① Solve $\begin{array}{l} 2x-5y=10 \\ 7x+2y=28 \end{array}$ in 3 ways:

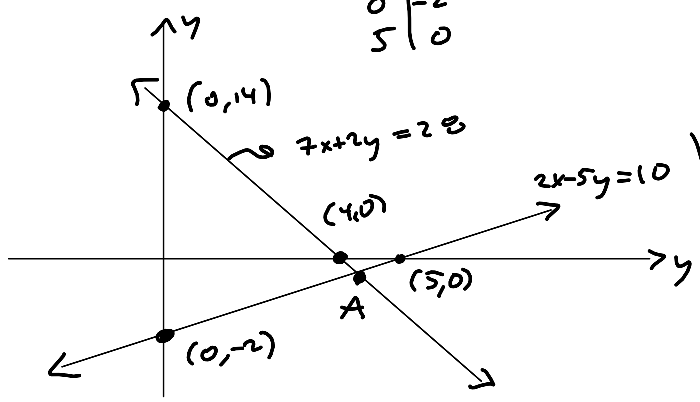
② Graphically

$2x-5y=10$

x	y
0	-2
5	0

$7x+2y=28$

x	y
0	14
4	0



$\boxed{A \approx (4.2, -.3)}$

(b) Substitution

$2x-5y=10 \Rightarrow -5y = 10-2x \Rightarrow y = \frac{10-2x}{-5} = \frac{2x-10}{5}$

$7x+2y=28 \Rightarrow$

$7x+2y = \left(7x+2\left(\frac{2x-10}{5}\right) = 28\right)(5)$

$\Rightarrow 35x+4x-20=140$

$\Rightarrow 39x = 160$

$\Rightarrow \boxed{x = \frac{160}{39}} \Rightarrow y = \frac{2\left(\frac{160}{39}\right)-10}{5}$

$= \frac{\frac{320}{39}-\frac{390}{39}}{5} = \frac{-70}{39(5)} = \boxed{\frac{-14}{39}=y}$

$\boxed{(x,y) = \left(\frac{160}{39}, -\frac{14}{39}\right)}$

ⓒ Elimination

E1 $2x-5y=10$

E2 $7x+2y=28$

$$\begin{array}{lrl} -7E1 & -14x+35y & =-70 \\ 2E2 & 14x+4y & =56 \\ \hline & 39y & =-14 \end{array}$$

$$\boxed{y=-\frac{14}{39}}$$

$$\Rightarrow 2x-5y=2x-5\left(\frac{-14}{39}\right)=2x+\frac{70}{39}=10$$

$$\Rightarrow 78x+70=390$$

$$78x=320$$

$$x=\frac{320}{78}=\boxed{\frac{160}{39}=x}$$

$$(x,y)=\left(\frac{160}{39},-\frac{14}{39}\right)$$

(4.10256410256 , - 0.358974358974)

An Independent System with a unique solution.

$$x+3y-z=4$$
$$-3x-8y+z=-16$$
$$4x+14y-7z=11$$

(Gaussian) Elimination

$3R1 \quad 3x+9y-3z=12$
$R2 \quad -3x-8y+z=-16$
$3R1+R2 \quad y-2z=-4$

$-4R1 \quad -4x-12y+4z=-16$
$R3 \quad 4x+14y-7z=11$
$2y-3z=-5$

New System
$$x+3y-z=4$$
$$y-2z=-4$$
$$2y-3z=-5$$

$-2R2 \quad -2y+4z=8$
$R3 \quad 2y-3z=-5$
$-2R2+R3 \quad z=3$

New (last) System
$$x+3y-z=4$$
$$y-2z=-4$$
$$z=3$$

$$y-2z=-4$$
$$y-2(3)=-4$$
$$y-6=-4$$
$$y=2$$

$$x+3y-z=4$$
$$x+3(2)-3=4$$
$$x+6-3=4$$
$$x+3=4$$
$$x=1$$

$$\boxed{(x,y,z)=(1,2,3)}$$

A dependent system with infinitely-many solutions.

$$x - y + 2z = 1$$
$$5x - 4y + 12z = 7$$
$$3x - 2y + 8z = 5$$

$-5R1 \quad -5x + 5y - 10z = -5$
$R2 \quad 5x - 4y + 12z = 7$
$-5R1 + R2 \quad y + 2z = 2$

$-3R1 \quad -3x + 3y - 6z = -3$
$R3 \quad 3x - 2y + 8z = 5$
$-3R1 + R3 \quad y + 2z = 2$

New System:

$$x - y + 2z = 1$$
$$y + 2z = 2$$
$$y + 2z = 2$$

$-R2 \quad -y - 2z = -2$
$R3 \quad y + 2z = 2$
$-R2 + R3 \quad 0 = 0$

New System:

$$x - y + 2z = 1$$
$$y + 2z = 2$$
$$0 = 0$$

$y + 2z = 2$

$\Rightarrow \boxed{y = -2z + 2}$

$x - y + 2z = 1$

$x - (-2z + 2) + 2z = 1$

$x + 2z - 2 + 2z = 1$

$x + 4z - 2 = 1$

$\boxed{x = -4z + 3}$

z is free!

Solution set:

$\boxed{(x,y,z) \in \{(-4z+3, -2z+2, z) \mid z \in \mathbb{R}\}}$ General Sol'n

Particular Sol'ns:

$z = 1 \Rightarrow (x,y,z) = (-4(1)+3, -2(1)+2, 1)$

$= (-1, 0, 1) \quad (z=1)$

A dependent system that has no solution.

$$x - y + 2z = 1$$
$$5x - 4y + 12z = 7$$
$$3x - 2y + 8z = 8$$

$$\begin{array}{lr} -5R1 & -5x + 5y - 10z = -5 \\ R2 & 5x - 4y + 12z = 7 \\ \hline -5R1 + R2 & y + 2z = 2 \end{array} \qquad \begin{array}{lr} -3R1 & -3x + 3y - 6z = -3 \\ R3 & 3x - 2y + 8z = 8 \\ \hline -3R1 + R3 & y + 2z = 5 \end{array}$$

New System:

$$x - y + 2z = 1$$
$$y + 2z = 2$$
$$y + 2z = 5$$

$$\begin{array}{lr} -R2 & -y - 2z = -2 \\ R3 & y + 2z = 5 \\ \hline -R2 + R3 & 0 = 3 \end{array}$$

?! You Kiddin'?!

New System:

$$x - y + 2z = 1$$
$$y + 2z = 2$$
$$0 = 3 \quad ?!$$

0 = 3 is absurd. Assuming our mechanics are all good, this absurdity tells us we're reasoning from a faulty premise, namely, that this system HAS a solution.

Therefore, there is no solution.

Apr 25-2:16 PM