1340 Final Mills

1 (10 pts) We sketch the feasible region. I scratch out the bad stuff.

$x-3y \le 15$
$2x+9y < 36$
$x > -2$

$x-3y \le 15$

x	y
0	-5
15	0

$\boxed{A=(0,-5) \quad B=(15,0)}$

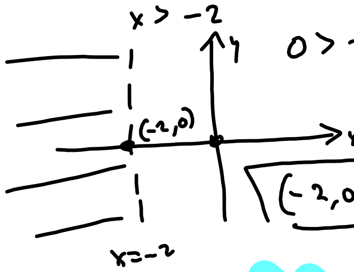
$0 \le 15$?
(0,0) Good

$2x+9y < 36$

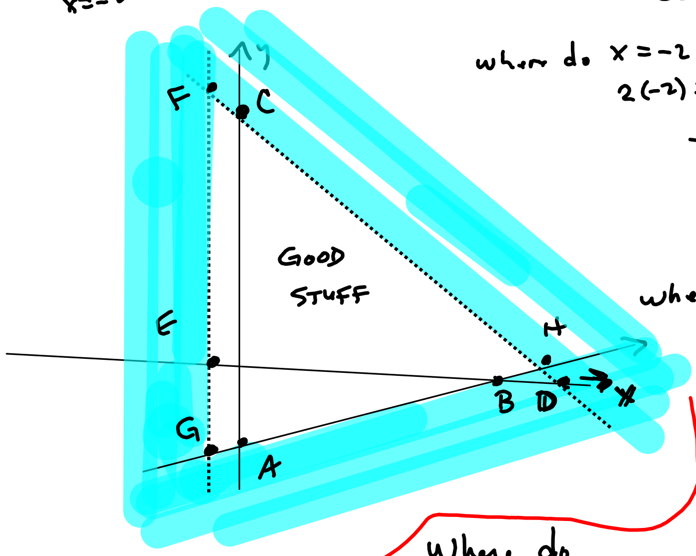
x	y
0	4
18	0

$\boxed{C=(0,4) \quad D=(18,0)}$

$x > -2$ $\quad 0 > -2$? Yes (0,0) Good



$\boxed{(-2,0)=E}$



This is #2 stuff:

CORNER POINTS

when do $x=-2$ & $2x+9y=36$ meet?

$2(-2)+9y = 9y-4 = 36$

$\Rightarrow 9y = 40 \Rightarrow y = \frac{40}{9}$

$\boxed{F = \left(-2, \frac{40}{9}\right) \text{ corner}}$

where do $x=-2$ & $x-3y=15$ meet?

$-2-3y=15$

$-3y=17$

$y=-\frac{17}{3}$

$\boxed{G=\left(-2,-\frac{17}{3}\right)}$ CORNER

Where do $x-3y=15$ & $2x+9y=36$ meet?

All #2 stuff.

$\Rightarrow x=3y+15$

$\Rightarrow 2(3y+15)+9y = 6y+30+9y=36$

$\Rightarrow 15y=6$

$\Rightarrow y=\frac{6}{15}=\boxed{\frac{2}{5}=y}$

$\Rightarrow x-3y = x-3\left(\frac{2}{5}\right) = x-\frac{6}{5}=15 \Rightarrow 5x-6=75$

$\Rightarrow 5x=81$

$\Rightarrow \boxed{x=\frac{81}{5}}$

2 (5 pts)

$F=\left(-2,\frac{40}{9}\right)=(-2,4.\overline{44})$

$G=\left(-2,-\frac{17}{3}\right)=(-2,-5.\overline{6})$

$H=\left(\frac{81}{5},\frac{2}{5}\right)=(16.2,.4)$

See work above

#1 the hard way

$x - 3y \le 15$

$-3y \le -x + 15$

$y \ge \frac{1}{3}x - 5 \overset{SET}{=} 0$

$\Rightarrow \frac{1}{3}x = 5 \Rightarrow$

$x = 15 \rightsquigarrow (15, 0)$ x-int

y-int: $(0, -5)$

Above line good

$x > -2$

Good to right of $x = -2$

$2x + 9y < 36$

$9y < -2x + 36$

$\boxed{y < -\frac{2}{9}x + 4}$

y-int: $(0, 4)$

$y = 0 \Rightarrow -\frac{2}{9}x = -4$

$-2x = -36$

$x = 18$

x-int: $(18, 0)$

Good BELOW the line

3 (a) 5pts $|-3x-9| \geq 6$

$\Rightarrow -3x-9 \geq 6$ OR $-3x-9 \leq -6$

$3x+9 \leq -6$ $\qquad$ $3x+9 \geq 6$

$3x \leq -15$ $\qquad$ $3x \geq -3$

$x \in \boxed{\{x \mid x \leq -5 \text{ OR } x \geq -1\}}$

$= \boxed{(-\infty, -5] \cup [-1, \infty)}$

Alternate

$-3x-9 \geq 6$ OR $-3x-9 \leq -6$

$-3x \geq 15$ $\qquad$ $-3x \leq 3$

$x \leq \frac{15}{-3} = -5$ $\qquad$ $x \geq -1$

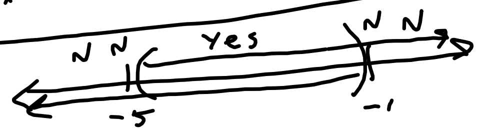
(b) 5pts $|-3x-9| < 6$

$\Rightarrow -3x-9 < 6$ AND $-3x-9 > -6$

$3x+9 > -6$ $\qquad$ $3x+9 < 6$

$3x > -15$ $\qquad$ $3x < -3$

$\Rightarrow \boxed{x \in \{x \mid x > -5 \text{ AND } x < -1\}}$

$=$ 

$= \boxed{(-5, -1)}$

Alt:

