(1) (10 pts) Feasible Region for

$2x - 7y \le 14$

$3x + 2y < 6$

$x \le 7$

$y \le 3$

Shading:

$x \le 7$
$0 \le 7$?
$(0,0)$ GOOD

$2x - 7y \le 14$
$0 \le 14$? Yes
$(0,0)$ Good

| x | y |
|---|---|
| 0 | -2 |
| 7 | 0 |

GOOD

$3x + 2y < 6$
$0 < 6$?
$(0,0)$ GOOD

| x | y |
|---|---|
| 0 | 3 |
| 2 | 0 |

$A = (0,3)$

GOOD

$y \le 3$
$0 \le 3$?
$(0,0)$ good

GOOD

Corners:
$A = (0,3)$
$C = \left(\frac{14}{5}, -\frac{6}{5}\right)$

Find E:

$2x - 7y = 14$
$3x + 2y = 6$

$6x - 21y = 42$
$-\ (6x + 4y = -12)$

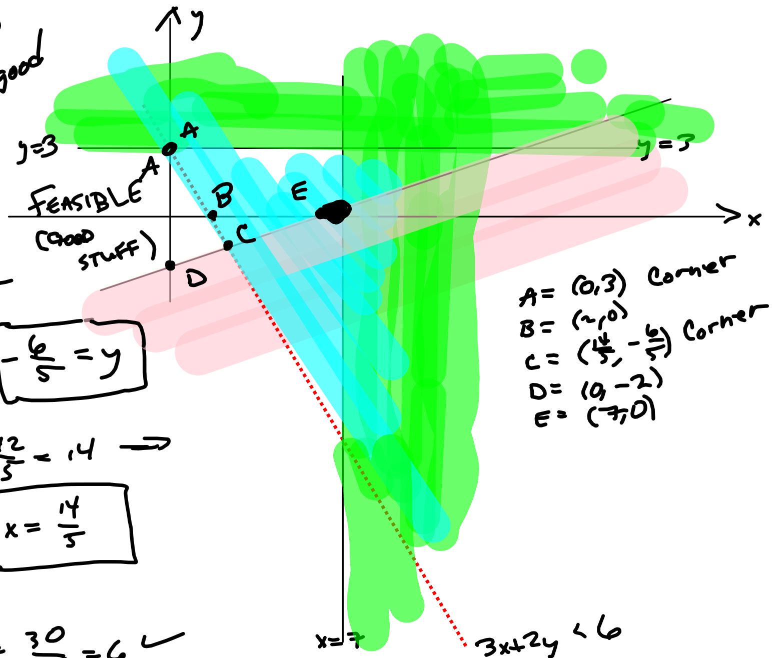
$-25y = 30$

$y = -\frac{30}{25} = \boxed{-\frac{6}{5} = y}$

$2x - 7y = 2x - 7\left(-\frac{6}{5}\right) = 2x + \frac{42}{5} = 14 \rightarrow$

$2x = -\frac{42}{5} + \frac{70}{5} = \frac{28}{5} \rightarrow \boxed{x = \frac{14}{5}}$

Check $3\left(\frac{14}{5}\right) + 2\left(-\frac{6}{5}\right) = \frac{42-12}{5} = \frac{30}{5} = 6$ ✓




$A = (0,3)$ Corner
$B = (2,0)$
$C = \left(\frac{14}{5}, -\frac{6}{5}\right)$ Corner
$D = (0,-2)$
$E = (7,0)$

2) 10 pts $A=(4,5), B=(-1,2), C=(10,-5)$

Let $d_1 = d(A,B)$, $d_2 = d(A,C)$, $d_3 = d(B,C)$

$$d_1^2 = (4+1)^2 + (5-2)^2 = 5^2 + 3^2 = 25+9 = 34$$

$$d_2^2 = (4-10)^2 + (5+5)^2 = 6^2 + 10^2 = 136$$

$$d_3^2 = (-1-10)^2 + (2+5)^2 = 11^2 + 7^2 = 121+49 = 170$$

$d_1^2 + d_2^2 = 34 + 136 = 170 = d_3^2 \longrightarrow \Delta ABC$ is a right triangle.

