1. (10 pts) Sketch the system of linear inequalities

$$x-3y \le 15$$
$$2x+9y<36$$
$$x>-2$$

Find and label all x- and y-intercepts with ordered-pair labels, such as (– 2, 0). Don't waste time on tick marks. Eyeball it and label things. Use the intercept method. The last inequality only has an x- intercept on its boundary.

2. (5 pts) Find the corner points of the feasible region. For full credit, give exact answers as fractions.

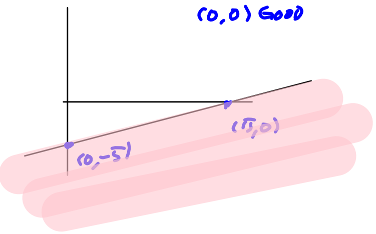
$x-3y \le 15$

x	y
0	-5
15	0

$-3y=15 \Rightarrow y=\frac{15}{-3}$

$0 \le 15$? Yes

(0,0) Good



$2x+9y<36$

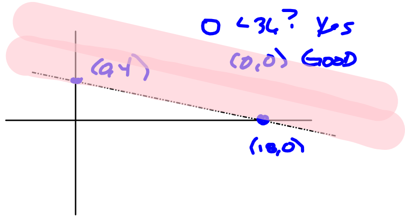
x	y
0	4
18	0

$9y=36$

$2x=36$

$0<36$? Yes

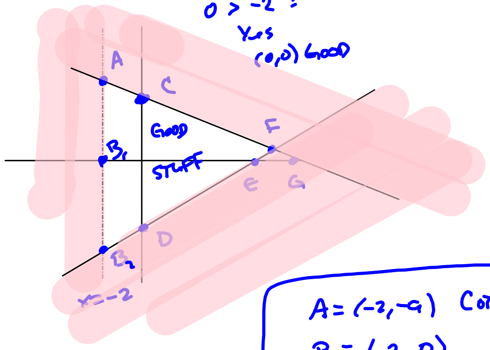
(0,0) Good



$x>-2$

$0>-2$? Yes

(0,0) Good



A: $x=-2$

$2x+9y=36$

$2(-2)+9y=36$

$-4+9y=36$

$9y=\frac{36}{-4}=-9$

$$\boxed{\begin{array}{l} A=(-2,-9) \text{ CORNER} \\ B_1=(-2,0) \\ ?!\ B_2\ (-2,-\frac{17}{3}) \text{ CORNER} \\ C=(0,4) \\ D=(0,-5) \\ E=(15,0) \\ F=(\frac{81}{5},\frac{2}{5}) \text{ CORNER} \\ G=(18,0) \end{array}}$$

B_2 $x=-2$

$x-3y=15$

$-2-3y=15$

$-3y=17$

$y=-\frac{17}{3}$

FIND F:

$2x+9y=36$

$x-3y=15 \Rightarrow x=3y+15$

$2(3y+15)+9y=6y+30+9y=15y+30=36$

$\Rightarrow 15y=6$

$y=\frac{6}{15}=\frac{2}{5}=y \Rightarrow$

$x=3y+15=3(\frac{2}{5})+15=\frac{6}{5}+15(\frac{5}{5})$

$=\frac{6+75}{5}=\frac{81}{5}$

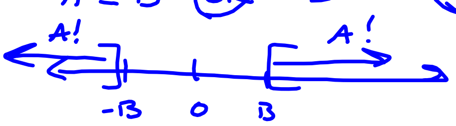
3. Solve the absolute-value inequalities.

