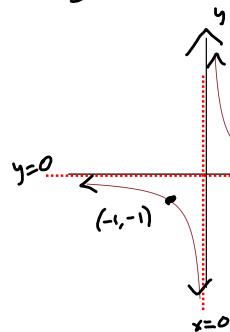


Graph the function  $g(x)$  by transforming the graph of a basic function,  $f(x)$ . Start with a basic function graph, with at least 2 – and preferably 3 – points labeled. Then track where each of those points is moved to at each step. Using the same points I always use is usually the easiest, because THEY are the easiest ones to obtain in your basic function graph that is always the first graph in these sequences.

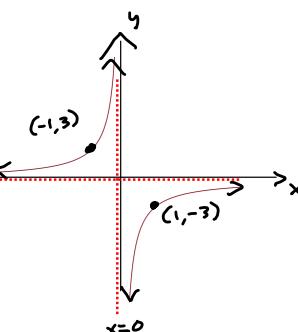
1. (5 pts)  $g(x) = \frac{-3}{4x-20} + 8$  (Use  $(1,1)$ , and  $(-1,-1)$  as the 3  $(x,y)$ 's in the 1<sup>st</sup> graph.). I hope and expect to see 2 asymptotes, clearly shown and labeled.

$$g(x) = \frac{-3}{4(x-5)} + 8$$

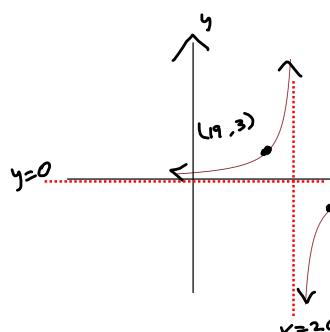
$\textcircled{1} f(x) = \frac{1}{x}$



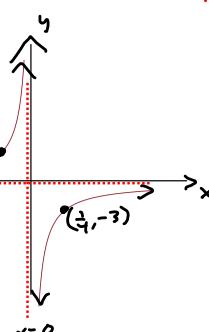
$\textcircled{1} -3f(x) = \frac{-3}{x}$



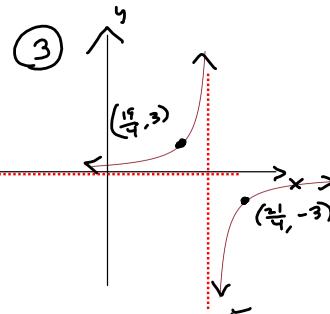
$\textcircled{2} M1 \quad -\frac{3}{x-20} = -3f(x-20)$   
 $x \mapsto x+20$



$\textcircled{2} M2 \quad -\frac{3}{4x} = -3f(4x)$   
 $x \mapsto \frac{1}{4}x$

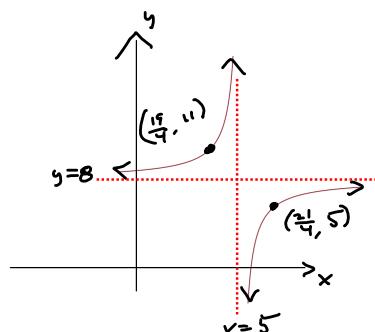


$\textcircled{1} \frac{-3}{4x-20}$   
 $= -3f(4x-20)$   
 $x \mapsto \frac{1}{4}x$



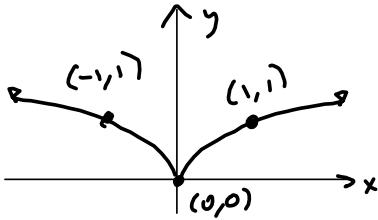
$\textcircled{2} \frac{-3}{4(x-5)} = -3f(4(x-5))$   
 $x \mapsto x+5$   
 $\frac{1}{4}+5=\frac{1+20}{4}=\frac{21}{4}$   
 $0+5=5$   
 $-\frac{1}{4}+5=\frac{-1+20}{4}=\frac{19}{4}$

$\textcircled{4} g(x) = \frac{-3}{4x-20} + 8 \quad \textcircled{m1} = -3f(4x-20) + 8 \quad \textcircled{m2} \quad \frac{-3}{4(x-5)} + 8 = -3f(4(x-5)) + 8$   
 $y \mapsto y+8$

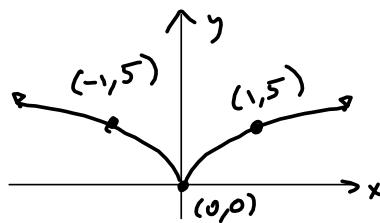


2. (5 pts)  $g(x) = 5(7x+21)^{2/3} - 8$  (Use  $(0,0)$ ,  $(1,1)$ , and  $(8,4)$  as the 3 points in the 1<sup>st</sup> graph.)

