1. Find the partial fraction decomposition.

Complex Variables
Integration

a. (5 pts) $\frac{5x+1}{x^2-1} = \boxed{\frac{5x+1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}}$

$\frac{(5x+1)\,LCD}{LCD} = \left(\frac{A}{x-1} + \frac{B}{x+1}\right) LCD$

$(x+1)(x-1)$

$5x+1 = A(x+1) + B(x-1)$

$x=-1$:

$5(-1)+1 = A(-1+1) + B(-1-1) = -2B$

$-4 = -2B$

$\Rightarrow \boxed{2 = B}$

$x=+1$:

$5(1)+1 = 6 = 2A$

$\Rightarrow \boxed{A = 3}$

b. (**Bonus** 5 pts) $\frac{x-8}{x^3+8x} = \boxed{\frac{x-8}{x(x^2+8)} = \frac{A}{x} + \frac{Bx+C}{x^2+8}}$

$\Rightarrow \quad x-8 = A(x^2+8) + (Bx+C)(x)$

$x=0 \Rightarrow$

$-8 = A(8)$

$\Rightarrow \boxed{A=-1}$

$x=1$:

$1-8 = A(1^2+8) + (B(1)+C)(1)$

$9A + B + C = -7$

$-9 + B + C = -7$

$B+C = 2$

$x=-1$:

$-1-8 = (-1)((-1)^2+8) + (B(-1)+C)(-1)$

$-9 = -(9) + (-B+C)(-1) = -9 + B - C = -9$

$B - C = 0$

$\boxed{B=C}$

$B+C = 2$

$2B = 2$

$\Rightarrow \boxed{B = 1 = C}$

(2)
$$2x - 7y \le 14$$
$$3x + 2y > 6$$
$$x \le 7$$
$$y \le 3$$

See Midterm Videos

We work a different one, since I've written the one on the left up for you, twice.

$3x + 5y \ge 15$

$5x - 3y < 30$

$x \ge 10$ $\quad 0 \ge 10$? No

$y \le 17$ $\quad 0 \le 17$? Yes

We graph the system of linear inequalities.

$3x+5y \ge 15$

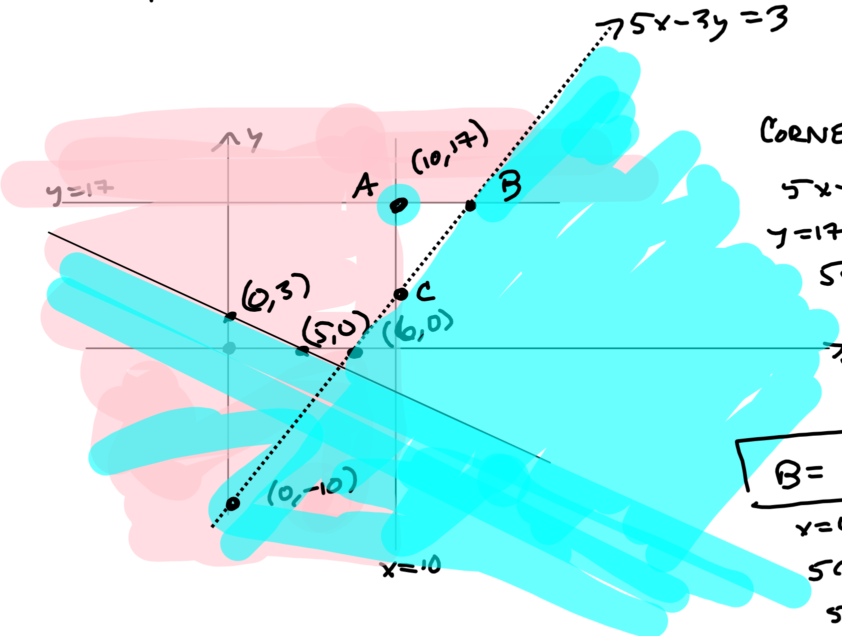
x	y
0	3
5	0

$0 > 15$? No

$5x - 3y < 30$

x	y
0	-10
6	0

$0 < 30$? Yes



CORNERS:

$5x - 3y = 30$

$y = 17$:

$5x - 3(17) = 30$

$5x - 51 = 30$

$5x = 81$

$x = \frac{81}{5}$

$\boxed{B = \left(\frac{81}{5}, 17\right)}$

$x = 10$:

$5(10) - 3y = 30$

$50 - 3y = 30$

$-3y = -20$

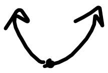
$y = \frac{20}{3}$

$\boxed{A = (10, 17)}$

$\boxed{C = \left(10, \frac{20}{3}\right)}$

(3) Find focal length, p, eq'n of directrix, focal diameter, sketch, endpts of latus rectum, vertex, focus, . . .