a. (5 pts) $|-3x-9| \geq 6$

$|A| \geq B \Rightarrow$

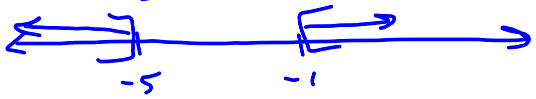
$A \geq B$ OR $-B$ UNION

$-3x-9 \geq 6$ OR $-3x-9 \leq -6$



$-3x \geq 15$ $\quad -3x \leq 3$

$x \leq \frac{15}{-3} = -5$ OR $x \geq -1$

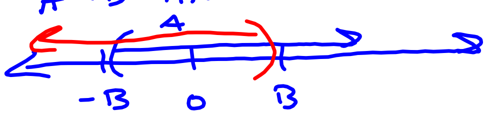


$= (-\infty, -5] \cup [-1, \infty)$

b. (5 pts) $|-3x-9| < 6$

$|A| < B \Rightarrow$

$A < B$ AND $A > -B$



$-3x-9 < 6$ AND $-3x-9 > -6$

$-3x < 15$ $\quad -3x > 3$

$x > -5$ AND $x < \frac{3}{-3} = -1$

$= \boxed{(-5, -1)}$

c. (5 pts) $|-3x-9| < -6$ $\quad \boxed{\text{No solution!}}$ $\quad |A| \geq 0$

d. (5pts) $|-3x-9| > -6$ $\quad \boxed{(-\infty, \infty)}$

3. Solve the absolute-value inequalities.

a. (5 pts) $|-3x-9| \geq 6$ ⇒

$|-3x-9| = |3x+9| \geq 6$ ⇒

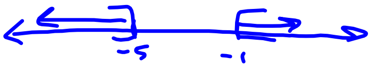
$3x+9 \geq 6$ OR $3x+9 \leq -6$

$-9 = -9$ $\quad$ $-9 = -9$

$3x \geq -3$ $\quad$ $3x \leq -15$

$x \geq -\frac{3}{3} = -1$ $\quad$ $\frac{3x}{3} \leq \frac{-15}{3} = -5$

OR $\quad x \leq -5$



OR

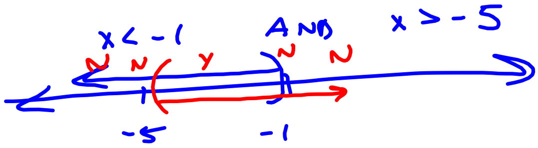
$(-\infty,-5] \cup [-1,\infty)$

b. (5 pts) $|-3x-9| < 6$

$3x+9 < 6$ AND $3x+9 > -6$

$3x < -3$ AND $3x > -15$

$x < -1$ AND $x > -5$



$= \boxed{(-5,-1)}$

c. (5 pts) $|-3x-9| < -6$ $\quad$ Nevah! No Sol'n

d. (5pts) $|-3x-9| > -6$ $\quad$ $\boxed{\text{Always! } (-\infty,\infty)}$

4. Let $f(x)=\dfrac{1}{x-6}$, $g(x)=\sqrt{x-15}$, and $h(x)=x^2-2x$

a. (5 pts) Find the domain of f, g, and h.

4/2

$D(f)$: Need $x-6 \neq 0 \Rightarrow x \neq 6$

$D(f) = \{x \mid x \neq 6\} = (-\infty, 6) \cup (6, \infty) = \boxed{\mathbb{R} \setminus \{6\} = D(f)}$

$D(g)$: Need $x-15 \geq 0$

$D(g) = \{x \mid x \geq 15\} = \boxed{[15, \infty) = D(g)}$

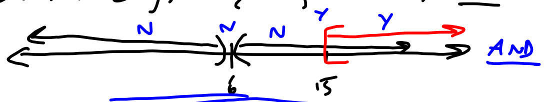
$\boxed{D(h) = \mathbb{R}} = (-\infty, \infty)$, b/c h is a polynomial.

b. (5 pts) Find $f+g$ and state its domain.

$(f+g)(x) = f(x) + g(x) = \dfrac{1}{x-6} + \sqrt{x-15}$

$D(f+g)$: Need to keep BOTH happy. "AND"

$D(f) \cap D(g) = \{x \mid x \in D(f)$ AND $x \in D(g)\}$



$\boxed{= [15, \infty) = D(f+g)}$

c. (5 pts) Find $g \circ h$ and state its domain.