$$\textcircled{0} \quad f(x) = x^{2/3} = \sqrt[3]{x^2}$$

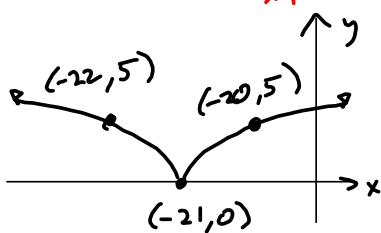


$$\textcircled{1} \quad 5f(x) = 5x^{2/3}$$



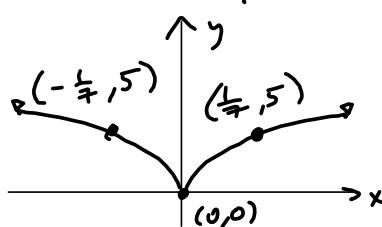
$$\textcircled{2 M1} \quad 5f(x+21) = 5(x+21)^{2/3}$$

$x \mapsto x-21$



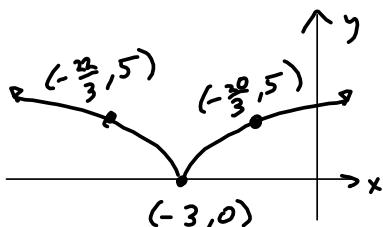
$$\textcircled{2 M2} \quad 5f(7x) = 5(7x)^{2/3}$$

$x \mapsto \frac{1}{7}x$



$$\textcircled{3 M1} \quad 5f(7x+21) = 5(7x+21)^{2/3}$$

$x \mapsto \frac{1}{7}x$



$$\textcircled{3 M2} \quad 5f(3(x+3))$$

$$= 5(3(x+3))^{2/3}$$

$x \mapsto x-3$

$$-\frac{1}{7} - 3 = -\frac{1-21}{7} = -\frac{22}{7}$$

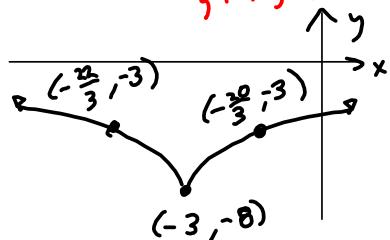
$$0 - 3 = -3$$

$$\frac{1}{7} - 3 = \frac{1-21}{7} = -\frac{20}{7}$$

$$\textcircled{4} \quad 5f(7x+21) - 8$$

$$= g(x) = 5(7x+21)^{2/3} - 8 \quad \textcircled{M1}$$

$y \mapsto y-8$

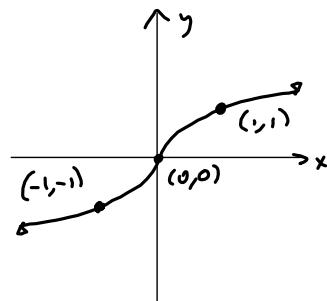


$$g(x) = 5(7(x+3))^{2/3} - 8 \quad \textcircled{M2}$$

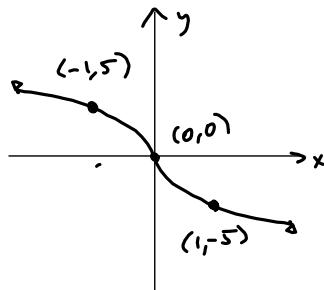
$y \mapsto y-8$

3. (5 pts)  $g(x) = -5\sqrt[3]{3x-18} + 7$

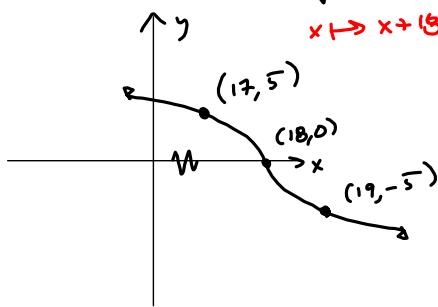
(1)  $f(x) = \sqrt[3]{x}$



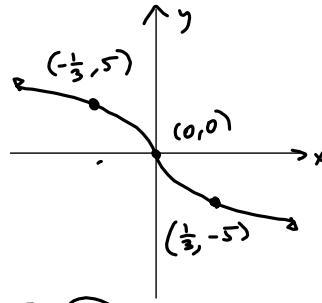
(1)  $-5\sqrt[3]{x} = -5f(x)$   
 $y \mapsto -5y$



