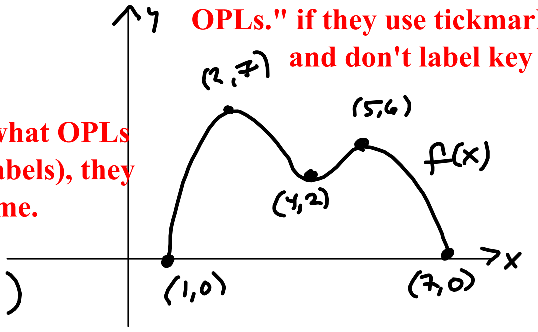
1340 WEEK 5 SOLUTIONS H. MILLS

"Please no tickmarks. Use OPLs." if they use tickmarks, and don't label key pts.

① (5 pts) we refer to the figure on the right to answer the following:



(a) $\mathcal{D}(f) = [1,7]$

"$\mathcal{D}$" means "domain."

If they don't know what OPLs are, (ordered-pair labels), they are welcome to ask me.

(b) $\mathcal{R}(f) = [0,7]$

"$\mathcal{R}$" means Range

(c) f is increasing on $(1,2)\cup(4,5)$

Closed or half-closed intervals are also OK.

(d) f is decreasing on $(2,4)\cup(5,7)$

(e) Local max @ $(2,7)$, $(5,6)$

" min " $(4,2)$.

Each of the above is worth 1 point, up to 4 points total. That, plus context/graph makes 5 pts.

The graph can be a copy-paste from the book or it can be a hand sketch with key points labeled. Context is 1 pt and it includes both some kind of opening statement, such as I did, referring them to the graph, AND the graph, itself, AND the key points labeled.

If they miss one or more, they can't get the full 4 points.

If they miss 4, they should still get 1 point for the one they got right. I hope that makes sense.

Your eBook says:

"By convention we write the intervals on which a function is increasing or decreasing as open intervals. (It would also be true to say that the function is increasing or decreasing on the corresponding closed interval.)"

I say that if you follow the strict definition, the intervals should be closed intervals, and there's an overlap of one point between intervals of increase at the local max and local min points. Some books require that to include a point as a point of increase, that there's a point to its right to compare it to within the interval. That makes it an open-interval proposition. I wish books would make up their minds, or make more precise definitions.

In Calculus I, they use the same definition, and the WebAssign is looking for closed intervals.

To their credit, the definition used doesn't belabor the point at the end points, but of course, the end points deserve better treatment.

I'm accepting either open or closed intervals for the intervals of increase or decrease.

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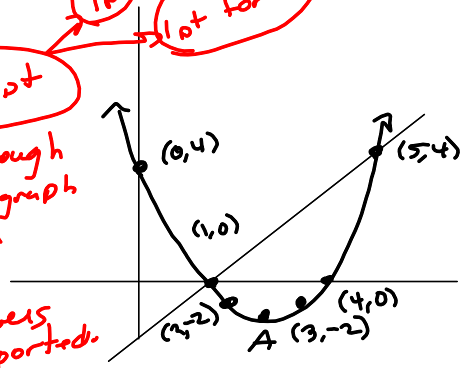
(2) Graphs of $f(x)=x^2-5x+4$ and $g(x)=x-1$ are shown. Solve $f(x)=g(x)$ in two ways

1pt for a

1pt for b *

Context 1pt

Need enough detail in graph to answer questions, OR else answers are unsupported.



(a) (5 pts) Using the graph

$f(x)=g(x) \Longrightarrow x \in \{1,5\}$

(b) (5pts) Algebraically:

$x^2-5x+4=x-1$

$\Longrightarrow x^2-6x+5=0$ 2pts

$\Longrightarrow (x-5)(x-1)=0$

$\Longrightarrow \boxed{x \in \{1,5\}}$ 2pts

Context * 1pt (May just write it once, as I did.)