$(g \circ h)(x) = g(h(x)) = g(☺) = \sqrt{☺ - 15} = \sqrt{h(x) - 15}$

$= \sqrt{x^2 - 2x - 15}$

No restriction

$D(g \circ h) = \{x \mid x \in D(h)$ and $h(x) \in D(g)\}$

$D(g \circ h) = \{x \mid x^2 - 2x \geq 15\}$

$= \{x \mid x^2 - 2x - 15 \geq 0\}$

Need $x^2 - 2x - 15 \geq 0$

x $a=1, b=-2, c=-15$

$b^2 - 4ac = 2^2 - 4(1)(-15) = 64$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{-(-2) \pm \sqrt{64}}{2(1)} = \dfrac{2 \pm 8}{2} = \dfrac{1 \pm 4}{1}$ $\boxed{\begin{array}{l} 5 = x \\ -3 = 3 \end{array}}$

$(x-5)(x-(-3))$

$(x+3)(x-5) = 0 \Rightarrow \boxed{x = 5, -3}$

$x^2 - 2x - 15 = x^2 - 2x + 1^2 - 1 - 15 = (x-1)^2 - 16 = 0$

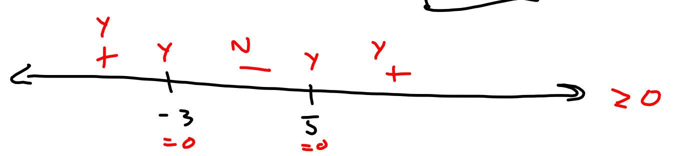
$\frac{2}{2} \to 1 \to 1^2$

$\Rightarrow (x-1)^2 = 16 \Rightarrow$

$x - 1 = \pm 4 \Rightarrow$

$x = 1 \pm 4$ $\boxed{\begin{array}{l} x = 5 \\ x = -3 \end{array}}$ $\Rightarrow (x-5)(x+3)$

$(x-5)(x+3)$



$\Rightarrow D(g \circ h) = (-\infty, -3] \cup [5, \infty)$

d. (**Bonus** 5 pts) Find $f \circ g \circ h$ and state its domain.

$f(x)=\frac{1}{x-6}$, $g(x)=\sqrt{x-15}$, $h(x)=x^2-2x$

$(f\circ g\circ h)(x)= f(g(h(x))) = f(g(x^2-2x)) = f(\sqrt{x^2-2x-15})$

$= \dfrac{1}{\sqrt{x^2-2x-15}-6}$

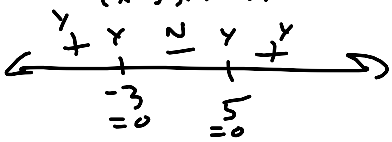
FORMALLY: $\{x \mid x\in D(h) \text{ and } h(x)\in D(g) \text{ and } g(h(x))\in D(f)\}$

$= \{x \mid x\in\mathbb{R} \text{ and } x^2-2x\ge 15 \text{ and } \sqrt{x^2-2x-15}\ne 6\}$

OR just inspect $f\circ g\circ h$ and observe:

Need $x^2-2x-15\ge 0$ and $\sqrt{x^2-2x-15}\ne 6$

$(x-5)(x+3)\ge 0$



$\underline{x\in(-\infty,-3]\cup[5,\infty)}$ and

$x^2-2x-15\ne 6$

$x^2-2x-21\ne 6$

$x^2-2x \ne 27$

$x^2-2x+1^2 \ne 27+1$

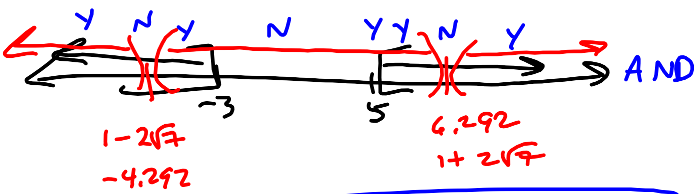
$(x-1)^2\ne 28$

$x-1\ne \pm\sqrt{28} = \pm 2\sqrt{7}$

$\underline{x\ne 1\pm 2\sqrt{7}}$

$\{x \mid x\ne 1\pm 2\sqrt{7}\}$

$= \mathbb{R}\setminus\{1\pm 2\sqrt{7}\}$



$\boxed{\begin{aligned} &= (-1, 1-2\sqrt{7})\cup(1-2\sqrt{7},-3] \\ &\quad \cup [5, 1+2\sqrt{7})\cup(1+2\sqrt{7},\infty) \\ &= D(f\circ g\circ h)\end{aligned}}$

