MAT 1340-G81, G82 | Writing Project #2 | Name____________________
50 Points | Spring, 2024

Graphing Functions by Transforming Basic Functions

Discussion

FORMATTING: See Writing Project #1 for instructions on formatting and submitting your work in PDF form in the drop box in Assignments on D2L.

Main Resources: Homework (Chapter 2) Notes and Videos, Writing Project 2 Videos (and notes), and a selection of Old Writing Projects.

Students may use either of the following 2 methods. For full credit, I need to see 5 graphs for each problem, 1 point each. I expect to see you arrive at the graph of g by steps, applying each move, one at a time. There are 4 moves:

1. Replace $f(x)$ by $af(x)$: Replace y by ay in next graph: $y \to ay$.
2. Replace $f(x)$ by $f(bx)$: Replace x by $\frac{1}{b}x$ in next graph: $x \to \frac{1}{b}x$
3. Replace $f(x)$ by $f(x+c)$: Replace x by $x-c$ in the next graph: $x \to x-c$
4. Replace $f(x)$ by $f(x)+d$: Replace y by $y+d$ in the next graph: $y \to y+d$

These 4 moves, in a proper sequence can get you from the graph of a basic function, $f(x)$, to the given function, $g(x) = af(bx+c)+d$

There are two generally accepts ways of going about stringing the moves together. The two methods are identical, except that steps 2 and 3 are executed in reverse order, and in slightly different ways.

The reason I like Method 2 so much is that's how you want to think about functions in Trig and Calculus. Factoring out the coefficient of x inside the function allows you to *see* the phase shift in a trig function, at a glance. So it's very good for your mathematical intuition. Method 1 avoids having to add fractions, which some college students fear. You need only use one method. I will display both in the solutions. An upwardly-mobile student should be able to do both.

Method 1: This method does the horizontal shift before the horizontal stretch/shrink. This method is easier, but does less to build your math intuition and relating the math to physics, down the road.

$$0.\ f(x) \Rightarrow 1.\ 3f(x) \Rightarrow 2.\ 3f(x+2) \Rightarrow 3.\ 3f(5x+2) \Rightarrow 4.\ 3f(5x+2)+7 = g(x)$$

$$1.\ (x,y) \mapsto (x,3y) \qquad 2.\ (x,y) \mapsto (x-2,y) \qquad 3.\ (x,y) \mapsto \left(\frac{1}{5}x, y\right) \qquad 4.\ (x,y) \mapsto (x,y+7)$$

Method 2: This method does the horizontal shrink/stretch before the horizontal shift. This will require you to factor out the coefficient of x inside the function to obtain the proper horizontal shift and stretch/shrink. This method, applied to trigonometric functions in your math future, will give you the correct phase shift.

$$0.\ f(x) \Rightarrow 1.\ 3f(x) \Rightarrow 2.\ 3f(5x) \Rightarrow 3.\ 3f\left(5\left(x+\frac{2}{5}\right)\right) \Rightarrow 4.\ 3f\left(5\left(x+\frac{2}{5}\right)\right)+7 = g(x)$$

$$1.\ (x,y) \mapsto (x,3y) \qquad 2.\ (x,y) \mapsto \left(\frac{1}{5}x, y\right) \qquad 3.\ (x,y) \mapsto \left(x-\frac{2}{5}, y\right) \qquad 4.\ (x,y) \mapsto (x,y+7)$$

Problem Set

Graph the function $g(x)$ by transforming the graph of a basic function, $f(x)$. Start with a basic function graph, with at least 2 – and preferably 3 – points labeled. Then track where each of those points is moved to at each step. Using the same points I always use is usually the easiest, because THEY are the easiest ones to obtain in your basic function graph that is always the first graph in these sequences.

1. (5 pts) $g(x)=\dfrac{-3}{4x-20}+8$ (Use $(1,1)$, and $(-1,-1)$ as the 3 $(x,y)'s$ in the 1st graph.). I hope and expect to see 2 asymptotes, clearly shown and labeled.

2. (5 pts) $g(x)=5(7x+21)^{2/3}-8$ (Use $(0,0),(1,1)$, and $(8,4)$ as the 3 points in the 1st graph.)

3. (5 pts) $g(x)=-5\sqrt[3]{3x-18}+7$

4. (5 pts) $g(x)=6\sqrt[4]{2x+14}+11$

5. (5 pts) $g(x)=5(6x-42)^3+8$

We treat lines and parabolas a little differently. They come up so often - plus the completing-the-square trick – we sidestep the whole $f(bx)$ issue and just work with $g(x)=a(x-h)^2+k$ and $g(x)=m(x-h)+k=m(x-x_1)+y_1$.

6. (5 pts) $g(x)=7(x-2)+6$

7. (5 pts) $g(x)=7(x-2)^2+6$

8. (5 pts) $g(x)=x^2-6x-27$

9. (5 pts) $g(x)=2x^2-4x+20$

10. (5 pts) $g(x)=2x^2-4x+20$

I'd rather you used a *process*, but here is a complete-the-square formula for #s 8 – 10 that's worth remembering as a check:

$$g(x)=ax^2+bx+c=a(x-h)^2+k=a\left(x-\frac{-b}{2a}\right)^2+g\left(-\frac{b}{2a}\right)=a\left(x+\frac{b}{2a}\right)^2+g\left(-\frac{b}{2a}\right), \text{ where}$$

$(x,y)=(h,k)=\left(-\dfrac{b}{2a},g\left(-\dfrac{b}{2a}\right)\right)$ is the vertex.