{16}{2} = 8 \rightsquigarrow (8,0)$

$\frac{5-11}{2} = -\frac{6}{2} = -3 \rightsquigarrow (-3,0)$

y-int: $f(0) = -24 \rightsquigarrow (0,-24)$

(2)* Find the zeros. The vertex is $\frac{1}{2}$-way between, by symmetry about its axis

$$x^2-5x-24 = (x-8)(x+3) = 0 \Longrightarrow x \in \{-3, 8\}$$

$\boxed{\text{x-ints: } (-3,0), (8,0)}$

$\boxed{\text{y-int: } f(0) = -24 \rightsquigarrow (0,-24)}$

$$h = \frac{-3+8}{2} = \frac{5}{2}$$

$$k = f(h) = \left(\frac{5}{2}\right)^2 - 5\left(\frac{5}{2}\right) - 24$$

$$= \frac{25}{4} - \frac{25}{2}\cdot\frac{2}{2} - \frac{24}{1}\cdot\frac{4}{4}$$

$$= \frac{25-50-96}{4} = \frac{-25-96}{4} = -\frac{121}{4} = k$$

$$\Longrightarrow \text{Vertex} = (h,k) = \left(\frac{5}{2}, -\frac{121}{4}\right)$$

* Sledgehammer also works for the zeros.

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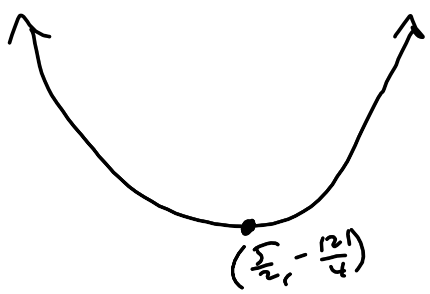
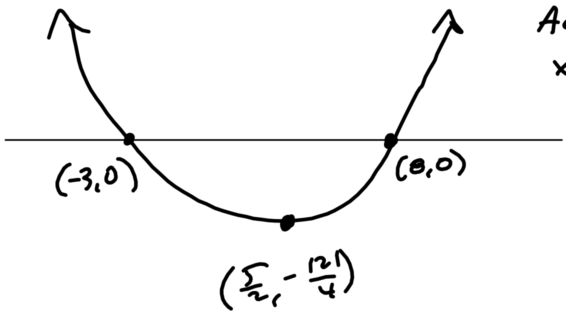
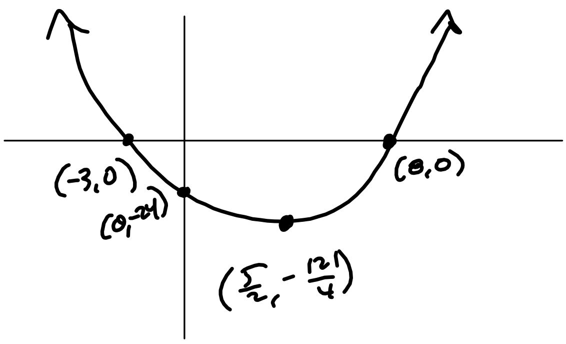
How I do the graphs, step by step, once I have all the pertinent information I need:



Draw the shape
Label vertex



Add x-axis &
x-ints



Put y-axis between
$x=-3$ & $x=\frac{5}{2}$
Label y-int

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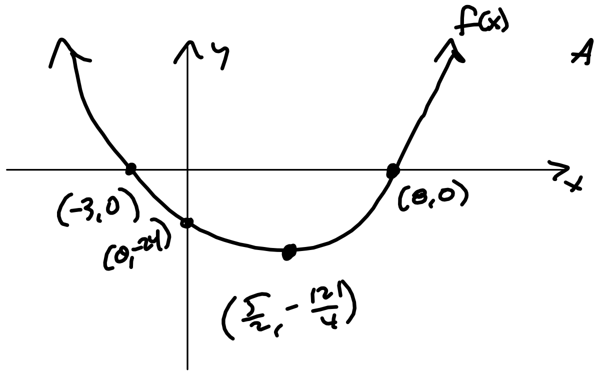

Add Final embellishments

$f(x) = x^2 - 5x - 24$

You can do all these steps on just one graph. I'm just trying to show you the best order in which to do the steps. Sometimes, it ends up pretty cramped, and I re-do it, once I'm sure where things go and how to position my labels for best clarity.

Notice how my graph is *relatively* or *qualitatively* correct, but there are no tickmarks and I'm not splitting hairs. Just getting the shape, general location of things, and precise labels. There's an art to making quick sketches that pass muster with the instructor....