$-3x-9 < 6$ AND $-3x-9 > -6$

$-3x < 15$ $\qquad$ $-3x > 3$

$x > \frac{15}{-3} = -5$ $\qquad$ $x < -1$

(c) 5pts $|-3x-9| < -6$ Nevah!

$\boxed{\text{No Sol'n}}$

$|A| \geq 0.$

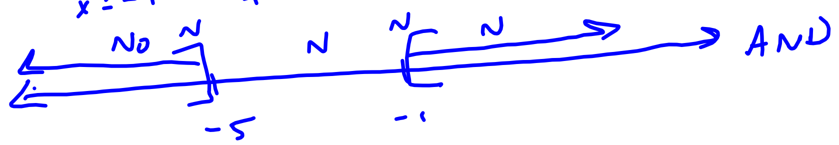
Never $< 0.$

Following rules w/o "seeing" it, immediately:

$-3x-9 \leq -6$ AND $-3x-9 \geq -(-6) = 6$

$-3x \leq 3$ $\qquad$ $-3x \geq 15$

$x \geq -1$ AND $x \leq -5$

$x \in$ 

$= \emptyset =$ Empty set, i.e. No Sol'n.

4) a) 5pts $f(x)=\frac{1}{x-6}$, $g(x)=\sqrt{x-15}$, $h(x)=x^2-2x$

$\mathcal{D}(f)=\{x \mid x\neq 6\}=\mathbb{R}\setminus\{6\}=\boxed{(-\infty,6)\cup(6,\infty)=\mathcal{D}(f)}$

$\mathcal{D}(g)=\{x \mid x\geq 15\}=\boxed{[15,\infty)=\mathcal{D}(g)}$

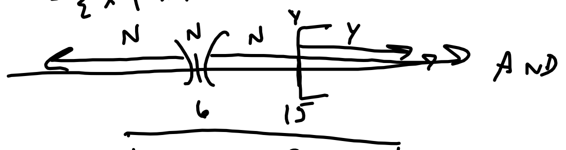
$\mathcal{D}(h)=\mathbb{R}=(-\infty,\infty)=\{x \mid x \text{ is real}\}=\mathcal{D}(h)$

b) 5pts $(f+g)(x)=f(x)+g(x)=\boxed{\frac{1}{x-6}+\sqrt{x-3}=f+g}$

$\mathcal{D}(f+g)=\mathcal{D}(f)\cap\mathcal{D}(g)$

$=\{x \mid x\in\mathcal{D}(f) \text{ and } x\in\mathcal{D}(g)\}$

$=\{x \mid x\neq 6 \text{ and } x\geq 15\}=\{x \mid x\geq 15\}$



$=\boxed{[15,\infty)=\mathcal{D}(f+g)}$

c) 5pts $(g\circ h)(x)=\sqrt{h(x)-15}=\boxed{\sqrt{x^2-2x-15}=g\circ h}=\sqrt{(x-5)(x+3)}$

METHOD 1

$\Rightarrow \mathcal{D}(g\circ h)=\{x \mid x\in\mathcal{D}(h) \text{ and } h(x)\in\mathcal{D}(g)\}$

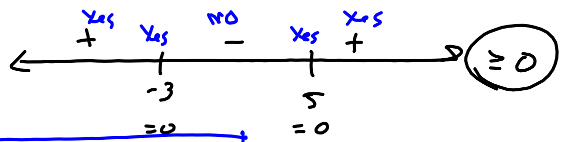
$=\{x \mid x\in\mathbb{R} \text{ and } h(x)\geq 15\}$

$=\{x \mid x^2-2x\geq 15\}$

$=\{x \mid x^2-2x-15\geq 0\}$

$=\{x \mid (x-5)(x+3)\geq 0\}$

$\Rightarrow$



$=\boxed{\begin{array}{l}(-\infty,-3]\cup[5,\infty)\\ =\mathcal{D}(g\circ h)\end{array}}$

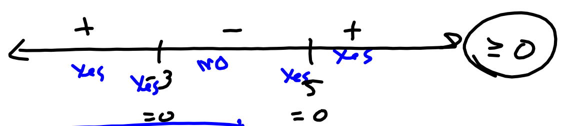
Method 2

$g\circ h=\sqrt{x^2-2x-15}=\sqrt{(x-5)(x+3)}$

$h(x)$ is no restriction

Need $x^2-2x-15=(x-5)(x+3)\geq 0$

$\Rightarrow$



$=\boxed{\begin{array}{l}(-\infty,-3]\cup[5,\infty)\\ =\mathcal{D}(g\circ h)\end{array}}$

