1340 Week 3 written SOLUTIONS H. MILLS

(1) We solve $\sqrt{7x} + x = 0$

$\Rightarrow \sqrt{7x} = -x$

$\Rightarrow 7x = (-x)^2 = x^2$

$\Rightarrow x^2 - 7x = 0$

$\Rightarrow x(x-7) = 0$

$\Rightarrow x \in \{0, 7\}$

Check: $\sqrt{0} + 0 = 0$ ✓

$\sqrt{49} + 7 = 14 \neq 0$ ✗

$x=7$ is extraneous

Soln is $\boxed{x=0}$

(2) (5pts) $2(x-5)^2 - 13(x-5) - 7 = 0$ is quadratic in $w = x-5$

$\Rightarrow 2w^2 - 13w - 7 = 0$

$\Rightarrow (2w+1)(w-7) = 0$

$\Rightarrow w = -\frac{1}{2} = x-5 \Rightarrow x = 5 - \frac{1}{2} = \boxed{\frac{9}{2} = x}$ or

or $w = 7 = x-5 \Rightarrow x = 5+7 = \boxed{12 = x}$

or write: $\boxed{x \in \{\frac{9}{2}, 12\}}$ is stylin'

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#s 3-6 We find all real solutions of the equation.

(3) (5pts) $x^5 - 18x^4 + 6x^3 = 0 \longrightarrow$

$x^3(x^2 - 18x + 6) = 0$

$\Rightarrow x = 0$ OR $x^2 - 18x + 6 = 0 \longrightarrow a = 1, b = -18, c = 6$

$b^2 - 4ac = 18^2 - 4(1)(6)$

$= 324 - 24 = 300$

$300 = 2 \cdot 150$

$150 = 2 \cdot 75$

$75 = 3 \cdot 25$

$25 = 5 \cdot 5$

$\Rightarrow \sqrt{300} = 2 \cdot 5\sqrt{3}$

$= 10\sqrt{3}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{18 \pm 10\sqrt{3}}{2} = \boxed{9 \pm 5\sqrt{3} = x}$

FINAL ANS: $\boxed{x \in \{0, 9 \pm 5\sqrt{3}\}}$

(4) (5pts) $2x^3 + 7x^2 - 8x - 28 = 0 \longrightarrow$

$x^2(2x + 7) - 4(2x + 7)$

$= (2x + 7)(x^2 - 4)$

$= (2x + 7)(x - 2)(x + 2) = 0$

$\Rightarrow \boxed{x \in \{-\frac{7}{2}, \pm 2\}}$

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(5) (5pts) $\frac{x+6}{x^2+6} = \frac{2}{x+2}$ LCD $= (x+2)(x^2+6)$

$\Rightarrow (x+2)(x^2+6)\left(\frac{x+6}{x^2+6} = \frac{2}{x+2}\right)$ or cross-multiply.

$\Rightarrow (x+6)(x+2) = 2(x^2+6)$

$\Rightarrow x^2+8x+12 = 2x^2+12$

$\Rightarrow x^2-8x = 0$

$\Rightarrow x(x-8) = 0$

$\Rightarrow \boxed{x \in \{0, 8\}}$

Check: $\frac{0+6}{0^2+6} \stackrel{?}{=} \frac{2}{0+2}$ ✓

$\frac{8+6}{64+6} \stackrel{?}{=} \frac{2}{8+2}$

$\frac{1}{5} = \frac{7}{35} = \frac{14}{70} = \frac{2}{10} = \frac{1}{5}$ ✓

(6) (5pts) $\frac{x+\frac{4}{x}}{\frac{4}{x}+5} = 2x$

$\Rightarrow \frac{x\left(x+\frac{4}{x}\right)}{x\left(\frac{4}{x}+5\right)} = \frac{x^2+4}{4+5x} = \frac{x^2+4}{5x+4} = 2x$

$\Rightarrow x^2+4 = 2x(5x+4) = 10x^2+8x$

$\Rightarrow 9x^2+8x-4 = 0$

$(-4)(9) = (-)(2)(-2)(3)(3)$

$\Rightarrow a=9, b=8, c=-4$

$\Rightarrow b^2-4ac = 8^2-4(9)(-4)$

$= 64+144$

$= 208$

2 | 208
2 | 104
2 | 52
2 | 26
13

$\sqrt{208} = 2^2\sqrt{13} = 4\sqrt{13}$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-8 \pm 4\sqrt{13}}{2(9)} = \boxed{\frac{-4 \pm 2\sqrt{13}}{9} = x}$

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(7) (spts) we solve $\frac{x+6}{x^2+6} \geq \frac{2}{x+2}$

By #5, we know they're equal @ $x=0,8$
But there's a lot more to it than that. We also have a sign change on the right-hand side @ $x=-2$.
This is how to beat these. Use $x=0,8$ as a check, but the idea is to get everything on one side & then analyze a SIGN PATTERN.