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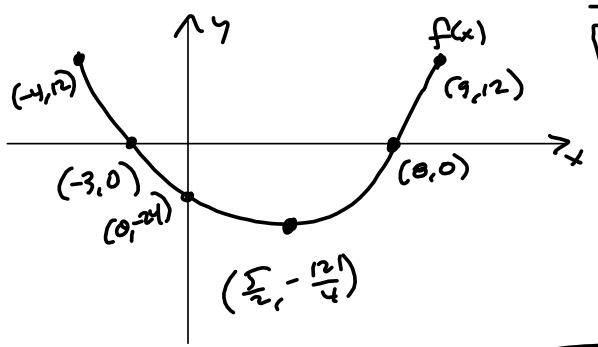
I was having so much fun, I left out part of the question. I left out the $-4 \le x \le 9$ part.

$$f(-4) = (-4)^2 - 5(-4) - 24 = 16 + 20 - 24 = 36 - 24 = 12 \rightarrow (-4, 12)$$

$$f(9) = 9^2 - 5(9) - 24 = 81 - 45 - 24 = 81 - 69 = 12 \rightarrow (9, 12)$$



$$\boxed{\begin{array}{l} \mathcal{D}(f) = [-4, 9] \\ \mathcal{R}(f) = [-\frac{121}{4}, 12] \end{array}}$$

(3) (5 pts) Let $g(x) = \sqrt{x^2 - 5x - 24}$. We find $\mathcal{D}(g)$:

$$\mathcal{D}(g) = \{x \mid g(x) \text{ is defined \& real}\}$$

$$= \{x \mid x^2 - 5x - 24 \ge 0\}.$$

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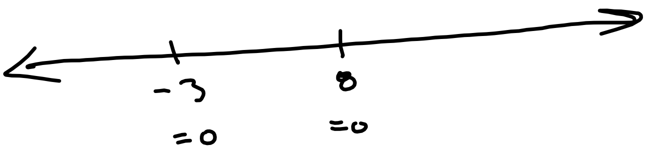
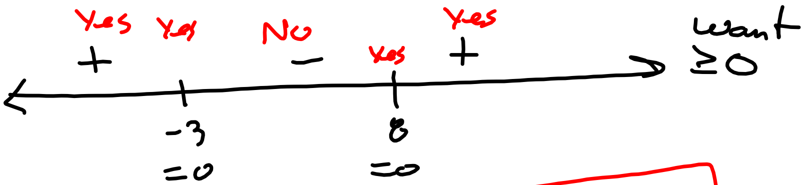
We solve $x^2-5x-24 \ge 0$

$\longrightarrow (x-8)(x+3) \ge 0$



Want ≥ 0

Interval	Test	$(x+3)(x-8)$
$(-\infty,-3)$	$x=-4$	$(-)(-) = +$
$(-3,8)$	$x=0$	$(3)(-8) = -$
$(8,\infty)$	$x=9$	$(12)(1) = +$



$\longrightarrow x \in \boxed{(-\infty,-3] \cup [8,\infty) = D(g)}$

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(6) Let $f(x) = x^2-5x-24$. Then

(a) (5pts) $f(x+2) = (x+2)^2-5(x+2)-24$

$$= x^2+4x+4-5x-10-24$$

$$= \boxed{x^2-x-30 = f(x+2)}$$

(b) (5pts) $f(x)+f(2) = x^2-5x-24-(2^2-5(2)-24)$

$$= x^2-5x-24-(4-10-24)$$

$$= x^2-5x-24-(-30)$$

$$= \boxed{x^2-5x+6} = f(x)+f(2)$$

The POINT is that $f(x+2) \neq f(x)+f(2)$!

(7) Let $s(x) = x^2-5x$. Then the

(a) (5pts) NET CHANGE in s from $x=1$ to $x=5$ is

$$f(5)-f(1) = 5^2-5(5)-(1^2-5(1))$$

$$= 25-25-(-4)$$

$$= \boxed{4 = f(5)-f(1)}$$

(b) (5pts) The AVERAGE RATE OF CHANGE from $x=1$ to $x=5$

is $\dfrac{f(5)-f(1)}{5-1} = \dfrac{4}{4} = 1$

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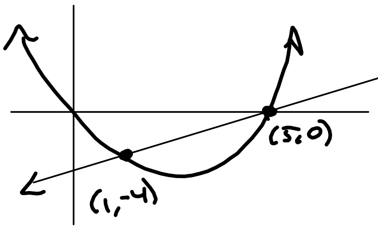
(c) 5pts $\dfrac{s(a+h)-s(a)}{h} = \dfrac{(a+h)^2-5(a+h)-(a^2-5a)}{h}$

$= \dfrac{a^2+2ah+h^2-5a-5h-a^2+5a}{h} = \dfrac{2ah+h^2-5h}{h}$

$= \dfrac{h(2a+h-5)}{h} = \boxed{2a+h-5}$ (as long as $h\neq 0$)

$= \dfrac{f(a+h)-f(a)}{h}$

Note: Part b picture:



Slope of this line is $\dfrac{f(5)-f(1)}{5-1}$

$(x_1,y_1)=(1,f(1))=(1,-4)$

$(x_2,y_2)=(5,f(5))=(5,0)$