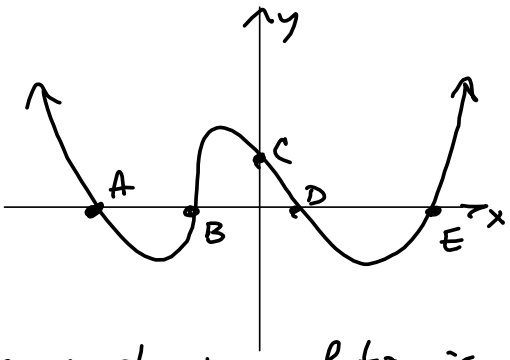
1340 WEEK 4 SOLUTIONS Hi MILLS

(1) Use the graph of $f(x) = x^4+4x^3-34x^2-76x+105$ to answer the following:

(a) (5pts) Solve $f(x) = 0$



Sketch Key:
A = (-7, 0)
B = (-3, 0)
C = (0, 105) y-int
D = (1, 0)
E = (5, 0)

From the graph, the solution is $x = -7, -3, 1, 5$. The solution set is $\{-7, -3, 1, 5\}$. We write

$x \in \{-7, -3, 1, 5\}$ or $x = -7, -3, 1, 5$, interchangeably.

This is a set. This is a condition x must satisfy.

Condition for club membership and the club, itself, are not quite the same thing.

(b) (5pts) Solve $f(x) > 0$. We report intervals of x where the graph's y-values are above the x-axis. This is true

for $x < -7$ OR $-3 < x < 1$ OR $x > 5$

$(-\infty, -7) \cup (-3, 1) \cup (5, \infty)$

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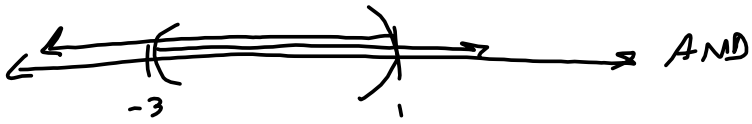
"OR" means "UNION."

"AND" means "INTERSECTION."

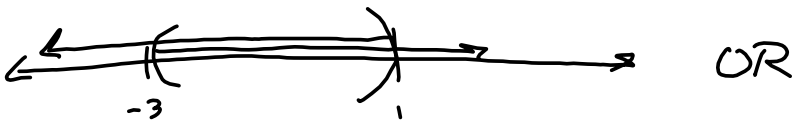
This page is more instruction than assignment.

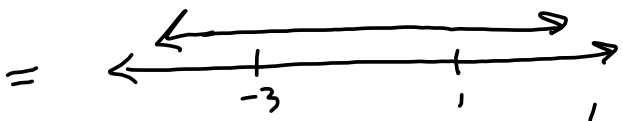
$-3 < x < 1$ means

$-3 < x$ AND $x < 1$. The intersection is the set contained in BOTH. It's the overlap



$-3 < x$ OR $x < 1$ LOOKS LIKE





It's the whole real line!

MIND YOUR AND'S and OR'S.

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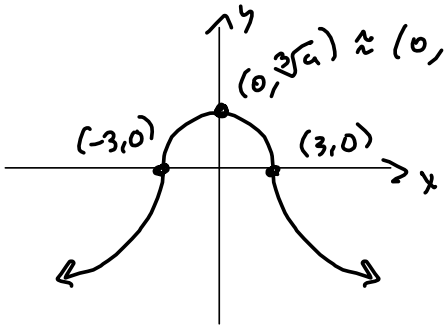
(2) Let $y = f(x) = \sqrt[3]{9-x^2}$, we employ a graphing utility to answer the following

(a) (5pts) A hand sketch with x- & y-intercepts shown.



$(0, \sqrt[3]{9}) \approx (0, 2.08008382305)$ from Desmos.

(b) (5pts) we check for symmetry

$y = \sqrt[3]{9-x^2}$

$\sqrt[3]{9-(-x)^2} = \sqrt[3]{9-x^2} = y$, so symmetric in or about the y-axis. The graph supports this.

