MAT 1340 Take-Home Test 3 – Spring, 2024 Name_________________

30 Points WRITING PROJECT #3

1. Please print your name at the top of the first page of your assignment.
2. This is a formative as well as a summative assessment, so please leave room on your pages for your grader's comments.
3. Use only white paper with no lines. No graph paper. No highlighters. Circle final answers where appropriate.
4. White background for your PDFs are all that will be accepted. This is your fourth attempt to generate clear, high-contrast work against a white background. (Most of you are doing an excellent job!)
5. Amateur photos/scans of written work that are gray or difficult to read, or have black or colored borders, will be docked 20%, and may not be graded at all.
6. Do the work early enough to access a good scanner (Kiefer Library, Learning Commons in Greeley, Windsor, Loveland, or Fort Lupton. There is also a FedEx copy service in Greeley, that can produce PDFs that are at least as good as the many examples of my written solutions. Consider my work to be a decent – albeit cramped – standard of quality. Students should leave more room for their and their teacher's annotations. I'm making one document for many, so I'm trying to save some paper.
7. A graphing utility (calculator or online or computer algebra system) will give you some quick, instant information about where zeros and asymptotes are located, but I'm looking for particular features and labels that are modeled in Old Writing Project #3's, which live in the Writing Project 3 Videos.

Upload your finished project as a multi-page, single-file PDF in one of the Writing Project #3 Drop-Boxes on D2L.

Early-Bird Deadline: 11:59 p.m., Friday, March 22nd. 5 Bonus Points (Possible 35/30 score.)

On-Time Deadline: 11:59 p.m., Monday, March 25th. No bonus

Late Deadline: 11:59 pm., Wednesday, March 27th. 20% deduction.

BEGIN TEST:

We will be working with $f(x) = 4x^5 + 13x^4 - 56x^3 + 281x^2 - 410x + 168$ for most of this test. We'll say everything about this polynomial that's worth saying.

1. (2 pts) Describe the end behavior of f with a simple graphic.

2. (2 pts) Use Descartes' Rule of Signs to determine the *possible* number of positive and negative zeros.

3. (2 pts) Use the Rational Zeros Theorem to determine the *possible* rational zeros (roots) of f.

4. (2 pts) Using the information, above, find all real zeros of f . Finding all zeros includes finding the multiplicity of each. This means performing multiple synthetic divisions. Always check for multiplicity greater than 1 with another synthetic division, just in case.

5. (2 pts) From your work, above, factor f over the real numbers. This will involve an irreducible quadratic factor that your grapher has no way of helping you to see. without the synthetic divisions in #4, bringing you closer and closer, step by step, to the irreducible quadratic.

6. (2 pts) Give a rough sketch of f from all of the above information. This is an *art* whose essence is really only found in my videos. If you're too tied to your grapher's output, you'll not capture the real essence of what's going on, or the key features I'm always looking for. Your picture will be more "vertical" than it should be.

7. (2 pts) Now we've covered everything *real* about f. Let's use that work to find *all* the roots of f and *split f into linear factors*. 5 roots (counting repetitions) are *guaranteed by the Fundamental Theorem of Algebra,* and we have found the 3 real ones. The other 2 are nonreal, hiding inside the irreducible quadratic polynomial that remains as the last, very very depressed piece that can't be factored with (or over) the reals.

   Now do your quadratic equation thing to find the 2 nonreal roots. Finally, apply the Factor Theorem to *all* the above work, and represent f as a product of linear factors, $f(x)=a(x-r_1)^{m_1}(x-r_2)^{m_2}\cdots(x-r_w)^{m_w}$.
   Don't forget the leading coefficient, a.
   This wrings (almost) every useful drop of the Theorems on Polynomials out of f, so now on to Rational Functions, which are *quotients* of polynomials!

8. (5 pts) Sketch the graph of $R(x)=\dfrac{10x^2-59x+84}{3x^2-7x-10}$, showing all intercepts, asymptotes, and capturing the *essential features* of the shape of the graph. If you're a slave to your grapher, and oblivious to the features I'm looking for, it'll jump off the page at me (and be bad).

Note: There *is* a subtle feature to this graph that I downplay on tests, but you should pick up on with a take-home, namely, the horizontal asymptote *does* intersect the graph of the function.

I'm willing to part with **5 bonus points** if you can find the point of intersection of $R(x)$ with its horizontal asymptote and label it with an ordered-pair label. I'm also looking for its effect on the graph. There's a little wiggle to this graph in the 1st quadrant.

9. (2 pts) Sketch the graph of $Q(x)=\dfrac{10x^3-159x^2+674x-840}{3x^3-37x^2+60x+100}$. Q has exactly the same graph as R, *except* for the *hole* in the graph of Q, which I expect you to find and clearly label in your graph. I'll give you full credit for #8 and #9, if you show the hole in the graph of Q on your sketch for R in #8 above.

10. (5 pts) Sketch the graph of $T(x)=\dfrac{10x^3-139x^2+556x-672}{3x^2-7x-10}$, showing all intercepts and asymptotes. This was also built off #8, so use the zeros you found for the numerator in #8 to help you find the 3rd zero of this new numerator.

These are two great examples of polynomial and rational inequalities.

11. (2 pts) What is the domain of $W(x)=\sqrt{(x+5)(x-2)^2(x+3)^4(x-7)^5}$?

12. (2 pts) What is the domain of $\mathrm{K}(x)=\sqrt{\dfrac{(x-2)^2(x-7)^5}{(x+5)(x+3)^4}}$?