3a) 5 pts $|4x-9| \le 7 \longrightarrow$

$4x-9 \le 7$ & $4x-9 \ge -7$

$4x \le 16$ & $4x \ge 2$

$x \le 4$ & $x \ge \frac{2}{4} = \frac{1}{2}$

$\Longrightarrow \boxed{x \in [\frac{1}{2}, 4]}$

b) 5 pts $|4x-9| > 7 \longrightarrow$

$4x-9 > 7$ OR $4x-9 < -7$

$4x > 16$ OR $4x < 2$

$x > 4$ OR $x < \frac{1}{2}$

$\longrightarrow \boxed{x \in (-\infty, \frac{1}{2}) \cup (4, \infty)}$

c) 5 pts $|4x-9| \le 0 \longrightarrow$

$4x-9 = 0 \longrightarrow$

$\boxed{x = \frac{9}{4}}$

d) 5 pts $|4x-9| > -7 \longrightarrow$

$\boxed{x = \text{any real \#}}$

b/c $|\Delta| \ge 0 > -7 \ \forall x \in \mathbb{R}$

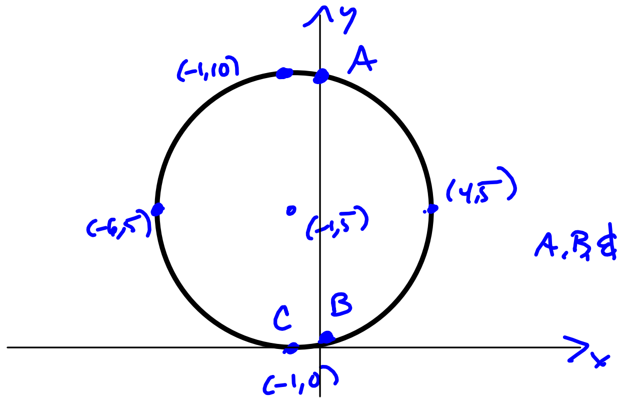
(4) $x^2+y^2+2x-10y=-1$ is a circle

(a) (5pts) write in stdd form:

$$x^2+2x+1^2+y^2-10y+5^2=-1+1+25$$

$$\boxed{(x+1)^2+(y-5)^2=25 \quad (h,k)=(-1,5)=\text{center} \quad \text{Radius}=\sqrt{25}=5=r}$$

(b)




A, B & C for Part c.

(c) (5pts) x- & y-ints:

$\boxed{\text{x-int}: (-1,0)}$

y-int: $(0+1)^2+(y-5)^2=25$

$(y-5)^2=25-1=24$

$y=5\pm\sqrt{24}=5\pm2\sqrt{6}$

$$\boxed{A=(0,5+2\sqrt{6}) \quad B=(0,5-2\sqrt{6}) \quad C=(-1,0)}$$

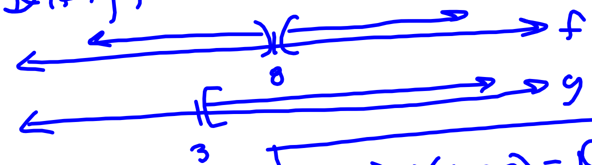
5) $f(x) = \frac{1}{x-8}$, $g(x) = \sqrt{x-3}$, $h(x) = x^2+2x$

a) 5pts

$$\boxed{\begin{aligned} &\mathcal{D}(f) = \mathbb{R} \setminus \{8\} \\ &\mathcal{D}(g) = [3,\infty) \\ &\mathcal{D}(h) = (-\infty,\infty) = \mathbb{R} \end{aligned}}$$

b) 5pts $(f+g)(x) = \frac{1}{x-8} + \sqrt{x-3}$

$\mathcal{D}(f+g) = \mathcal{D}(f) \cap \mathcal{D}(g) = \{x \mid x \in \mathcal{D}(f) \text{ AND } x \in \mathcal{D}(g)\}$



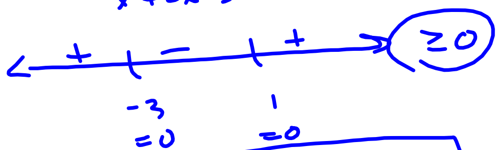

$\mathcal{D}(f) \cap \mathcal{D}(g) = \boxed{[3,8) \cup (8,\infty) = \mathcal{D}(f+g)}$

c) $(g \circ h)(x) = \sqrt{x^2+2x-3}$

Need $h(x) \in \mathcal{D}(g)$, i.e.