4 e 5pts $f \circ g \circ h = f(g(h(x))) = f\left(\sqrt{x^2+2x-15}\right) =$

$$\boxed{\dfrac{1}{\sqrt{(x^2-2x-15)}-6}}$$

$\mathcal{D}(f\circ g\circ h) = \{x \mid x\in \mathcal{D}(h) \text{ and } h(x)\in\mathcal{D}(g) \text{ and } g(h(x))\in\mathcal{D}(f)\}$

$= \{x \mid \underset{\text{No restriction}}{x\in\mathbb{R}} \text{ and } x^2-2x\geq 15 \text{ and } \sqrt{x^2-2x-15}\neq 6\}$

$= \{x \mid x\in(-\infty,-3]\cup[5,\infty) \text{ and } x^2-2x-15\neq 36\}$

by part c. Now we need $x^2-2x-15\neq 36$

$x^2-2x \neq 51$

$x^2-2x+1\neq 52$

$(x-1)^2\neq \pm\sqrt{52} = \pm 2\sqrt{13}$

$x-1\neq \pm 2\sqrt{13}$

$x\neq 1\pm 2\sqrt{13}\approx$

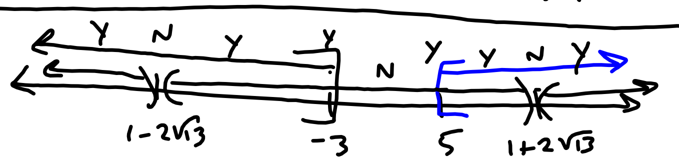
8.211102550 OR - 6.211102550

$2\,|\,52$, $2\,|\,26$, 13

$2\,|\,96$, $2\,|\,48$, $2\,|\,24$, $2\,|\,12$, $2\,|\,6$, 3

$$\boxed{\mathcal{D} = \{x \mid x\in(-\infty,-3]\cup[5,\infty) \text{ and } x\neq 1\pm 2\sqrt{13}\}}$$

=



AND

$= (-\infty, 1-2\sqrt{13})\cup(1-2\sqrt{13},-3]\cup[5,1+2\sqrt{13})\cup(1+2\sqrt{13},\infty)$

$\approx \boxed{(-\infty,-6.2)\cup(-6.2,-3]\cup[5,8.2)\cup(8.2,\infty)}$

(5) $A=(-3,4)$, $B=(8,17)$

(a) (5pts) $m = \frac{17-4}{8-(-3)} = \frac{13}{11} \rightarrow \boxed{y=\frac{13}{11}(x+3)+4 \quad \text{or } y=\frac{13}{11}(x-8)+17}$

For obstinate people

$y=\frac{13}{11}x+\frac{83}{11}$ or $y=1.\overline{18}x+7.\overline{54}$

(b) (5pts) Parallel to part (a) thru $\left(\frac{3}{13},\frac{5}{117}\right)$:

$$\boxed{y=\frac{13}{11}\left(x-\frac{3}{13}\right)+\frac{5}{117}}$$

For obstinate people:

$y=\frac{13}{11}x-\frac{296}{1287}$ or $y=1.\overline{18}x-0.22\overline{999222}$

(c) (5pts) Perpendicular to part (a) thru $\left(\frac{3}{13},\frac{5}{117}\right)$:

$$y=-\frac{11}{13}\left(x-\frac{3}{13}\right)+\frac{5}{117}$$

For obstinate people:

$y=-\frac{11}{13}x+\frac{362}{1521}$ OR

$y=-0.\overline{846153}x$

$+\ 0.2380013149243918474687705456936226166995397764628533859303090072320841551610 78$

6 a 5pts $f(x) = x^2 - 3x - 1 = ax^2 + bx + c$,

where $a = 1, b = -3, c = -1 \longrightarrow$

$b^2 - 4ac = 3^2 - 4(1)(-1) = 9 + 4 = 13 \longrightarrow$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \boxed{\frac{3 \pm \sqrt{13}}{2} = x}$$

b 5pts $f(x) = x^2 - 3x - 1 = x^2 - 3x + \left(\frac{3}{2}\right)^2 - \frac{9}{4} - \frac{4}{4}$

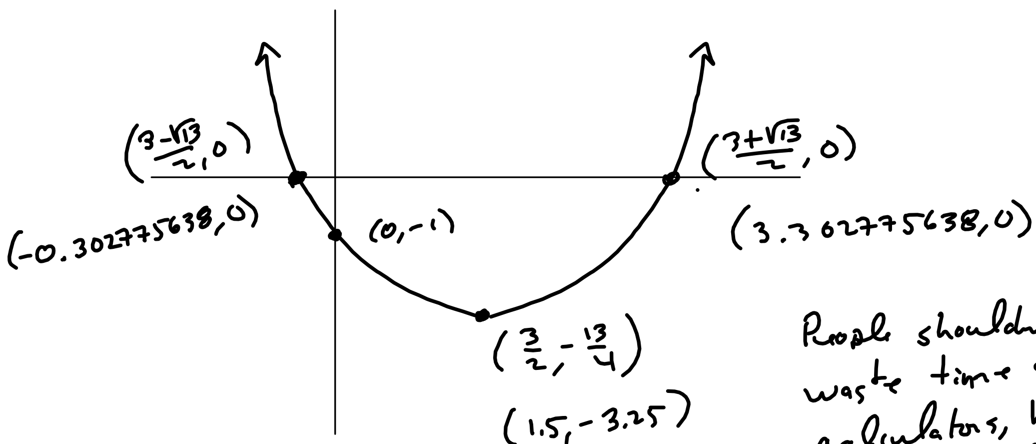
$= \boxed{\left(x - \frac{3}{2}\right)^2 - \frac{13}{4} = f(x) \text{ for part c.}} \quad = 0 \longrightarrow$