$\frac{x+6}{x^2+6} - \frac{2}{x+2} \geq 0$ LCD $= (x+2)(x^2+6)$, as in #5. But here, we ADD FRACTIONS

$$\left(\frac{x+6}{x^2+6}\right)\left(\frac{x+2}{x+2}\right) - \left(\frac{2}{x+2}\right)\left(\frac{x^2+6}{x^2+6}\right)$$

$$= \frac{x^2+8x+12-2x^2-12}{(x+2)(x^2+6)} = \frac{-x^2+8x}{(x+2)(x^2+6)}$$

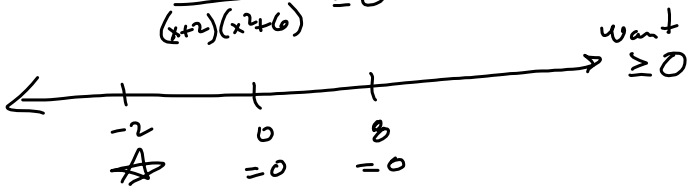
$$= \frac{-x(x-8)}{(x+2)(x^2+6)} = 0 \text{ when } x=0,8 \checkmark$$

Also, $=$ ☆ (Blow-up!) when $x=-2$

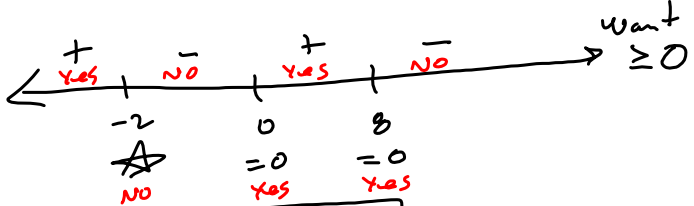
$x=-2,0,8$ break up the # line into 3 intervals

We want $\frac{x+6}{x^2+6} - \frac{2}{x+2} \geq 0$ so we want

$$\frac{-x(x-8)}{(x+2)(x^2+6)} \geq 0$$




| Interval | Test | |
|---|---|---|
| $(-\infty,-2)$ | $-3$ | $\frac{-(-3)(-3-8)}{(-3+2)((-3)^2+6)} = \frac{(+)(-)}{(-)(+)} = + > 0$ |
| $(-2,0)$ | $-1$ | $\frac{-(-1)(-1-8)}{(-1+2)((-1)^2+6)} = \frac{(+)(-)}{(+)(+)} = - < 0$ |
| $(0,8)$ | $1$ | $\frac{-(1)(1-8)}{(1+2)(1^2+6)} = \frac{(-)(-)}{(+)(+)} = + > 0$ |
| $(8,\infty)$ | $9$ | $\frac{(-9)(9-8)}{(9+2)(9^2+6)} = \frac{(-)(+)}{(+)(+)} = - < 0$ |




→ $x \in (-\infty,-2) \cup [0,8]$

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(8) Solve the absolute-value inequalities.

(a) (2 pts) $|2x-3| > 7 \rightarrow$ KEY

$2x-3 > 7$ OR $2x-3 < -7$

$2x > 10$ $2x < -4$

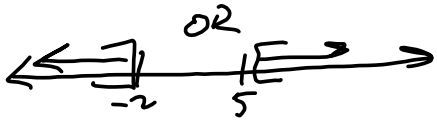
$x > 5$ OR $x < -2$

Solution-Set Answer

$(-\infty, -2) \cup (5, \infty)$

or WRITE

$\{x \mid x > 5 \text{ OR } x < -2\}$




(b) (5 pts) $|3x+8| \leq 35 \rightarrow$

$3x+8 \leq 35$ and $3x+8 \geq -35$

$3x \leq 27$ $3x \geq -43$

$x \leq 9$ and $x \geq -\frac{43}{3}$

$\{x \mid x \leq 9 \text{ and } x \geq -\frac{43}{3}\}$


$=$


$= \left[-\frac{43}{3}, 9\right]$

(c) (5 pts) $|2x-3| > -7$

$\rightarrow$ x = any real # (ALWAYS)

(d) (5 pts) $|3x+8| \leq -35$ (NEVER!)

No Soc'n