$x^2+2x \geq 3 \longrightarrow$

$x^2+2x-3 = (x+3)(x-1) \geq 0$




$\mathcal{D} = \boxed{(-\infty,-3] \cup [1,\infty)} = \mathcal{D}(h \circ g)$

d) 5pts $(f \circ h)(x) = f(h(x)) = \frac{1}{x^2+2x-8}$

$\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(h) \text{ and } h(x) \in \mathcal{D}(f)\}$

Need $x^2+2x \neq 8 \longrightarrow$

$x^2+2x-8 = (x+4)(x-2) \neq 0$

$\longrightarrow x \neq -4, 2$ → Least preferred version.

$\Rightarrow \mathcal{D}(f \circ g) = \mathbb{R} \setminus \{-4,2\} = \{x \mid x \neq -4 \text{ and } x \neq 2\}$

$= \{x \mid x \neq 4 \text{ or } 2\} = \{x \mid x \notin \{-4,2\}\}$

$= (-\infty,-4) \cup (-4,2) \cup (2,\infty)$

Any version

(6) $A=(3,-5),\ B=(7,8) \Longrightarrow$

(a) 5pts $m = \text{slope} = \dfrac{8-(-5)}{7-3} = \dfrac{13}{4}$

$y = m(x-x_1)+y_1$

$\boxed{y = \frac{13}{4}(x-3)-5} = \frac{13}{4}x - \frac{39}{4} - \frac{20}{4} = \frac{13}{4}x - \frac{59}{4} = 3.25x - 14.75$

(b) 5pts parallel to (a), thru $(-11,7) \Longrightarrow$

$\boxed{y = \frac{13}{4}(x+11)+7} = \frac{13}{4}x + \frac{143}{4} + \frac{28}{4} = \frac{13}{4}x + \frac{171}{4} = 3.25x + 42.75$

$\boxed{y = 3.25x + 42.75}$

(c) 5pts Perpendicular to (a), thru $(-11,7) \Longrightarrow$

Because people waste time.

$\boxed{y = -\frac{4}{13}(x+11)+7} = -\frac{4}{13}x - \frac{44}{13} + \frac{91}{13} = -\frac{4}{13}x + \frac{47}{13}$

$\approx -0.3076923077x + 3.615384461$

$\approx -0.308x + 3.615$

(7) $f(x) = 2x^2-3x-1$

(a) 5pts $a=2,\ b=-3,\ c=-1 \Longrightarrow$

$b^2-4ac = 9-4(2)(-1) = 9+8 = 17 \Longrightarrow$

$x = \dfrac{3 \pm \sqrt{17}}{2(2)} = \boxed{\dfrac{3\pm\sqrt{17}}{4} = x}$

$-1 - \frac{9}{8} = \frac{-8-9}{8} = \frac{-17}{8}$

(b) 5pts $2x^2-3x-1$

$= 2\left(x^2 - \frac{3}{2}x + \left(\frac{3}{4}\right)^2\right) - 1 - 2\left(\frac{9}{16}\right)$

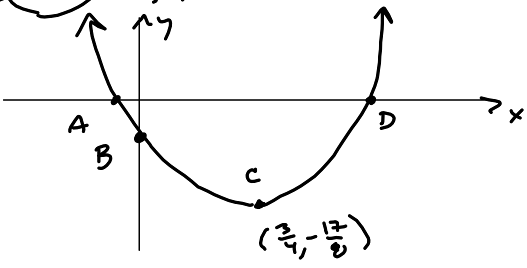
$= 2\left(x-\frac{3}{4}\right)^2 - \frac{17}{8} \overset{\text{SET}}{=} 0 \Longrightarrow$