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(3) We solve the eqn $\sqrt{3x+22}+2=x$ in two ways:

(2) (Sp15) with a graphing utility:

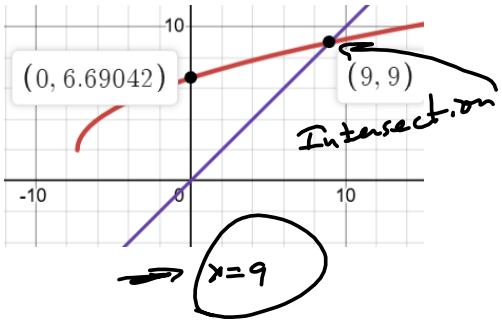
Two ways:

① Graph $\sqrt{3x+22}+2=y$ & $y=x$, separately and find the intersection(s).

② Graph $y=\sqrt{3x+22}+2-x$ and find the x-intercepts.

Method 1: Graph both sides, separately

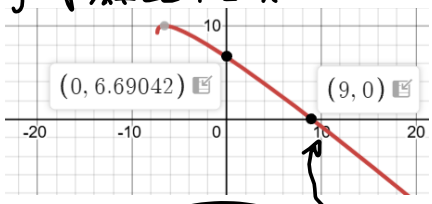
$y=\sqrt{3x+22}+2$ & $y=x$.



$\Rightarrow x=9$

METHOD 2: Get everything on one side

$y=\sqrt{3x+22}+2-x$



x-intercept

$\Rightarrow x=9$

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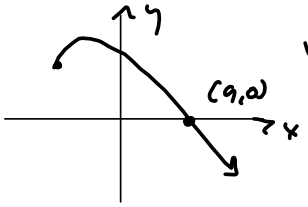
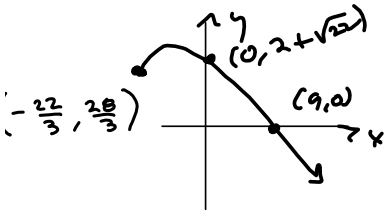
Teacher's notes on #3a. #3b on next page.

A hand sketch, with key point & general "shape" of the graphs, will suffice, here.

$y=\sqrt{3x+22}+2-x$



would've been OK



This would be BONUS LAND for excellence.

$3x+22=0$

$3x=-22$

$x=-\frac{22}{3}$ ⟹

$y=\sqrt{3\left(-\frac{22}{3}\right)+22}+2-\left(-\frac{22}{3}\right)$

$=2+\frac{22}{3}=\frac{6+22}{3}=\frac{28}{3}$

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(6)(5pts) Algebraic Solution:

$\sqrt{3x+22} + 2 = x \longrightarrow$

Need to see → $\sqrt{3x+22} = x-2 \longrightarrow$

$\longrightarrow \left(\sqrt{3x+22}\right)^2 = (x-2)^2$ → OPTIONAL TO SHOW.

If you must see this step, write an extra line, as I did. Don't cram it into the previous step.

Need to see → $3x+22 = x^2-4x+4$

$-3x-22 = -3x-22$

$0 = x^2-7x-18$

$= (x-9)(x+2) = 0 \longrightarrow$

$x = 9$ OR $x = -2$

Check:

$\sqrt{3(9)+22} + 2 \stackrel{?}{=} 9$

$\sqrt{27+22} + 2 \stackrel{?}{=} 9$

$\sqrt{49} + 2 = 7+2 = 9$ ✓

$\sqrt{0+22} + 2 = 0$? NO

$x=0$ is extraneous solution

$x=9$ is the solution

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(4) If S is proportional to the product of x and the square of y and inversely proportional to the square root of z, then

$$\boxed{S = K\frac{xy^2}{\sqrt{z}}}, \text{ for some } K > 0.$$

If $S=3$, when $x=3, y=2$, and $z=16$, then

$$S = 3 = \frac{3(2)^2}{\sqrt{16}}K = \frac{3(4)}{4}K = 3 = S$$

$$\Rightarrow 3K = 3$$

$$\Rightarrow \boxed{K=1}!$$

What is $S(2,3,4)$? I.e., what is S when $x=2, y=3$, and $z=4$?

$$S = K\left(\frac{2(3)^2}{\sqrt{4}}\right) = \frac{2(9)}{2} = \boxed{9 = S}$$