(a) $y = \frac{1}{12}x^2$



$x^2 = 4py$ $p =$ focal length

$(x-h)^2 = 4p(y-k)$

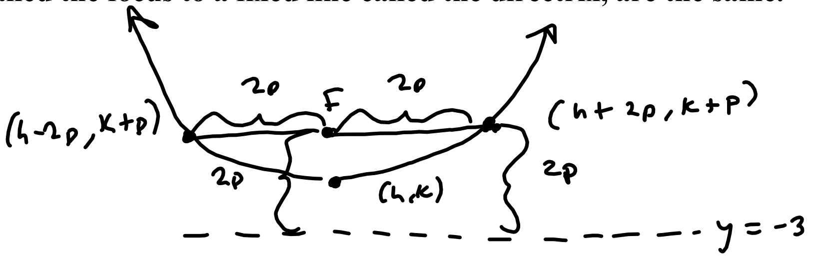
$(h,k) =$ vertex $= (0,0)$

$\Rightarrow x^2 = 12y = 4py \Rightarrow$

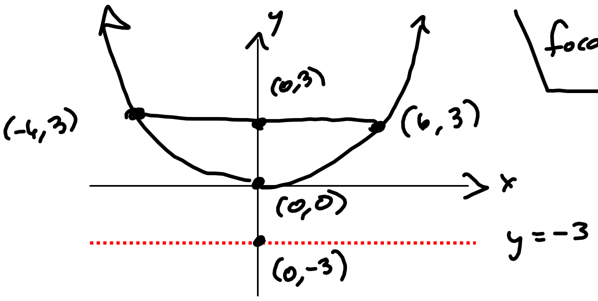
$12 = 4p \Rightarrow$

$\boxed{3 = p}$ = focal length $\Rightarrow$ focal diameter $= 4p$

Geometric Description: A parabola is the set of all points whose distance from a fixed point called the focus to a fixed line called the directrix, are the same.



$(h,k) = (0,0)$, $p = 3$



$\boxed{\text{focal diam} = 12 =}$ $4p = 4 \cdot 3 = 12$

(b) $x = -\frac{1}{20}y^2$ $(h,k) = (0,0)$ opens left

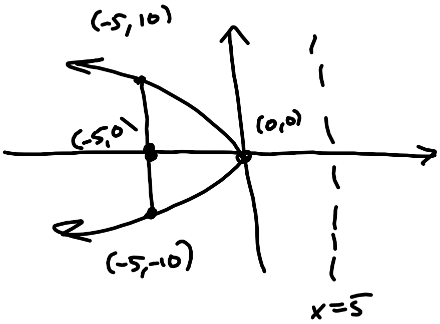
$20x = -y^2$

$-20x = y^2$

$4p = -20$

$4|p| = 20 \rightarrow \boxed{p = 5 = \text{focal length}}$

$4|p| = \boxed{\text{focal diameter} = 20}$



(c) $\left(y = \frac{1}{8}x^2 - \frac{5}{4}x - \frac{71}{8}\right)(8)$

$\Rightarrow \quad 8y = x^2 - 10x - 71$

$= x^2 - 10x + 5^2 - 25 - 71$

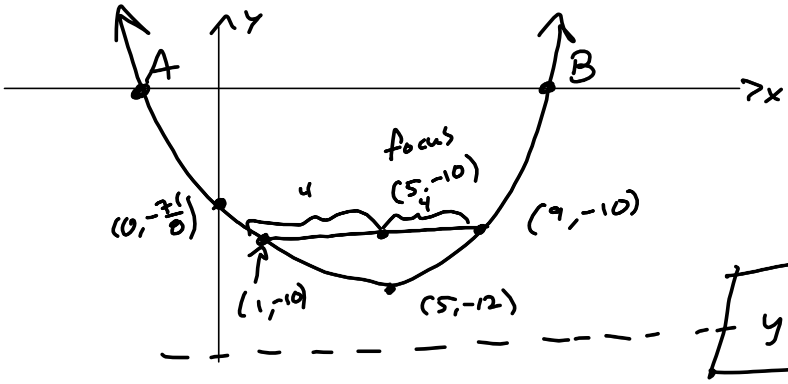
$= (x-5)^2 - 96 \rightarrow$

$8y + 96 = (x-5)^2$

$8(y+12) = (x-5)^2$

$(h,k) = (5,-12)$

$\boxed{\begin{array}{l} 4p = 8 = \text{focal diameter} \\ p = 2 = \quad \text{" length} \end{array}}$ &