$$\left(x - \frac{3}{2}\right)^2 = \frac{13}{4}$$

$$x - \frac{3}{2} = \pm \frac{\sqrt{13}}{2}$$

$$\boxed{x = \frac{3 \pm \sqrt{13}}{2}}$$

c 5pts We sketch $f(x) = \left(x - \frac{3}{2}\right)^2 - \frac{13}{4}$



People shouldn't waste time w/ calculators, here, but they will, because they resist using their intelligence.

7

a) 5 pts
$$\left(\frac{5x+25}{x+3} = \frac{4x+20}{x+2}\right)(x+3)(x+2)$$

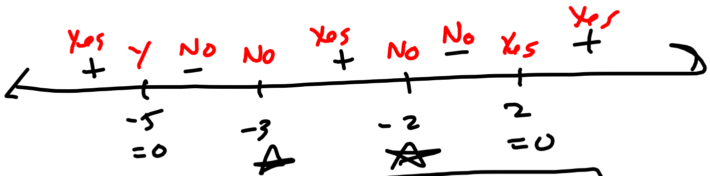
$$\Rightarrow (x+2)(5x+25) = (x+3)(4x+20)$$
$$5x^2+25x+10x+50 = 4x^2+20x+12x+60$$
$$5x^2+35x+50 = 4x^2+32x+60$$
$$x^2+3x-10=0$$
$$(x+5)(x-2)=0$$
$$x \in \{-5, 2\}$$

b) 5 pts
$$\frac{5x+25}{x+3} \geq \frac{4x+20}{x+2}$$

Using Previous work, but keeping denominators.

$$\Rightarrow \frac{5x^2+35x+50}{(x+3)(x+2)} \geq \frac{4x^2+32x+60}{(x+3)(x+2)}$$

$$\Rightarrow \frac{x^2+3x-10}{(x+3)(x+2)} = \frac{(x+5)(x-2)}{(x+3)(x+2)} \geq 0$$



Want ≥ 0
("+" or "= 0")

$$\Rightarrow \boxed{x \in (-\infty,-5] \cup (-3,-2) \cup [2,\infty)}$$

9) $f(x) = x^4 - 2x^3 + x^2 + 8x - 20$

a) 5pts $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

b) 5pts

$$\begin{array}{r|rrrrr} 1 & 1 & -2 & 1 & 8 & -20 \\ & & 1 & -1 & 0 & \text{NO} \\ \hline & 1 & -1 & 0 & 8 & \end{array} \qquad \begin{array}{r|rrrrr} -1 & 1 & -2 & 1 & 8 & -20 \\ & & -1 & 3 & -4 & \text{NO} \\ \hline & 1 & -3 & 4 & 4 & \end{array}$$

$$\begin{array}{r|rrrrr} 2 & 1 & -2 & 1 & 8 & -20 \\ & & 2 & 0 & 2 & 20 \\ \hline & 1 & 0 & 1 & 10 & 0 \end{array} \text{ sweet!} \qquad \begin{array}{r|rrrrr} -2 & 1 & -2 & 1 & 8 & -20 \\ & & -2 & 8 & & \\ \hline & 1 & -4 & & & \end{array}$$

try again:
$$\begin{array}{r|rrrr} 2 & 1 & 0 & 1 & 10 \\ & & 2 & 4 & \\ \hline & 1 & 2 & \text{NO} & \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 1 & 10 \\ & & -2 & +4 & -10 \\ \hline & 1 & -2 & 5 & 0 \end{array}$$

$b^2 - 4ac = 2^2 - 4(1)(5) = 4 - 20 = -16 < 0$ No real zeros

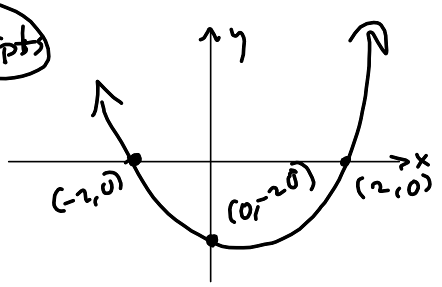
$$\boxed{f(x) = (x+2)(x-2)(x^2 - 2x + 5)}$$

c) 5pts $x^2 - 2x + 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{-16}}{2(1)} = \frac{2 \pm 4i}{2} = \boxed{1 \pm 2i = x \\ = \text{nonreal zeros}}$$