$2\left(x-\frac{3}{4}\right)^2 = \frac{17}{8} \Longrightarrow$

$\left(x-\frac{3}{4}\right)^2 = \frac{17}{16} \Longrightarrow$

$\boxed{x = \dfrac{3\pm\sqrt{17}}{4}} \approx 1.780776406$ OR $-0.2807764060$

(c) 5pts $f(x) = 2\left(x-\frac{3}{4}\right)^2 - \frac{17}{8}$ is standard form




$A = \left(\frac{3-\sqrt{17}}{4}, 0\right)$

$B = (0,-1)$

$C = \left(\frac{3}{4}, -\frac{17}{8}\right)$

$D = \left(\frac{3+\sqrt{17}}{4}, 0\right)$

(d) 5pts $\mathcal{D}(f) = \mathbb{R} = (-\infty, \infty) = \{x \mid x \in \mathbb{R}\}$

= the set of all real numbers. ← any of these

Any version of the above.

$\boxed{\text{Range is } \left[-\frac{17}{8}, \infty\right)} = \mathcal{R}(f)$

(8) Factor $140x^2 - 111x - 198$

$a = 140,\ b = -111,\ c = -198 \Rightarrow$

$b^2 - 4ac = 111^2 - 4(140)(-198)$

$= 12321 + 110880$

$= 123201$

$\sqrt{b^2-4ac} = \sqrt{123201} = 351 \Rightarrow$

$x = \dfrac{111 \pm 351}{280}$ → $\dfrac{462}{280} = \dfrac{231}{140} = \dfrac{33}{20}$ ; → $\dfrac{-240}{280} = -\dfrac{6}{7}$

$2 \,|\, 140$, $2 \,|\, 70$, $5 \,|\, 35$, $7$ ; $7 \,|\, 231$, $3 \,|\, 33$, $11$

$2 \cdot 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11$

Subtract to obtain

$-111$ $\quad 4620$

$\Rightarrow f(x) = 140\left(x - \frac{33}{20}\right)\left(x + \frac{6}{7}\right) (7)(20)\left(x - \frac{33}{20}\right)\left(x + \frac{6}{7}\right)$

$= \boxed{(20x - 33)(7x + 6)}$

9 5pts $\dfrac{x}{x-1} = \dfrac{-6}{x-7} \Rightarrow$

$x^2 - 7x = -6x + 6 \Rightarrow$

$x^2 - x - 6 = (x-3)(x+2) = 0 \Rightarrow$

$\boxed{x \in \{-2, 3\}}$

10 $\dfrac{x}{x-1} \geq \dfrac{-6}{x-7} \Rightarrow$

$\left(\dfrac{x}{x-1}\right)\left(\dfrac{x-7}{x-7}\right) + \left(\dfrac{6}{x-7}\right)\left(\dfrac{x-1}{x-1}\right)$

$= \dfrac{x^2 - 7x + 6x - 6}{(x-1)(x-7)} = \dfrac{x^2 - x - 6}{(x-1)(x-7)} = \dfrac{(x-3)(x+2)}{(x-1)(x-7)} \geq 0$

