1340 Harry Mills

Harry's my 1st name, so it's what would be on the roster, so that's how I print my name. I don't mind cursive, but I don't want a cryptic signature.

① We graph the region for the system of inequalities:

$x > 1$
$y \geq 0$
$2x + 3y \leq 6$

I use "scratch out the bad stuff" method. Note key details I include and busy work I do not include (Like tick marks).

Get things generally right. Label things the way I do & get an intuition for what matters and what doesn't. This is the highest order of human intelligence.

$x = 1$ is a vertical line with x-intercept $\boxed{(1,0) \text{ x-int}}$ No y-int

$x > 1$ is all points to the right of the line $x = 1$.

$y = 0$ is the x-axis!

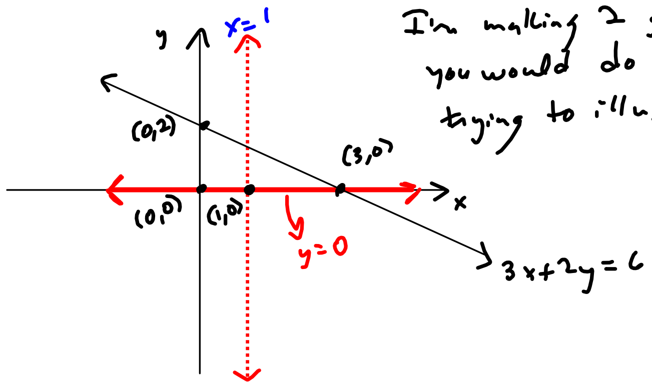
$y \geq 0$ is all points on or above the x-axis. Its x- & y-ints are just $\boxed{(0,0)}$

$2x+3y = 6$:

x	y
0	2
3	0

$2(0)+3y = 6 \Rightarrow y = \frac{6}{3} = 2 \rightarrow$ (0,2) y-int

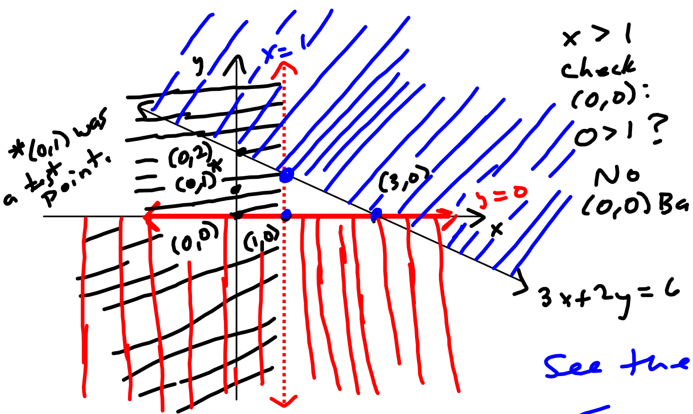
$2x+3(0) = 6 \Rightarrow y = \frac{6}{2} = 3 \rightarrow$ (3,0) x-int



I'm making 2 graphs, but you would do it on one graph. I'm trying to illustrate the process.

"scratch out the bad stuff"

After building the lines and labeling them, as above, I would then add the shading layers for the inequalities.



x > 1
check (0,0):
0 > 1 ?
No
(0,0) Bad

y ≥ 0
check any point not on y-axis:
(0,1):
1 ≥ 0?
Yes.
(0,1) Good

3x+2y ≤ 6
check (0,0):
3(0)+2(0) ≤ 6?
0 ≤ 6 ?
Yes
(0,0) GOOD

See the triangle in the middle?

THAT is the "feasible region."

& that's what we want!

we have 2 of the 3 corners:
(1,0) & (3,0).
The 3rd corner is where $x=1$ and $3x+2y=6$ intersect:

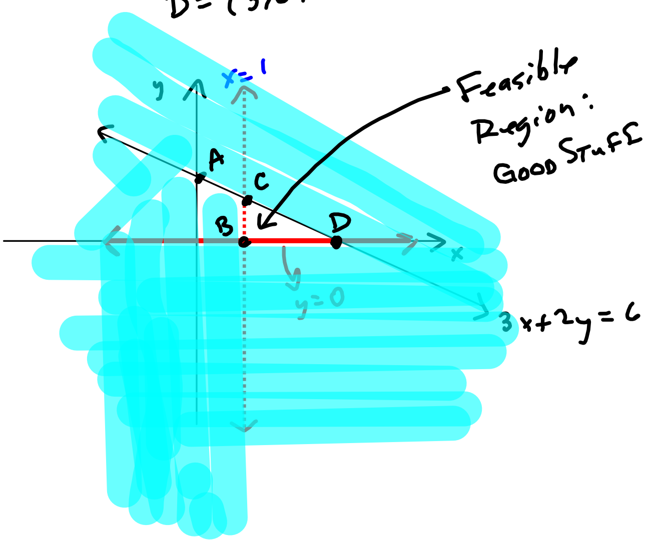
$x=1 \Longrightarrow 3x+2y = 3(1)+2y = 3+2y=6$

$\Longrightarrow 2y=3$

$\Longrightarrow \boxed{y=\frac{3}{2}}$. So $(1,\frac{3}{2})$ is that top corner.