EXTRA: A: x^2-5x+4

$= x^2-5x+\left(\frac{5}{2}\right)^2-\frac{25}{4}-\frac{16}{4}$

$= \left(x-\frac{5}{2}\right)^2-\frac{41}{4}$

$A=(h,k)=\boxed{\left(\frac{5}{2},-\frac{41}{4}\right)=A}$

Context – 1pt

(3) (5pts) Still using same figure,

1pt (a) $f(2)=-2$ (I added this point, later, b/c #3 asked)

1pt (b) $g(3)=2$ (" " " " " " " ")

(c) The set of all x such that $f(x)=4$ is $\underbrace{\{x \mid f(x)=4\}}_{\text{Teaching note. Not needed.}} = \{0,5\}$

1pt words

1pt final answer

Deduct 0.5pt if it's not a set.

If they only write $x=0,5$, that's not a set.

If they just write "-2" for a, without saying "$f(2)=$", that's $\frac{1}{2}$ pt, etc.

1340 Skip to page 6 for final product and points breakdown. MILLS

(4) (5pts) We sketch $f(x) = x^2 - 5x - 24$, $-4 \le x \le 9$

Skip to Page 5 for evaluation notes on the graph.

A couple ways to do this:

(1) Preferred: Complete the square for the vertex.

$$x^2 - 5x - 24 = x^2 - 5x + \left(\frac{5}{2}\right)^2 - \frac{25}{4} - \frac{24}{1}\cdot\frac{4}{4}$$

$$= \left(x - \frac{5}{2}\right)^2 - \frac{25}{4} - \frac{96}{4}$$

$$= \left(x - \frac{5}{2}\right)^2 - \frac{121}{4} \longrightarrow (h,k) = \left(\frac{5}{2}, -\frac{121}{4}\right)$$

x-ints: $f(x) = 0 \longrightarrow$

$$\left(x - \frac{5}{2}\right)^2 = \frac{121}{4}$$

$$\Longrightarrow x - \frac{5}{2} = \pm\sqrt{\frac{121}{4}} = \pm \frac{11}{2}$$

$$\Longrightarrow x = \frac{5 \pm 11}{2}$$

x-ints: $\frac{5+11}{2} = \frac{16}{2} = 8 \rightsquigarrow (8,0)$

$\frac{5-11}{2} = -\frac{6}{2} = -3 \rightsquigarrow (-3,0)$ x-ints

y-int: $f(0) = -24 \rightsquigarrow (0,-24)$

(2)* Find the zeros. The vertex is $\frac{1}{2}$-way between, by symmetry about its axis

$$x^2 - 5x - 24 = (x-8)(x+3) = 0 \Longrightarrow x \in \{-3, 8\}$$

$\boxed{\text{x-ints: } (-3,0), (8,0)}$

$\boxed{\text{y-int: } f(0) = -24 \rightsquigarrow (0,-24)}$

$$h = \frac{-3+8}{2} = \frac{5}{2}$$

$$k = f(h) = \left(\frac{5}{2}\right)^2 - 5\left(\frac{5}{2}\right) - 24$$

$$= \frac{25}{4} - \frac{25}{2}\cdot\frac{2}{2} - \frac{24}{1}\cdot\frac{4}{4}$$

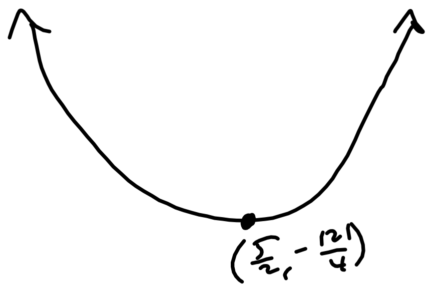
$$= \frac{25 - 50 - 96}{4} = \frac{-25-96}{4} = -\frac{121}{4} = k$$

$$\Longrightarrow \text{Vertex} = (h,k) = \left(\frac{5}{2}, -\frac{121}{4}\right)$$

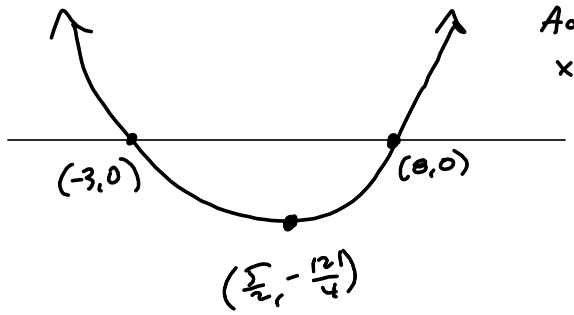
* Sledgehammer also works for the zeros.