5. Let $A = (-3,4)$ and $B = (8, 17)$. If you mastered point-slope as instructed, this is easy. If you always go back to slope-intercept, this is a lot of work.

a. (5 pts) Find an equation $y = m(x - x_1) + y_1$ of the line passing through A and B.

$A = (-3,4) = (x_1, y_1)$, $B = (8,17) = (x_2, y_2)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{17-4}{8-(-3)} = \frac{13}{11}$

$y - y_1 = m(x - x_1)$

$y = m(x - x_1) + y_1$

$y = m(x - x_1) + y_1$

$\boxed{y = \frac{13}{11}(x - (-3)) + 4}$

$y = \frac{13}{11}(x - 8) + 17$

b. (5 pts) Find an equation of the line $y = m(x - x_1) + y_1$ passing through $\left(\frac{3}{13}, \frac{5}{117}\right)$ that is parallel to the line from part a.

$\boxed{y = \frac{13}{11}\left(x - \frac{3}{13}\right) + \frac{5}{117}}$

c. (5 pts) Find an equation of the line $y = m(x - x_1) + y_1$ passing through $\left(\frac{3}{13}, \frac{5}{117}\right)$ that is perpendicular to the line from part a.

$m_\perp = -\frac{1}{m} = \frac{1}{\frac{-13}{11}} = 1 \cdot \frac{-11}{13} = -\frac{11}{13}$

$\boxed{y = -\frac{11}{13}\left(x - \frac{3}{13}\right) + \frac{5}{117}}$

6. Consider the quadratic function $f(x) = x^2 - 3x - 1$.

a. (5 pts) Solve the equation $f(x) = 0$ using the Quadratic Formula. For full credit, evaluate the discriminant and its square root, first. I'm looking for exact answers, not decimal approximations.

$a = 1, b = -3, c = -1$

$b^2 - 4ac = 3^2 - 4(1)(-1) = 9 + 4 = 13 \rightsquigarrow \sqrt{13}$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-(-3) + \sqrt{13}}{2(1)} = \boxed{\dfrac{3 \pm \sqrt{13}}{2} = x}$

b. (5 pts) Solve the equation $f(x) = 0$ by completing the square. Find the exact solution. Do not use a calculator.

$f(x) = x^2 - 3x - 1 = x^2 - 3x + \left(\frac{3}{2}\right)^2 - \frac{9}{4} - 1\left(\frac{4}{4}\right)$

$\left(\frac{3}{2} \to \left(\frac{3}{2}\right)^2 = \frac{9}{4}\right)$ scratch

$= \left(x - \frac{3}{2}\right)^2 - \frac{13}{4} \overset{\text{set}}{=} 0 \longrightarrow \left(x - \frac{3}{2}\right)^2 = \frac{13}{4}$

$x - \frac{3}{2} = \pm\sqrt{\frac{13}{4}} = \pm \frac{\sqrt{13}}{2}$

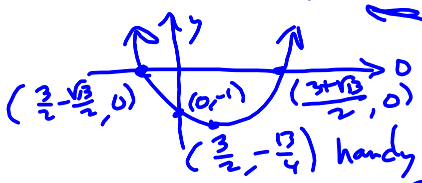
$\boxed{x = \frac{3}{2} \pm \frac{\sqrt{13}}{2}}$

c. (5 pts) Write $f(x)$ in standard form $f(x) = a(x-h)^2 + k$ and sketch its graph. Include the x- and y-interepts, as well as the vertex in your graph.

$f(x) = \left(x - \frac{3}{2}\right)^2 - \frac{13}{4}$

$(h,k) = \left(\frac{3}{2}, -\frac{13}{4}\right)$

$a(x-h)^2 + k$



handy for graphs!

d. (5 pts) What is the domain of f? What is the range of f?