Best way to present it?
I'd give all key points a letter label.

A = (0,2)
B = (1,0)
C = $(1,\frac{3}{2})$ OR (1,1.5)
D = (3,0)



Shade the bad stuff & the good stuff is revealed.

(0,0) (1,0)

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(2) The distance between $(x_1, y_1) = (2,-3)$ & $(x_2, y_2) = (-7,2)$ is

$$\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2} = \sqrt{(2-(-7))^2 + (-3-2)^2} = \sqrt{9^2+5^2} = \sqrt{81+25}$$

$$\boxed{=\sqrt{106}}$$

$2 \,\underline{)106}$
53

(3) The midpoint of $(x_1, y_1) = (\frac{\pi}{2}, \sqrt{3})$ & $(x_2, y_2) = (\frac{\pi}{3}, -2)$ is

$$\text{midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{\frac{\pi}{2}+\frac{\pi}{3}}{2}, \frac{\sqrt{3}-2}{2}\right)$$

$$= \left(\frac{\frac{3\pi+2\pi}{6}}{2}, \frac{\sqrt{3}-2}{2}\right) = \boxed{\left(\frac{5\pi}{12}, \frac{\sqrt{3}-2}{2}\right) = \text{midpoint}}$$

(4) We use Pythagorus to prove that the points $A=(1,2)$, $B=(-5,0)$, & $C=(-4,-3)$ form the vertices of a right triangle, i.e., the sum of the squares of the two shorter sides equals the square of the longest side.

$$d(A,B)=\sqrt{(1-(-5))^2+(2-0)^2}=\sqrt{6^2+2^2}=\sqrt{36+4}=\sqrt{40}=\sqrt{4\cdot 10}=2\sqrt{10}$$
$$=2\sqrt{10}$$

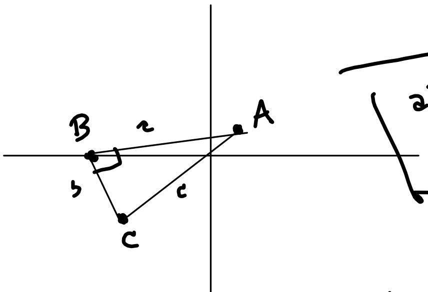
$$d(A,C)=\sqrt{(1-(-4))^2+(2+3)^2}=\sqrt{5^2+5^2}=\sqrt{50}=5\sqrt{2}=d(A,C)$$

$$d(B,C)=\sqrt{(-5-(-4))^2+(0+3)^2}=\sqrt{1^2+3^2}=\sqrt{10}$$

I shouldn't have simplified the radicals, yet.

$$d(A,B)=\sqrt{40},\ \underbrace{d(A,C)=\sqrt{50}}_{\text{longest}},\ d(B,C)=\sqrt{10}$$

$$d(A,C)^2=d(A,B)^2+d(B,C)^2 \ ?$$
$$\sqrt{50}^2=\sqrt{40}^2+\sqrt{10}^2 \ ?$$
$50=40+10$? Yep. $\Longrightarrow$ It's a right triangle! $\blacksquare$



$a^2+b^2=c^2 \Longrightarrow$ Right triangle

This is the converse of the Pythagorean Theorem, of which the textbook speaks

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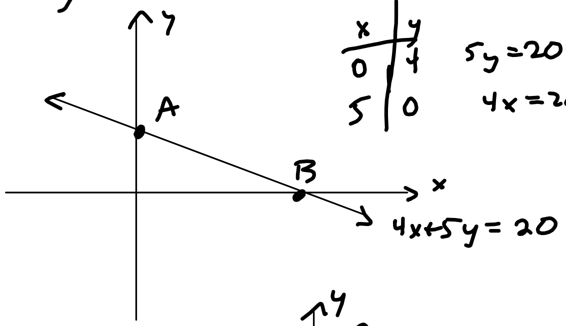
(5) We graph $4x+5y=20$ by the intercept method