$+ \quad - \quad + \quad - \quad +$ $\quad \geq 0$

$-2$: $=0$ $\quad 1$: $\cancel{A}$ $\quad 3$: $=0$ $\quad 7$: $\cancel{A}$

$\boxed{x \in (-\infty, -2] \cup (1, 3] \cup (7, \infty)}$

(11) $f(x) = \frac{1}{x-2}$

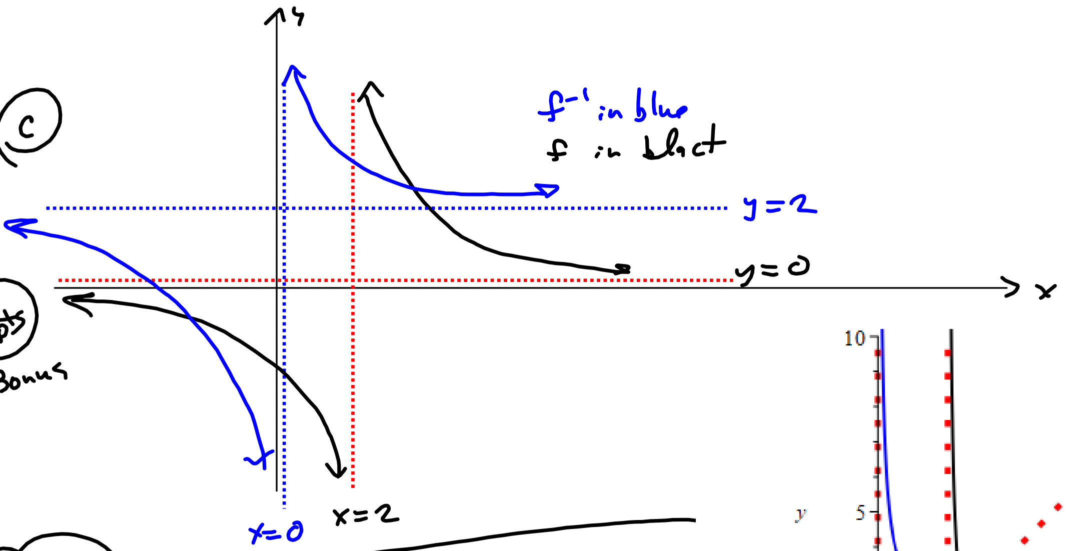
(a) 5pts Suppose $f(x_1)=f(x_2)$. We show $x_1=x_2$

$$\frac{1}{x_1-2} = \frac{1}{x_2-2} \Rightarrow x_2-2 = x_1-2 \Rightarrow x_2 = x_1 \quad \blacksquare$$

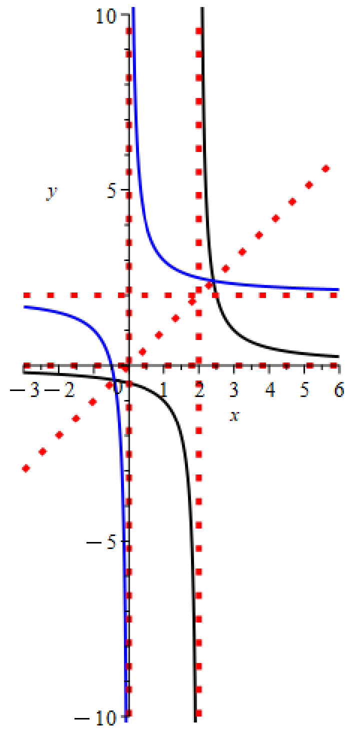
(b) 5pts $f^{-1}(x)$:

$$x = \frac{1}{y-2} \Rightarrow y-2 = \frac{1}{x} \Rightarrow y = \boxed{\frac{1}{x}+2 = f^{-1}(x)}$$

(c)




5pts Bonus




(d) 5pts

$$\boxed{\begin{aligned} \mathcal{D}(f) &= \mathcal{R}(f^{-1}) = \mathbb{R}\setminus\{2\} \\ \mathcal{R}(f) &= \mathcal{D}(f^{-1}) = \mathbb{R}\setminus\{0\} \end{aligned}}$$

12 $f(x) = x^4 - 7x^3 + 21x^2 - 37x + 30$

a 5pts Rational zeros possible:

$\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

b 5pts

$$\begin{array}{r|rrrrr} 1 & 1 & -7 & 21 & -37 & 30 \\ & & 1 & -6 & 15 & 22 \\ \hline & 1 & -6 & 15 & 22 & \end{array}$$

$$\begin{array}{r|rrrrr} -1 & 1 & -7 & 21 & -37 & 30 \\ & & -1 & 8 & -29 & \\ \hline & 1 & -8 & 29 & \text{NO} & \end{array}$$

$$\begin{array}{r|rrrrr} 2 & 1 & -7 & 21 & -37 & 30 \\ & & 2 & -10 & 22 & -30 \\ \hline 2 & 1 & -5 & 11 & -15 & 0 \\ & & 2 & -6 & 10 & \\ \hline & 1 & -3 & 5 & \text{NO} & \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & -5 & 11 & -15 \\ & & -2 & 14 & \text{No} \\ \hline & 1 & -7 & 25 & \end{array}$$

$$\begin{array}{r|rrrr} 3 & 1 & -5 & 11 & -15 \\ & & 3 & -6 & 15 \\ \hline & 1 & -2 & 5 & 0 \end{array} \quad !$$