$\boxed{D(f) = \mathbb{R}} = (-\infty, \infty)$

$\boxed{R(f) = \left[-\frac{13}{4}, \infty\right)}$

7. (5 pts) Solve the equation $\dfrac{5x+25}{x+3} = \dfrac{4x+20}{x+2}$ for x.

$\Rightarrow (5x+25)(x+2) = (4x+20)(x+3)$ cross-multiply is fine

$\Rightarrow 5x^2+35x+50 = 4x^2+32x+60$

$\Rightarrow x^2+3x-10=0 \Rightarrow$

$(x+5)(x-2)=0 \Rightarrow x \in \{-5, 2\}$

LCD $= (x+3)(x+2)$

Alternate, looking ahead to #8:

$\left(\dfrac{5x+25}{x+3}\right)\left(\dfrac{x+2}{x+2}\right) - \left(\dfrac{4x+20}{x+2}\right)\left(\dfrac{x+3}{x+3}\right) = 0 \Rightarrow$

$\dfrac{(5x+25)(x+2) - (4x+20)(x+3)}{\text{LCD}} = 0 \Rightarrow$

$5x^2+35x+50-(4x^2+32x+60)=0 \Rightarrow$

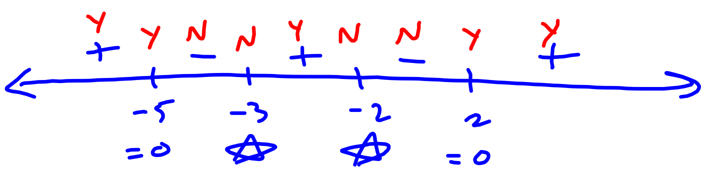
$5x^2+35x+50-4x^2-32x-60=0 \Rightarrow$

$x^2+3x-10=0$, etc.

8. (5 pts) Solve the inequality $\dfrac{5x+25}{x+3} \geq \dfrac{4x+20}{x+2}$ for x.

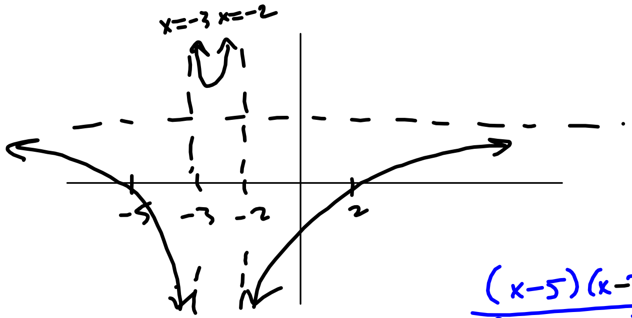
$\dfrac{x^2+3x-10}{(x+3)(x+2)} \geq 0$

$\dfrac{(x+5)(x-2)^1}{(x+3)(x+2)} \geq 0$ is positive to the right.



≥ 0 want "+" " "=0"

$= \boxed{(-\infty,-5] \cup (-3,-2) \cup [2,\infty)}$



y=1

How?

$\dfrac{(x-5)(x-2)}{(x+3)(x+2)} = \dfrac{x^2+\text{lower}}{x^2+\text{lower}} \xrightarrow{x\to\infty} \dfrac{x^2}{x^2} = 1$

$\boxed{y=1 \text{ is H.A.}}$

9. Let $f(x) = x^4 - 2x^3 + x^2 + 8x - 20$.

a. (5 pts) List all possible rational zeros of f.

factors of 20

$\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

b. (5 pts) Use synthetic division and the rational zeros of f to split f into the product of 2 linear factors and one irreducible quadratic factor.

$$\begin{array}{r|rrrrr} 1 & 1 & -2 & 1 & 8 & -20 \\ & & 1 & -1 & 0 & 8 \\ \hline & 1 & -1 & 0 & 8 & \end{array}$$

$$\begin{array}{r|rr} -1 & 1 & -2 \end{array}$$ Nah

$$\begin{array}{r|rrrrr} 2 & 1 & -2 & 1 & 8 & -20 \\ & & 2 & 0 & 2 & 20 \\ \hline & 1 & 0 & 1 & 10 & 0 \end{array}$$ Sweet!