d) 5pts $$\boxed{f(x) = (x-2)(x+2)(x-1+2i)(x-1-2i)}$$

e) 5pts



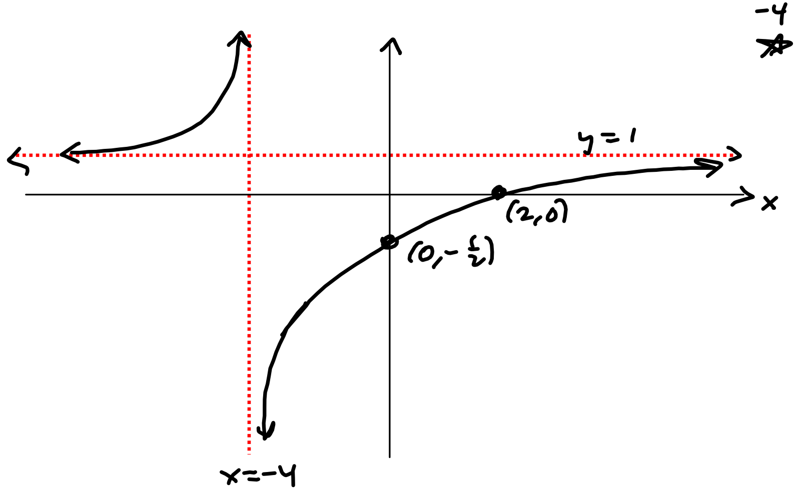
10 5pts $R(x) = \frac{x-2}{x+4}$ $\longrightarrow$ $\boxed{\begin{array}{l} x=-4 \text{ is V.A.} \\ \text{\& } (2,0) \text{ is } x\text{-int} \end{array}}$ $\boxed{\begin{array}{l} (0,-\frac{2}{4}) = (0,-\frac{1}{2}) \\ \text{is } y\text{-int} \end{array}}$

Also $\boxed{\text{H.A.: } y=1}$ from $\frac{x}{x} \xrightarrow{x\to\infty} 1$

+ − +

−4 2

$\cancel{\infty}$ =0



11 5pts $T(x) = \dfrac{x^2+3x-28}{x+5} = \dfrac{(x+7)(x-4)}{x+5}$

$\boxed{\text{y-int } \left(0, -\tfrac{28}{5}\right)}$

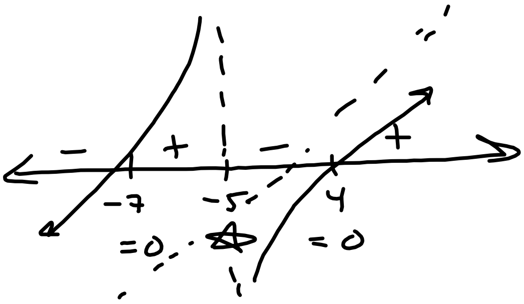
VA: $x=-5$

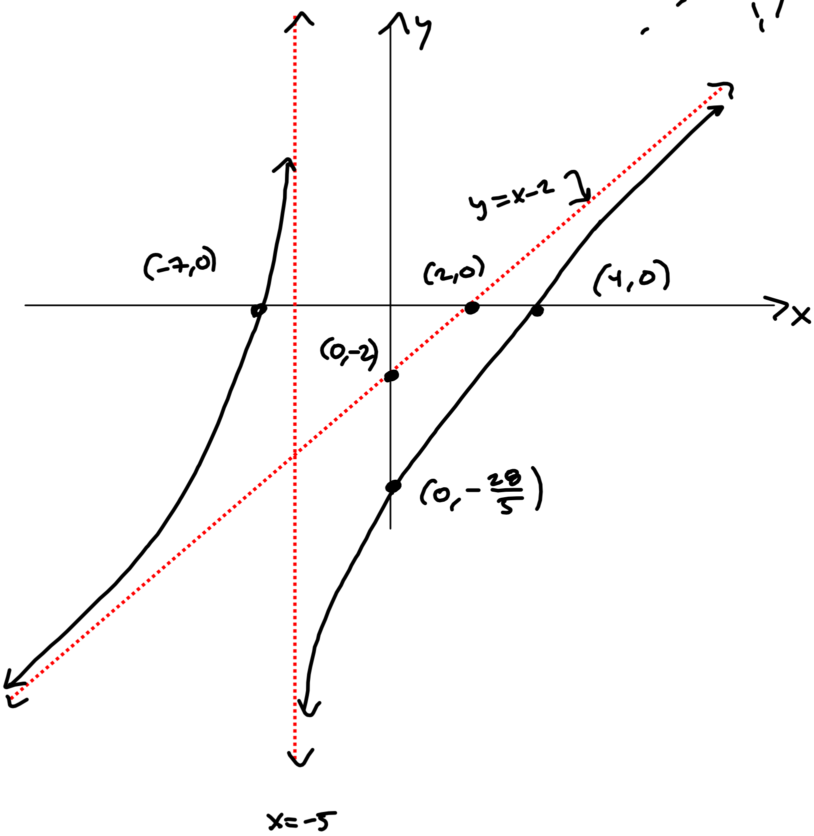
HA: NONE

x-int: $(-7,0), (4,0)$

SLANT Asymptote $y = x - 2$

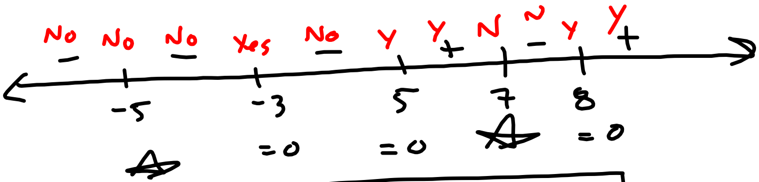
$$\begin{array}{r|rrr} -5 & 1 & 3 & -28 \\ & & -5 & \\ \hline & 1 & -2 & \end{array}$$