x	y
0	4
5	0

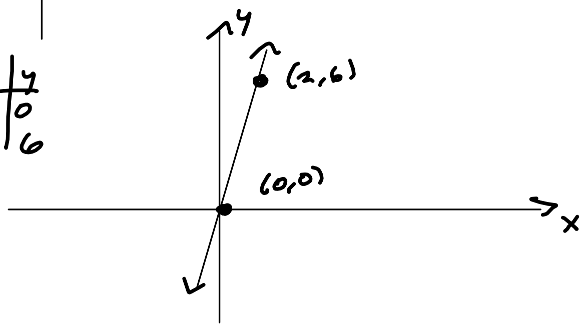
$5y=20 \Rightarrow y=4$

$4x=20 \Rightarrow x=5$

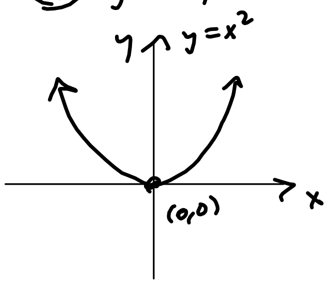
$\boxed{\begin{array}{l} A=(0,4) \\ B=(5,0) \end{array}}$

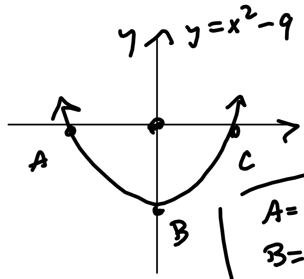
(6) Graph $y=3x$:

x	y
0	0
2	6



(7) $y=x^2-9$





$\boxed{\begin{array}{l} A=(-3,0) \\ B=(0,-9) \\ C=(3,0) \end{array}}$

$y(0) = 0^2-9 = -9 \rightsquigarrow \boxed{(0,-9) = B}$

$y=0 \Rightarrow$

$x^2-9=0 \Rightarrow$

$(x-3)(x+3)=0 \Rightarrow$

$x \in \{\pm 3\} \rightsquigarrow \boxed{\begin{array}{l} (3,0)=C \\ (-3,0)=A \end{array}}$

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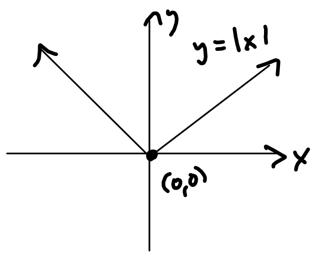
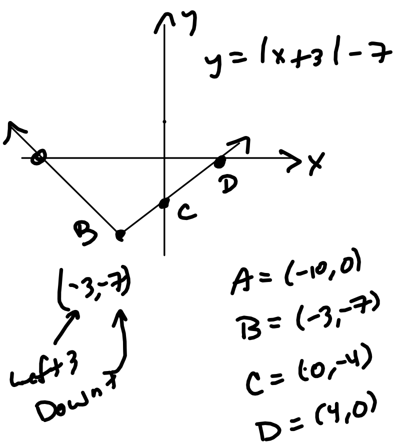
8 We graph $y = |x+3| - 7$ by shifting $y = |x|$

Left 3 Down 7





$A = (-10,0)$

$B = (-3,-7)$

$C = (0,-4)$

$D = (4,0)$

$y(0) = |3| - 7 = -4$

$(0,-4)$

$y = 0$

$|x+3| - 7 = 0$

$|x+3| = 7$

$x+3 = \pm 7$

$x = -3 \pm 7$

$-3+7 = 4 \rightsquigarrow (4,0)$

$-3-7 = -10 \rightsquigarrow (-10,0)$

(9) we graph the $\frac{1}{2}$-circle $y = -\sqrt{16-x^2}$

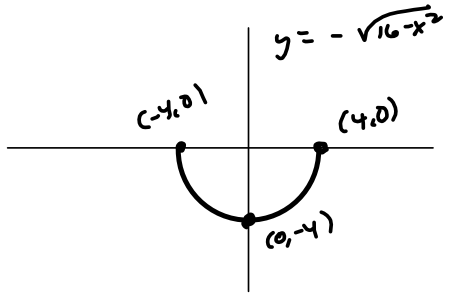
I really want you to recognize that square root structure as a circle.

$y = -\sqrt{16-x^2} \Longrightarrow$

$y^2 = 16-x^2 \Longrightarrow$

$x^2+y^2=16$ so it's evidently a circle of radius $r=4$

Now $y=\sqrt{16-x^2}$ is its top half, so...