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 1 & 10 \\ & & 2 & 4 & 10 \\ \hline & 1 & 2 & 5 & \end{array}$$ Nope

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 1 & 10 \\ & & -2 & 4 & -10 \\ \hline & 1 & -2 & 5 & 0 \end{array}$$ Sweet!

Part b

$$\boxed{(x-2)(x+2)(x^2-2x+5)}$$

c. (**Bonus** 5 pts) Find the nonreal zeros of f, using your work from part b.

$x^2 - 2x + 1^2 - 1 + 5 = (x-1)^2 + 4 \overset{\text{SET}}{=} 0 \implies$

$(x+1)^2 = -4$

$x + 1 = \pm\sqrt{-4} = \pm 2i$

$\boxed{x = -1 \pm 2i}$

d. (**Bonus** 5 pts) Split f into the product of 4 linear factors.

$$\boxed{(x-2)(x+2)(x-(-1+2i))(x-(-1-2i))}$$

$x = \pm 2$

10. (5 pts) Sketch the graph of $R(x)=\dfrac{x-2}{x+4}$ Show all intercepts and asymptotes.

$D(R) = \mathbb{R}\setminus\{-4\}$

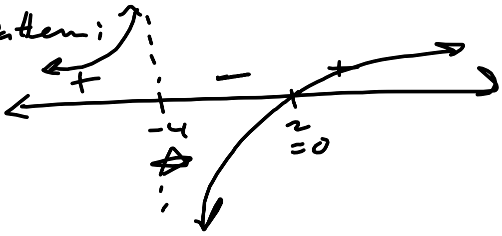
$\boxed{\text{V.A. } x=-4}$

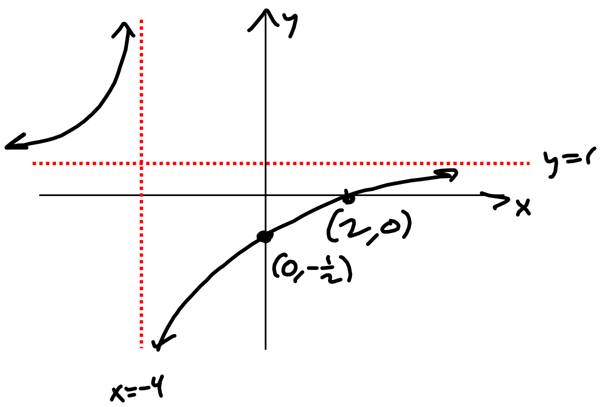
$\dfrac{x^1+\dots}{x^1+\dots} \xrightarrow{x\to\infty} \dfrac{x}{x} = \boxed{1=y \text{ is H.A.}}$

Zeros: $x-2=0 \implies x=2$

$R(0) = \dfrac{-2}{4} = -\dfrac{1}{2}$

Sign Pattern:





11. (**Bonus** 5 pts) Sketch the graph of $T(x) = \dfrac{x^2+3x-28}{x+5}$. Show all intercepts and asymptotes.

$D(T) = \mathbb{R} \setminus \{-5\}$

$\boxed{\text{V.A.}: \ x = -5}$

H.A. : None $\quad \dfrac{x^2 + \sim}{x^1 + \sim}$

$\boxed{(0, -\frac{28}{5}) \text{ is y-int}}$

Oblique Asymptote:

$x+5 \overline{)\, x^2+3x-28}$

$$\begin{array}{r|rrr} -5 & 1 & 3 & -28 \\ & & -5 & 10 \\ \hline & 1 & -2 & -18 \end{array}$$