$\boxed{y = -14}$ Direc.

(d) Add x- & y-ints.

$\boxed{\left(0, -\frac{71}{8}\right) = y\text{-int}}$

x-int:

$8y = (x-5)^2 - 96$

$0 = (x-5)^2 - 96 \rightarrow$

$(x-5)^2 = 96$

$x - 5 = \pm\sqrt{96} = \pm 4\sqrt{3}$

$\Rightarrow \quad x = 5 \pm 4\sqrt{3}$

$\boxed{A = (5-4\sqrt{3}, 0), \quad (5+4\sqrt{3}, 0) = B}$

2 | 96
2 | 48
2 | 24
2 | 6
3

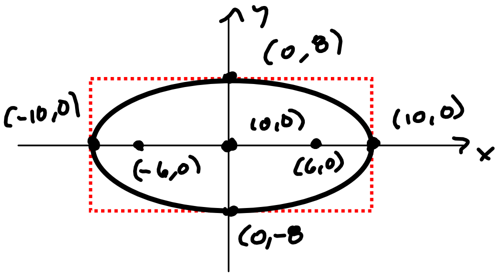
4 a Graph $\frac{x^2}{100}+\frac{y^2}{64}=1$. Center, foci, endpts of Major & Minor Axes.

$$\frac{x^2}{10^2}+\frac{y^2}{8^2}=1$$

$$\overset{\leftrightarrow}{a}=10,\ \overset{\updownarrow}{b}=8$$

$$c^2=a^2-b^2=100-64=36 \Rightarrow$$

$$\boxed{c=6}$$



(b) $49x^2 + 25y^2 + 196x - 150y - 804 = 0$

$49x^2 + 196x + 25y^2 - 150y = 804$

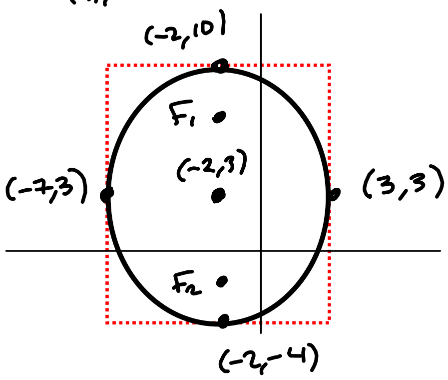
$49(x^2 + 4x + 2^2) + 25(y^2 - 6y + 3^2) = 804 + 25(9) + 49(4)$

$49(x+2)^2 + 25(y-3)^2 = 804 + 225 + 196 = 1225$

$\frac{(x+2)^2}{25} + \frac{(y-3)^2}{49} = \frac{1225}{(49)(25)} = 1$

$a = 5$, $b = 7$

$(h, k) = (-2, 3)$



$c^2 = b^2 - a^2 = 49 - 25 = 24$

$c = \sqrt{24} = 2\sqrt{6} \approx$

4.89897948557

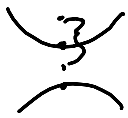
$F_1 = (-2, 3 + 2\sqrt{6})$

$F_2 = (-2, 3 - 2\sqrt{6})$

(5) Graph hyperbola. Center, foci, asymptotes.