1340 **Teacher stuff, to help develop the art of sketching graphs.** MILLS

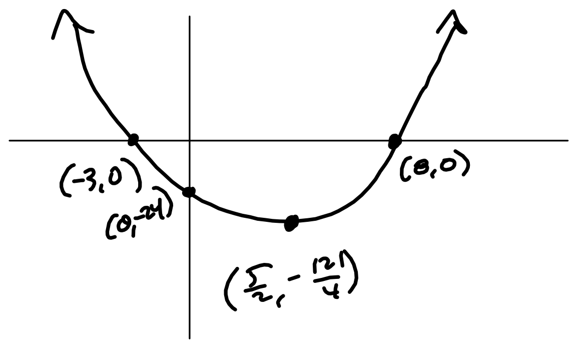
How I do the graphs, step by step, once I have all the pertinent information I need:



Draw the shape
Label vertex



Add x-axis &
x-ints



Put y-axis between
$x=-3$ & $x=\frac{5}{2}$
Label y-int

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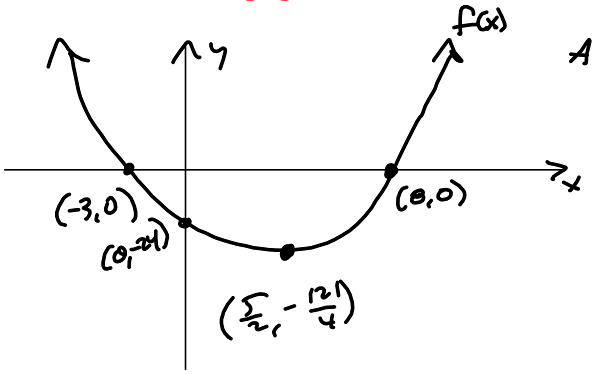
See next page, which includes the domain and range support.



Add Final embellishments

$f(x)=x^2-5x-24$

You can do all these steps on just one graph. I'm just trying to show you the best order in which to do the steps. Sometimes, it ends up pretty cramped, and I re-do it, once I'm sure where things go and how to position my labels for best clarity.

Notice how my graph is *relatively* or *qualitatively* correct, but there are no tickmarks and I'm not splitting hairs. Just getting the shape, general location of things, and precise labels. There's an art to making quick sketches that pass muster with the instructor....

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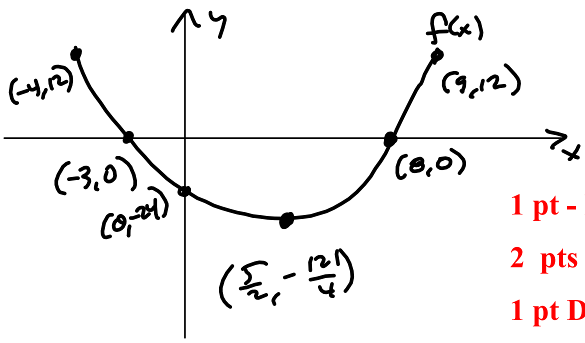
I was having so much fun, I left out part of the question. I left out the $-4 \le x \le 9$ part.

$$f(-4) = (-4)^2 - 5(-4) - 24 = 16 + 20 - 24 = 36 - 24 = 12 \to (-4, 12)$$

$$f(9) = 9^2 - 5(9) - 24 = 81 - 45 - 24 = 81 - 69 = 12 \to (9, 12)$$



$$\boxed{\begin{aligned} \mathcal{D}(f) &= [-4, 9] \\ \mathcal{R}(f) &= [-\tfrac{121}{4}, 12] \end{aligned}}$$

1 pt - Problem statement

2 pts - Vertex and the two endpoints at y = 12

1 pt Domain

1 pt Range

If they get the answer right, but they're not supporting their work, half credit.

BONUS Give them a point for getting the x-intercepts

BONUS Give them a point for the y-intercept.

Points must be given ordered-pair labels. Tickmarks just waste our time.

5 5pts Let $g(x) = \sqrt{x^2 - 5x - 24}$. We find $\mathcal{D}(g)$:

$$\mathcal{D}(g) = \{x \mid g(x) \text{ is defined \& real}\}$$

$$= \{x \mid x^2 - 5x - 24 \ge 0\}.$$

Teacher teaching.