$\boxed{y = x-2 \text{ is O.A.}}$

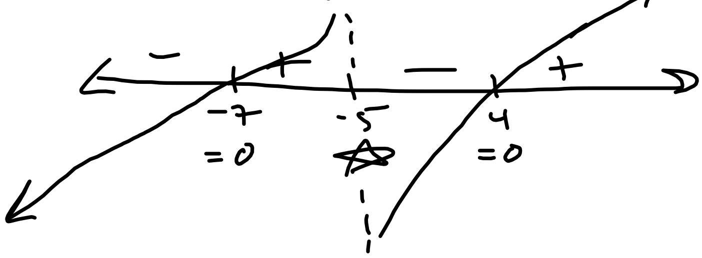
x-int: $x^2+3x-28=0$

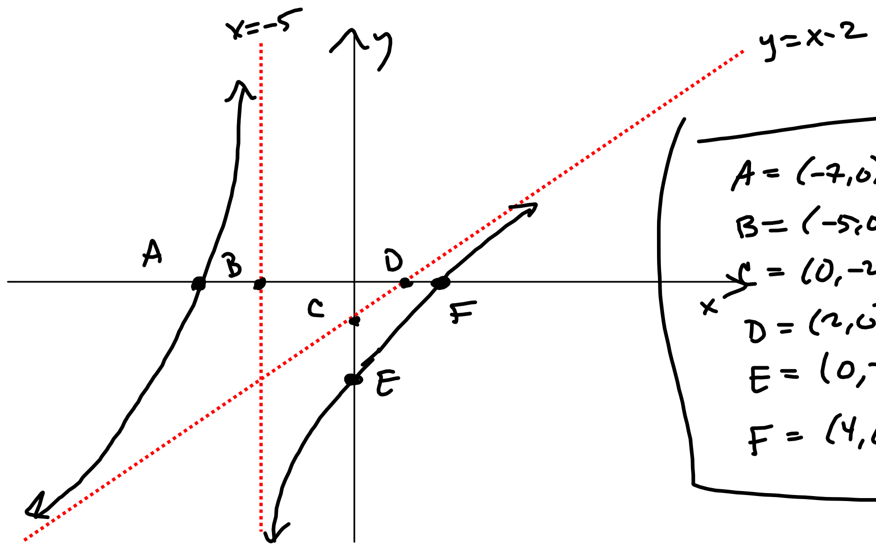
$(x+7)(x-4) = 0 \Rightarrow$

Or quadratic formula.

" complete the square.

$x \in \{-7, 4\}$





$A = (-7, 0)$

$B = (-5, 0)$

$C = (0, -2)$

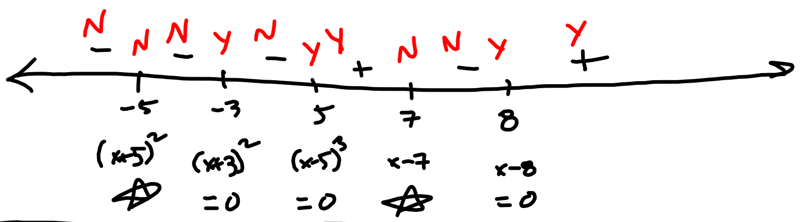
$D = (2, 0)$

$E = (0, -\frac{28}{5})$

$F = (4, 0)$

12. (**Bonus** 5 pts) Let $U(x) = \dfrac{(x+3)^2(x-5)^3(x-8)}{(x+5)^2(x-7)} =$ What is the domain of $V(x) = \sqrt{U(x)}$?

Need $u(x) \geq 0$ and $x \neq -5, 7$ $\qquad -5, -3, 5, 7, 8$



≥ 0

want "+" and "= 0"

$$\boxed{D(V) = \{-3\} \cup [5, 7) \cup [8, \infty)}$$

13. (5 pts) Sketch the graph of $g(x) = 4 \cdot 3^{2x-10}$ in 4 steps, counting the graph of $f(x) = 3^x$ as the first step. Label the key points $\left(-1, \frac{1}{3}\right), (0,1)$, and $(1,3)$ in the first graph, and track their movements through each step. For instance, $\left(-1, \frac{1}{3}\right)$ will move to $\left(-1, \frac{4}{3}\right)$ in the 2nd graph. Then $\left(-1, \frac{4}{3}\right)$ will move to another point in the 3rd graph, etc.