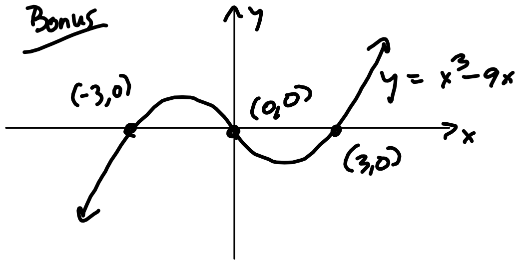
(10) Check $f(x) = x^3-9x$ for symmetry

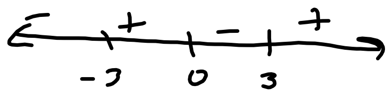
$f(-x) = (-x)^3-9(-x) = -x^3+9x = -(x^3-9x) = -f(x)$

This function is $\boxed{\text{symmetric wrt the origin.}}$

$x^3-9x = x(x^2-9) = x(x-3)(x+3)$

Bonus





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10 $x^2+y^2-6x+4y=23$

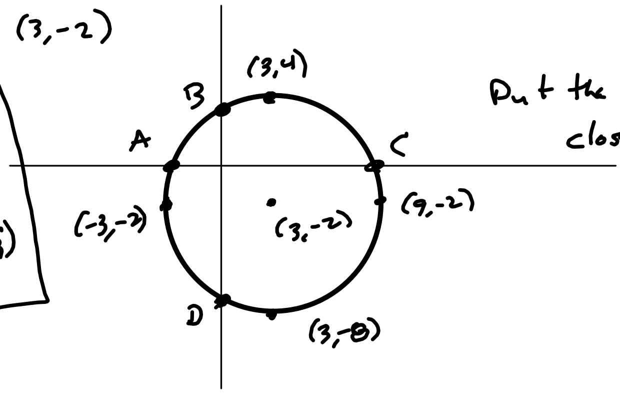
$x^2-6x \quad +y^2+4y \quad =23$

$x^2-6x+3^2+y^2+4y+2^2=23+9+4$

$(x-3)^2+(y+2)^2=36$ Circle of radius $6=r$, centered at

$(h,k)=(3,-2)$

$$\boxed{\begin{aligned}A&=(3-4\sqrt{2},0)\\ B&=(0,-2+3\sqrt{3})\\ C&=(3+4\sqrt{2},0)\\ D&=(0,-2-3\sqrt{3})\end{aligned}}$$



Put the letter labels close to the graph

x-int; $y=0$:

$(x-3)^2+(y+2)^2=36$

$(x-3)^2+(2)^2=36$

$(x-3)^2+4=36$

$(x-3)^2=32$

$2\,|\,32$, $2\,|\,16$, $2\,|\,8$, $2\,|\,4$, 2 $\quad \sqrt{2^5}$

$x-3=\pm\sqrt{32}=\sqrt{2^5}=\sqrt{2^4\cdot 2}=2^2\sqrt{2}=4\sqrt{2}$

$x=3\pm4\sqrt{2}$

$$\boxed{\begin{aligned}(3+4\sqrt{2},0)&=C\\ (3-4\sqrt{2},0)&=A\end{aligned}}$$

Bonus

y-int: $x=0$

$(-3)^2+(y+2)^2=36$

$9+(y+2)^2=36$

$(y+2)^2=27$

$3\,|\,27$, $3\,|\,9$, 3

$y+2=\pm\sqrt{27}=\pm3\sqrt{3}$

$y=-2\pm3\sqrt{3}$

$$\boxed{\begin{aligned}(0,-2+3\sqrt{3})&=B\\ (0,-2-3\sqrt{3})&=D\end{aligned}}$$

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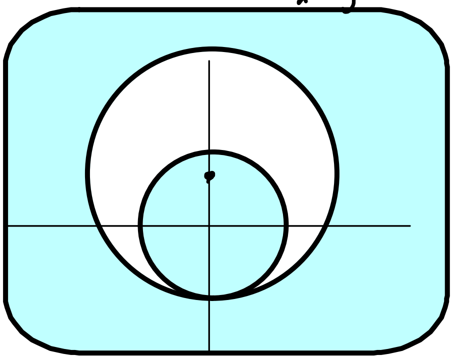
(12) Find area OUTSIDE $x^2+y^2=16$ and INSIDE $x^2+(y-3)^2=49$

$x^2+y^2>16$

$x^2+(y-3)^2<49$



If you shade the bad stuff, the good stuff is clean.

By the graph, the little circle is entirely contained in the big circle, so

Area = Area BIG − Area little.

$= \pi(7)^2 - \pi(4)^2$

$= \pi(49-16) = \pi(33)$ $\boxed{= 33\pi = \text{AREA}}$