$$\frac{y^2}{64} - \frac{x^2}{100} = 1$$

$\boxed{(h,k) = (0,0)}$



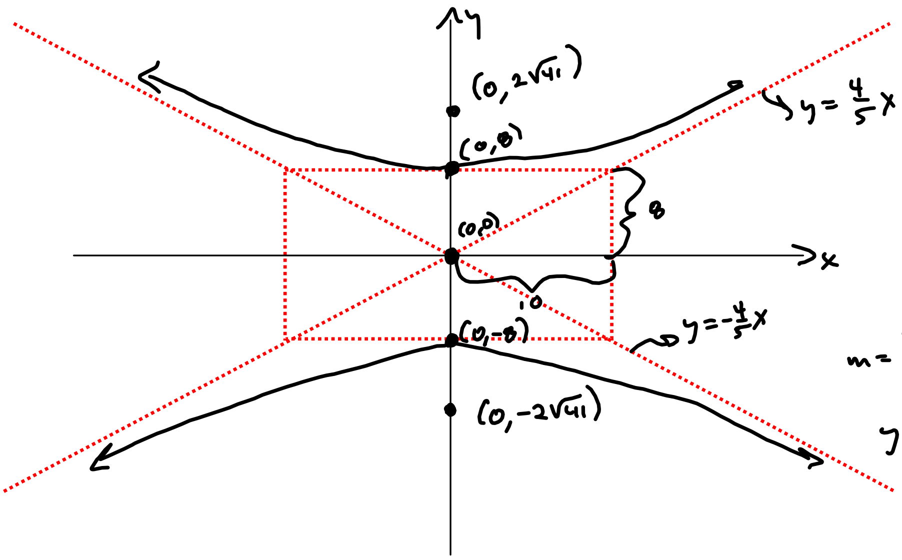
$a^2 = 64$, $b^2 = 100$

$a = 8$, $b = 10$

$c^2 = a^2 + b^2 = 164 = c^2$

$2 \mid 164$, $2 \mid 82$, 41

$\rightarrow c = 2\sqrt{41} \approx$ 12.8062484749



Actual Eq'ns of Asymptotes

$m = \frac{8}{10} = \frac{4}{5} = m$

$y = \frac{4}{5}(x-0) + 0$

$y = \frac{4}{5}x$

$y = -\frac{4}{5}x$

(b) $25x^2 - 36y^2 - 150x - 144y - 819 = 0$

$25x^2 - 150x \quad -36y^2 - 144y \quad = 819$

$25(x^2 - 6x + 3^2) - 36(y^2 + 4y + 2^2) = 819 + 25(9) - 144$

$25(x-3)^3 - 36(y+2)^2 = 819 - 144 + 225 = 675 + 225 = 900$

$$\frac{(x-3)^2}{36} - \frac{(y+2)^2}{25} = 1$$

$(h,k) = (3,-2)$

$a = 6, b = 5$

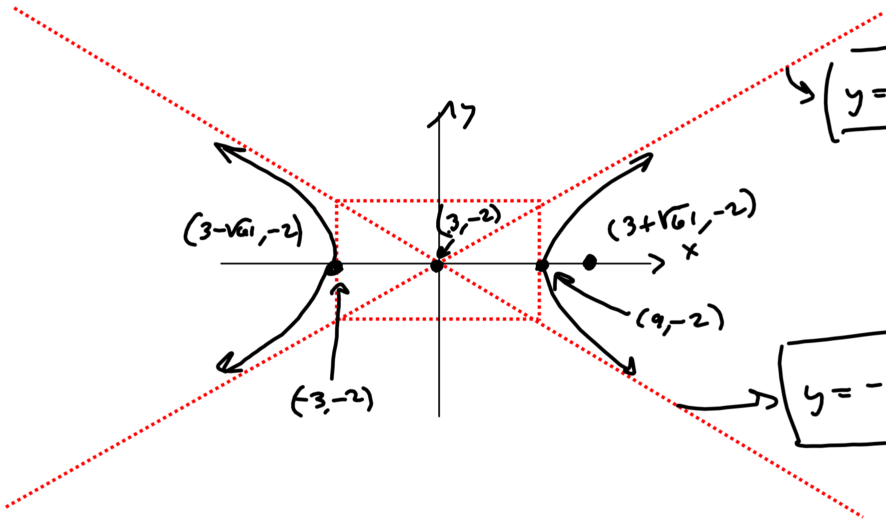
$c^2 = a^2 + b^2 = 36 + 25 = 61$

$\to c = \sqrt{61} \approx$ 7.something

336
25
180
720
900

819
144
225
1188

819
−144
675



$\boxed{y = \frac{5}{6}(x-3) - 2}$

$\boxed{y = -\frac{5}{6}(x-3) - 2}$

6 a) Use Pascal's Triangle for $(2x-3y)^4$

$\to$ Actual Question: $(3x-2y)^4$

$$\begin{array}{ccccccccc} &&&&1&&&& \\ &&&1&&1&&& \\ &&1&&2&&1&& \\ &1&&3&&3&&1& \\ 1&&4&&6&&4&&1 \end{array} \Rightarrow$$