(12) 5pts $u(x) = \dfrac{(x+3)^2(x-5)^3(x-8)}{(x+5)^2(x-7)}$

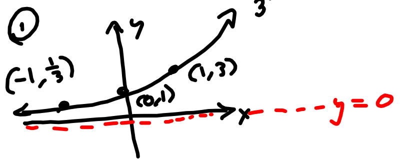
$v(x) = \sqrt{u(x)} \Longrightarrow \mathcal{D}(v) = \{x \mid u(x) \geq 0\}$



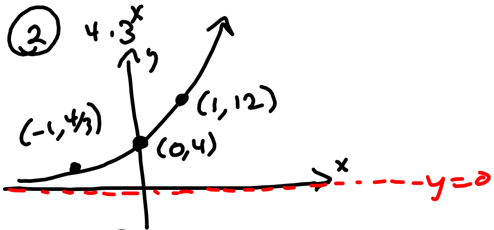
$$= \boxed{\{-3\} \cup [5,7) \cup [8,\infty) = \mathcal{D}(v)}$$

(13) 5pts Sketch $4 \cdot 3^{2x-10} = g(x)$

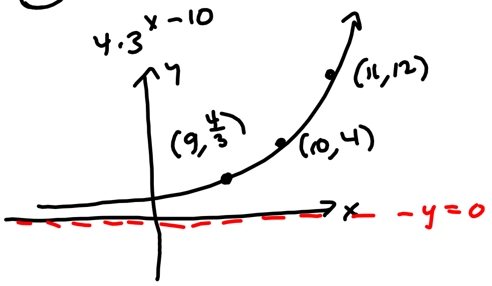
(1)



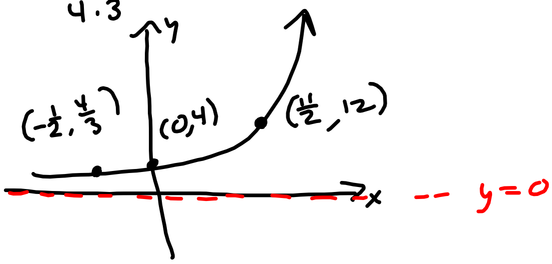
(2) $4 \cdot 3^x$



(3) M1 : $4 \cdot 3^{x-10}$



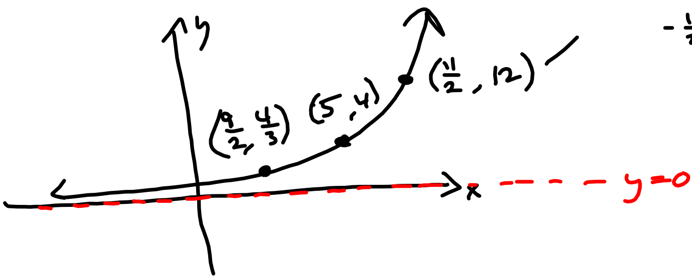
(3) M2 $4 \cdot 3^{2(x-5)}$



(4) $g(x) = 4 \cdot 3^{2x-10} = 4 \cdot 3^{2(x-5)}$

M1

$x \longmapsto \frac{1}{2}x$



M2

$x \longmapsto x+5$

$-\frac{1}{2}+5 = \frac{9}{2}$ ✓

$0+5 = 5$ ✓

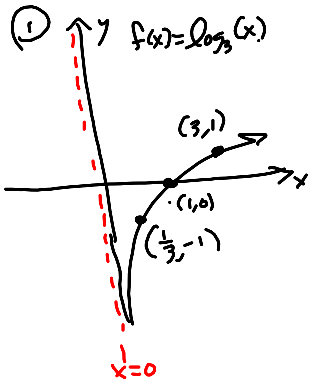
$\frac{1}{2}+5 = \frac{11}{2}$ ✓

14 a 5pts Find f^{-1} from #13

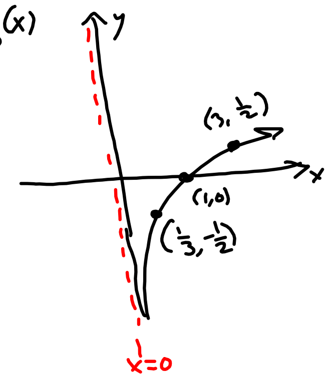
$$4 \cdot 3^{2x-10} = y$$
$$3^{2x-10} = \frac{y}{4}$$
$$\log_3\left(3^{2x-10}\right) = \log_3\left(\frac{y}{4}\right)$$
$$2x-10 = \log_3\left(\frac{y}{4}\right)$$
$$2x = \log_3\left(\frac{y}{4}\right) + 10$$
$$x = \frac{\log_3\left(\frac{y}{4}\right) + 10}{2}$$
$$\boxed{g^{-1}(x) = \frac{1}{2}\log_3\left(\frac{1}{4}x\right) + 5}$$