So $\boxed{f(x) = (x-2)(x-3)(x^2-2x+5)}$

c 5pts Nonreal zeros

$x^2 - 2x + 5 = x^2 - 2x + 1^2 - 1 + 5 = (x-1)^2 + 4 = 0 \Rightarrow$

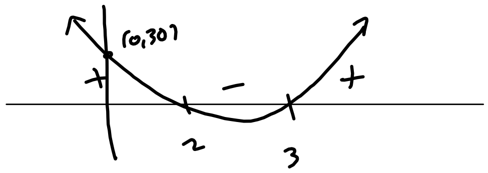
$(x-1)^2 = -4$

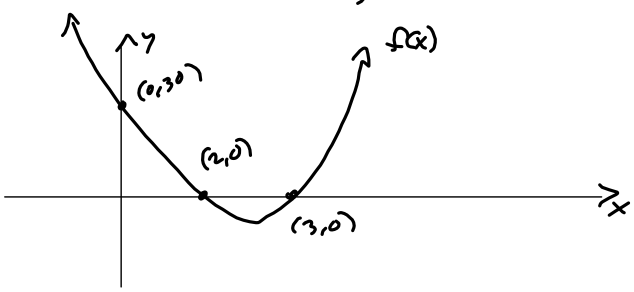
$x - 1 = \pm\sqrt{-4} = \pm 2i$

$\boxed{x = 1 \pm 2i}$

d 5pts $\boxed{f(x) = (x-2)(x-3)(x-1+2i)(x-1-2i)}$

e 5pts



(13) we sketch, showing all intercepts & asymptotes.

(a) 10pts $\frac{x-2}{x+4}$

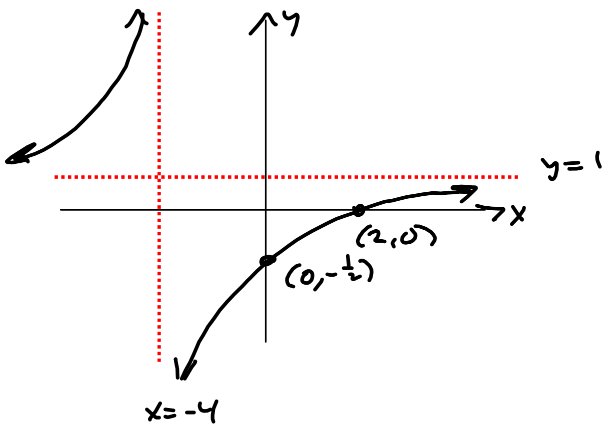
$\mathcal{D} = \mathbb{R} \setminus \{-4\}$

$\boxed{x=-4 \text{ is V.A.}}$

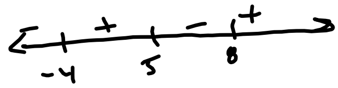
$\boxed{y=1 \text{ is H.A.}}$

$(0,-\frac{1}{2})$ is y-int

$(2,0)$ is x-int



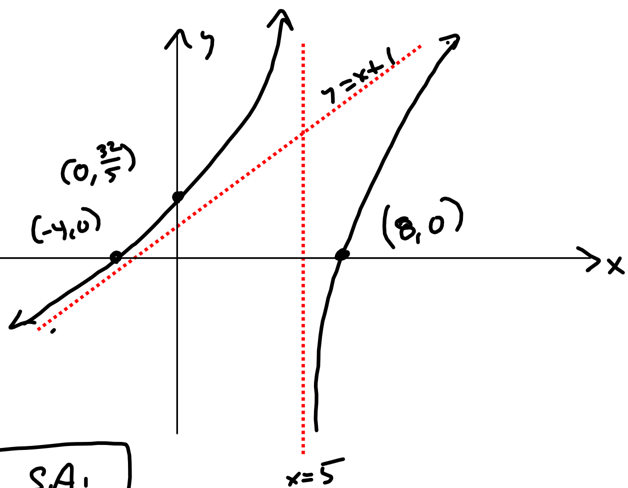

(b) 10pts $\frac{x^2-4x-32}{x-5} = \frac{(x-8)(x+4)}{x-5}$




V.A.: $x=5$

$(-4,0), (8,0)$ x-ints.