$$(2x-3y)^4 = 1(2x)^4(-3y)^0 + 4(2x)^3(-3y)^1 + 6(2x)^2(-3y)^2 + 4(2x)^1(-3y)^3 + (2x)^0(-3y)^4$$

$$= 16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$$

$$\begin{array}{r} {}^{5}27 \\ \underline{8} \\ 216 \end{array}$$

b) what's the coefficient of x^9 in $(2x-3y)^{16}$?

"n choose k" $= \binom{n}{k}$

$\{a,b,c\}$

$\{a,b\}, \{a,c\}, \{b,c\} \Rightarrow \binom{3}{2} = 3$

FORMULA: $\dfrac{n!}{k!(n-k)!} = \dfrac{3!}{2!1!} = \dfrac{3\cdot2\cdot1}{(2\cdot1)(1)} = 3$

$P(n,k)$ = Permutations on n things taken k at a time.

$P(3,2)$

$\{a,b,c\}$

$(ab), (ba), (ac), (ca), (bc), (cb)$

6 of 'em.

$(3)(2) = 6$ (By multiplication principle.

$n!$

$P(n,k) = \overline{(n-k)!}$

$$C(n,k) = \binom{n}{k} = \frac{P(n,k)}{\#\text{ of rearrangements}}$$

Binomial Theorem Says:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$$= \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{n-2} a^2 b^{n-2} + \binom{n}{n-1} a^1 b^{n-1} + \binom{n}{n} a^0 b^n$$

1, 1, n, n, 1, 1, 1, 1

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$\binom{4}{0}$ $\binom{4}{1}\binom{4}{2}\binom{4}{3}\binom{4}{4}$

$\binom{n}{k} = \binom{n}{n-k}$

$\binom{4}{1} = \binom{4}{3}$

$\binom{17}{6} = \binom{17}{11}$

(b) what's the coefficient of x^9 in $(2x-3y)^{16}$?

$\binom{16}{9}(2x)^9(-3y)^7$ is "x^9" term.

$$\frac{\cancel{16}\cdot\cancel{15}\cdot\cancel{14}\cdot 13\cdot\cancel{12}\cdot 11\cdot\cancel{10}\cdot 9\cdots}{(9\cdot 8\cdots)(\cancel{7}\cdot\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2})} = 8\cdot 5\cdot 13\cdot 2\cdot 11$$

8 5 2 2 1 3 1 1

$= 40 \cdot 26\cdot 11 = 11440$

11440

$(11440)(2^9)(-3)^7$

$= -11440(2^9)(3)^7 = -1.2809871\,36 \times 10^{10}\, x^9 y^7$

-12809871360

$-12809871360\, x^9 y^7$

coefficient of x^7 is

$\boxed{-12809871360\, y^7 \text{ is the coefficient of } x^9}$

"What's the coefficient of $x^9 y^7$?" on final.

$\boxed{-12809871360 \text{ is the coefficient of } x^9 y^7}$

7 **How many ways can you pick a starting lineup consisting of a point guard, shooting guard, small forward, power forward, and center, from a roster of 15 players?**

15 choose 5, when order (position) matters -
Permutation

$$P(15,5) = \frac{15!}{(15-5)!} = \frac{15!}{10!} = 15\cdot 14\cdot 13\cdot 12\cdot 11$$ 360360

$$\underset{SG}{\underline{15}}\cdot\underset{PG}{\underline{14}}\cdot\underset{PF}{\underline{13}}\cdot\underset{SF}{\underline{12}}\cdot\underset{C}{\underline{11}}$$

8 How many ways can you pick 4 volunteers out of a group of 10 volunteers?

$$\binom{10}{4} = \text{"10 choose 4"} = \frac{10!}{6!4!} = \frac{10\cdot\cancel{9}^{3}\cdot\cancel{8}^{1}\cdot 7}{\cancel{4\cdot 3\cdot 2}} = (30)(7) = \boxed{210}$$

210

$$\frac{1680}{8} = 210$$

56
30
1680