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We need $x^2-5x-24 \geq 0$ 1pt

1pt Context of question.

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$\rightarrow (x-8)(x+3) \geq 0$

-3 8

$=0$ $=0$

Want ≥ 0

$=0$ is also good

Interval	Test	$(x+3)(x-8)$
$(-\infty,-3)$	$x=-4$	$(-)(-) = +$
$(-3, 8)$	$x=0$	$(3)(-8) = -$
$(8,\infty)$	$x=9$	$(12)(4) = +$

There are more intuitive ways to do a sign pattern, based on the controlling factors at the x-intercept or vertical asymptote.

It's important that the student show their decision-making process, by laying out a number line and arriving at a +/- pattern and then write the solution.

Yes Yes No Yes Yes

$+$ $-$ $+$

-3 8

$=0$ $=0$

want ≥ 0 $\rightarrow$ 1pt

$\rightarrow x \in \boxed{(-\infty,-3] \cup [8,\infty) = D(g)}$ 1pt

They need to make it clear that it's the domain. write "Domain" or use $\mathcal{D}$ or D to indicate it. Either one.

They may also write

Charge .5pt for not making it a set. $\leftarrow$ $x \leq 3$ OR $8 \leq x$ but I would prefer to see $\boxed{\{x \mid x \leq 3 \text{ OR } 8 \leq x\}}$

without putting it as a set or in Interval notation, it's not a domain. It's only conditions to be in the set. so charge .5pts.

Domain and range are sets.

The bar isn't the sign that says "21 or over."

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Context – 1 pt

(6) Let $f(x) = x^2 - 5x - 24$. Then

(a) (5pts) $f(x+2) = (x+2)^2 - 5(x+2) - 24$ 2pts

$= x^2 + 4x + 4 - 5x - 10 - 24$

$= \boxed{x^2 - x - 30 = f(x+2)}$ 3pts

(b) (5pts) $f(x) + f(2) = x^2 - 5x - 24 + (2^2 - 5(2) - 24)$ 2pts

$= x^2 - 5x - 24 + (4 - 10 - 24)$

$= x^2 - 5x - 24 + (-30)$

$= \boxed{x^2 - 5x - 54} = f(x) + f(2)$ 3pts

The POINT is that $f(x+2) \neq f(x) + f(2)$!

(7) Let $s(x) = x^2 - 5x$. Then the

(a) (5pts) NET CHANGE in s from $x=1$ to $x=5$ is 1pts context

2pts $f(5) - f(1) = 5^2 - 5(5) - (1^2 - 5(1))$

$= 25 - 25 - (-4)$

$= \boxed{4 = f(5) - f(1)}$ 2 pts

(b) (5pts) The AVERAGE RATE OF CHANGE from $x=1$ to $x=5$

is $\dfrac{f(5) - f(1)}{5 - 1} = \dfrac{4}{4} = 1$ = Avg. Rate of change

3pts 2pts

1pt context

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(c) 5pts

$$\frac{s(a+h)-s(a)}{h} = \frac{(a+h)^2-5(a+h)-(a^2-5a)}{h}$$

2pts

$$= \frac{a^2+2ah+h^2-5a-5h-a^2+5a}{h} = \frac{2ah+h^2-5h}{h}$$

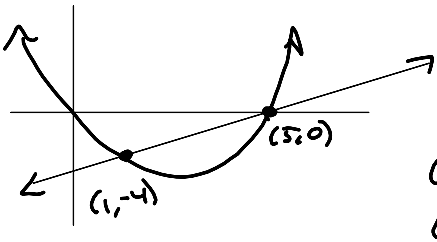
$$= \frac{h(2a+h-5)}{h} = \boxed{2a+h-5} \quad (\text{as long as } h \neq 0)$$

2pts

1pt context (from the top)

$$= \frac{f(a+h)-f(a)}{h}$$

Note: Part b picture:



slope of this line is $\dfrac{f(5)-f(1)}{5-1}$

$(x_1,y_1) = (1,f(1)) = (1,-4)$

$(x_2,y_2) = (5,f(5)) = (5,0)$