$(0, 32/5)$ y-int




Slant Asymptote:

$$\begin{array}{r|rrr} 5 & 1 & -4 & -32 \\ & & 5 & 5 \\ \hline & 1 & 1 & \end{array}$$

$\boxed{y = x+1 \text{ is S.A.}}$

14 10pts $u(x) = \dfrac{(x+3)(x-5)(x-8)}{(x+5)(x-7)} = \dfrac{\sim\sim +120}{\sim -35}$

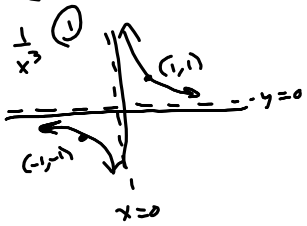
$-\quad -5 \quad + \quad -3 \quad - \quad 5 \quad + \quad 7 \quad - \quad 8 \quad +$

$-5$: $\cancel{\Delta}$, $-3$: $=0$, $5$: $=0$, $7$: $\cancel{\Delta}$, $8$: $=0$

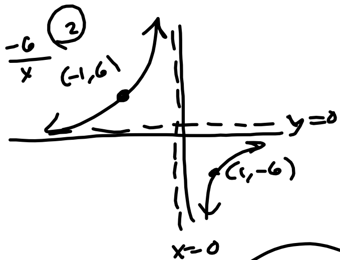
$$\boxed{\mathcal{D}\left(\sqrt{u(x)}\right) = \{x \mid u(x) \geq 0\} = (-5,-3] \cup [5,7) \cup [8,\infty)}$$

15 10pts $g(x) = \dfrac{-6}{(3x+21)^3} + 10 = \dfrac{-6}{(3(x+7))^3} + 10$

1 $\dfrac{1}{x^3}$




2 $\dfrac{-6}{x}$

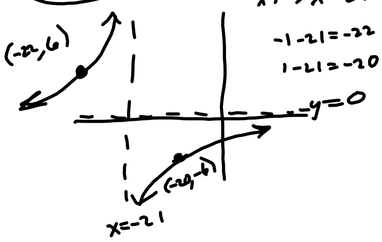



3 Method 1

$x \mapsto x-21$

$-1-21=-22$

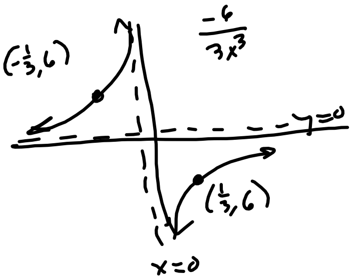
$1-21=-20$




3 Method 2

$x \mapsto \frac{1}{3}x$

$\dfrac{-6}{3x^3}$




4

METHOD 1

$x \mapsto \frac{x}{3}$

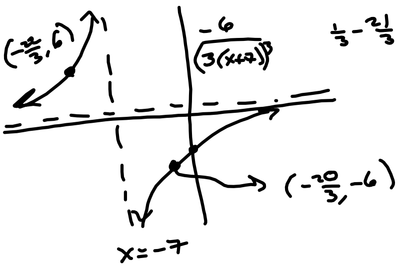
$-\frac{22}{3}$

$-\frac{20}{3}$

METHOD 2 $x \mapsto x-7$

$-\frac{1}{3} - \frac{21}{3} = -\frac{22}{3}$

$\frac{1}{3} - \frac{21}{3} = -\frac{20}{3}$




5

$g(x) = \dfrac{-6}{(3(x+7))^3} + 10$