15 5pts Sketch g^{-1}

1 $f(x) = \log_3(x)$

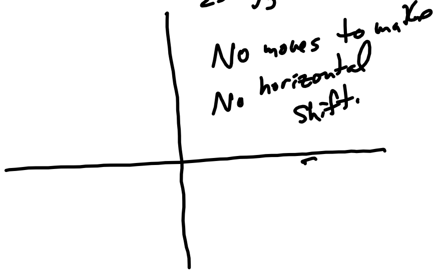


2 $\frac{1}{2}\log_3(x)$



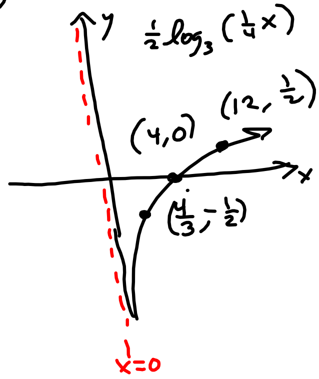
m1

$\frac{1}{2}\log_3($

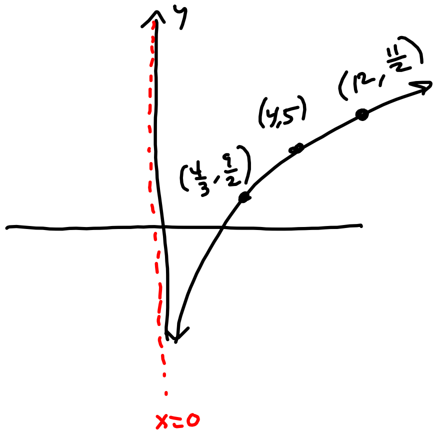


3 m2 or m1

$\frac{1}{2}\log_3\left(\frac{1}{4}x\right)$



$$g^{-1}(x) = \frac{1}{2}\log_3\left(\frac{1}{4}x\right) + 5$$



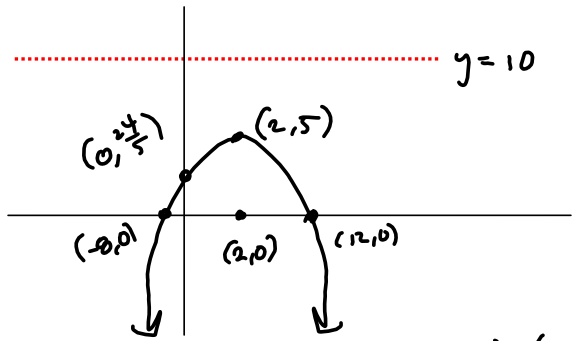
$$-\frac{1}{2}+5 = \frac{9}{2}$$
$$\frac{1}{2}+5 = \frac{1+10}{2}$$

16 5pts $y-5 = -\frac{1}{20}(x-2)^2$

$\Rightarrow y = -\frac{1}{2}(x-2)^2+5 \Rightarrow \boxed{(h,k) = (2,5)}$, opens down,

$(x-2)^2 = -20(y-5) = -4p(y-5)$

$\Rightarrow -20 = 4p \Rightarrow p = -5 =$ distance from vertex to focus & from vertex to directrix.



y-int:

$(-2)^2 = -20(y-5) = -20y+100$

$4-100 = -96 = -20y$

$y = \frac{-96}{-20} = \frac{48}{10} = \frac{24}{5}$

$\boxed{(0, \frac{24}{5}) = y\text{-int}}$

$y = -\frac{1}{20}(x-2)^2+5 \overset{\text{SET}}{=} 0$

$\Rightarrow (x-2)^2 - 100 = 0$

$(x-2)^2 = 100$

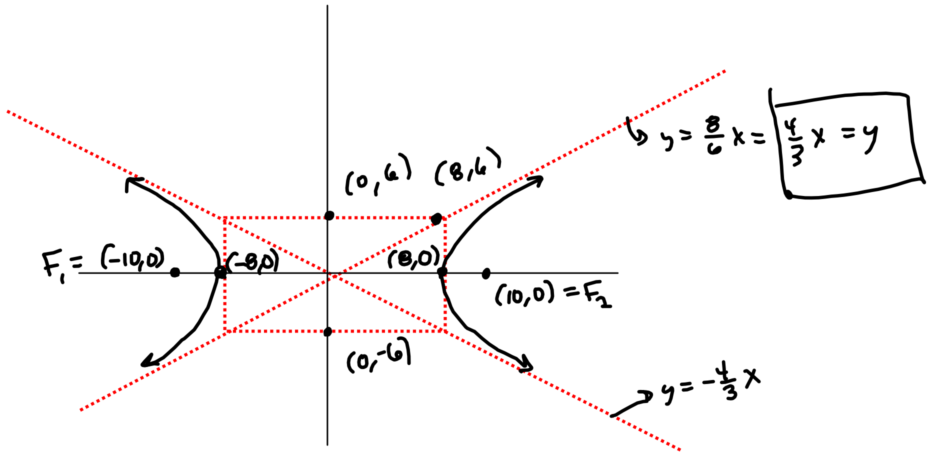
$x-2 = \pm 10$

$x = 2 \pm 10$ → 12, −8

17 5pts $\frac{x^2}{64} - \frac{y^2}{36} = 1$

$a = 8, b = 6$

$a^2 + b^2 = c^2 = 100 \implies c = 10$



18 5pts $36x^2 + 144x \quad + 25y^2 - 350y \quad = -469$

$36(x^2 + 4x \quad) + 25(y^2 - 14y \quad) = -469$

$36(x^2 + 4x + 2^2) + 25(y^2 - 14y + 7^2) = -469 + 36(4) + 25(49)$

$36(x+2)^2 + 25(y-7)^2 = -469 + 144 + 1225 = -469 + 1369$

$36(x+2)^2 + 25(y-7)^2 = 900$

$\frac{(x+2)^2}{25} + \frac{(y-7)^2}{36} = \frac{900}{36(25)} = \frac{36}{36} = 1$

(12)(12)
(36)(4)

14
25
70
28
350

25
50
1250

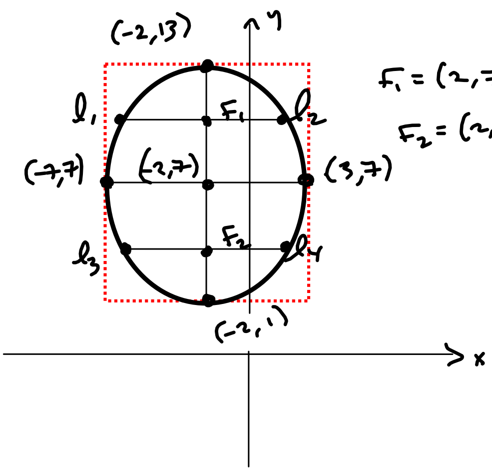
19 5pts $a = 5, b = 6$

$b^2 - a^2 = c^2$

$36 - 25 = 11 = c^2$

$\boxed{c = \sqrt{11}} \approx 3.316624790 \approx c$

$(h,k) = (-2,7)$



$F_1 = (2, 7+\sqrt{11}) \approx (2, 10.31662479)$

$F_2 = (2, 7-\sqrt{11}) \approx (2, 3.683375210)$

(20) (5pts)

$$\frac{(x+2)^2}{25} + \frac{(y-7)^2}{36} = 1$$

$\begin{array}{r}25\\ 250\end{array}$

$36(x+2)^2 + 25(y-7)^2 = 900$

$y = 7 \pm \sqrt{11} \longrightarrow$

$36(x+2)^2 + 25(7 \pm \sqrt{11} - 7)^2 = 36(x+2)^2 + 25(11) = 36(x+2)^2 + 275 = 900$

$36(x+2)^2 = 625$

$(x+2)^2 = \frac{625}{36}$

$x+2 = \pm\sqrt{\frac{625}{36}} = \pm \frac{25}{6}$

$x = -2 \pm \frac{25}{6} = \frac{-12 \pm 25}{6} \; \begin{array}{l} \nearrow \frac{13}{6} \\ \searrow -\frac{37}{6} \end{array}$

That gives us l_1 & l_2:

$l_1 = \left(-\frac{37}{6}, 7+\sqrt{11}\right)$

$l_2 = \left(\frac{13}{6}, 7+\sqrt{11}\right)$

And by symmetry:

$l_3 = \left(-\frac{37}{6}, 7-\sqrt{11}\right)$

$l_4 = \left(\frac{13}{6}, 7-\sqrt{11}\right.$

21 5pts $(x-2y)^5$

$= x^5 - 5(x)^4(2y) + 10x^3(2y)^2 - 10x^2(2y)^3 + 5x(2y)^4 - x^0(2y)^5$

$= \boxed{x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5}$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

22 5pts Coefficent of x^5y^7 in $(x-2y)^{12}$ is

$\binom{12}{5}(x^5)(-2y)^7 = \frac{12!}{5!7!}x^5(-2)^7y^7 = \frac{12\cdot 11\cdot \cancel{10}\cdot 9\cdot 8}{\cancel{5}\cdot 4\cdot 3\cdot \cancel{2}}(-2)^7x^5y^7$

$= (11\cdot 9\cdot 8)(128)x^5y^7 = -101376\, x^5y^7 \longrightarrow$

32
4
128

$\boxed{\text{-101376 is coefficient of } x^5y^7 \text{ in the expansion.}}$

23 5pts $\binom{12}{4} = \frac{12!}{8!4!} = \frac{12\cdot 11\cdot 10\cdot 9}{4\cdot 3\cdot 2} = 11\cdot 5\cdot 9 = 99\cdot 5 = \boxed{495}$

24 5pts $P(30,4) = \frac{30!}{26!} = 30\cdot 29\cdot 28\cdot 27 = \boxed{657{,}720}$

25 5pts

$3x+2y-7z=11$
$2y-5z=12$
$4y-10z=20$

$\rightsquigarrow$

$3x+2y-7z=11$
$2y-5z=12$
$2y-5z=10$

$\frac{160}{5} = 32$

$\rightsquigarrow$

$3x+2y-7z=11$
$2y-5z=12$
$0=-2$? Absurd!

$\longrightarrow$ No sol'n!

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
0 1 2 3 4 5