(2) M1  $-5f(x-18) = -5\sqrt[3]{x-18}$   
 $x \mapsto x+18$

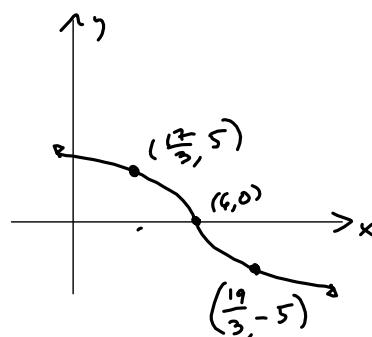


(2) M2  $-5f(3x) = -5\sqrt[3]{3x}$   
 $x \mapsto \frac{1}{3}x$



(3) M1  $-5f(3x-18) = -5\sqrt[3]{3x-18}$   
 $x \mapsto \frac{1}{3}x$

$17 \mapsto \frac{17}{3}$   
 $18 \mapsto \frac{18}{3} = 6$   
 $19 \mapsto \frac{19}{3}$



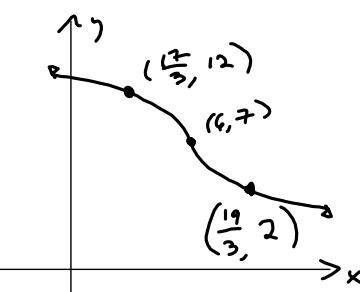
(3) M2  $-5f(3(x-6)) = -5\sqrt[3]{3(x-6)}$   
 $x \mapsto x+6$

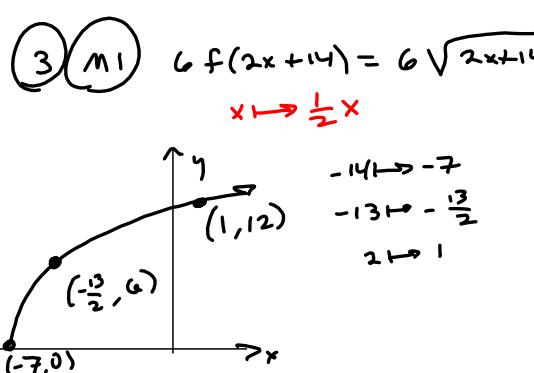
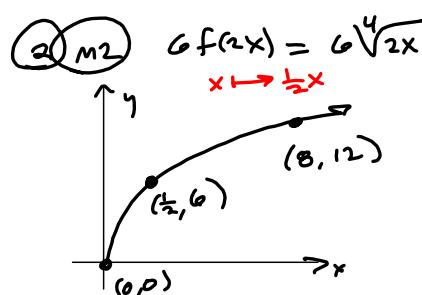
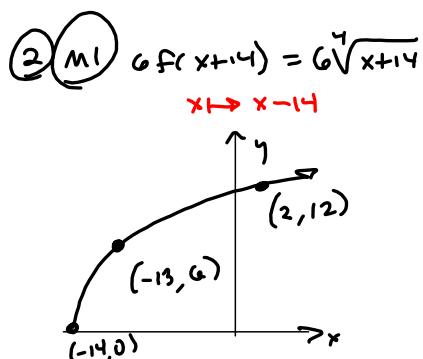
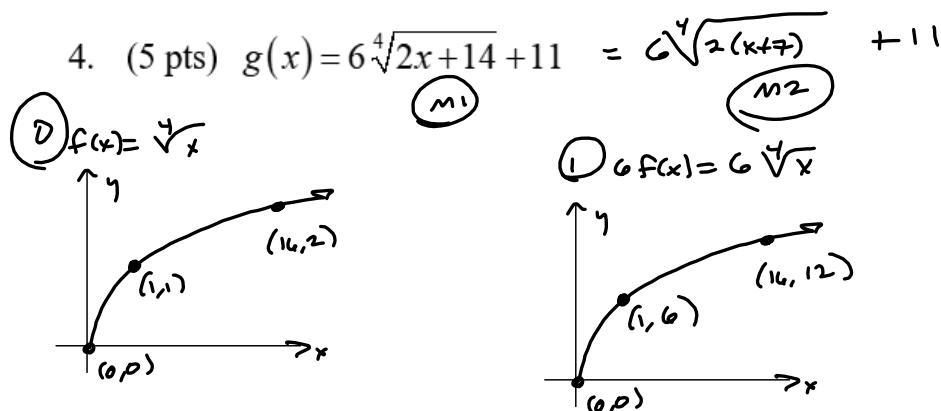
$$\begin{aligned} -\frac{1}{3} + 6 &= \frac{-1+18}{3} = \frac{17}{3} \\ 0+6 &= 6 \\ \frac{1}{3} + 6 &= \frac{1+18}{3} = \frac{19}{3} \end{aligned}$$

(4) M1  $-5f(3x-18) + 7 = g(x)$   
 $= -5\sqrt[3]{3x-18}$   
 $y \mapsto y+7$

M2  $-5f(3(x-6)) + 7 = g(x)$   
 $= -5\sqrt[3]{3(x-6)} + 7$   
 $y \mapsto y+7$  (same)

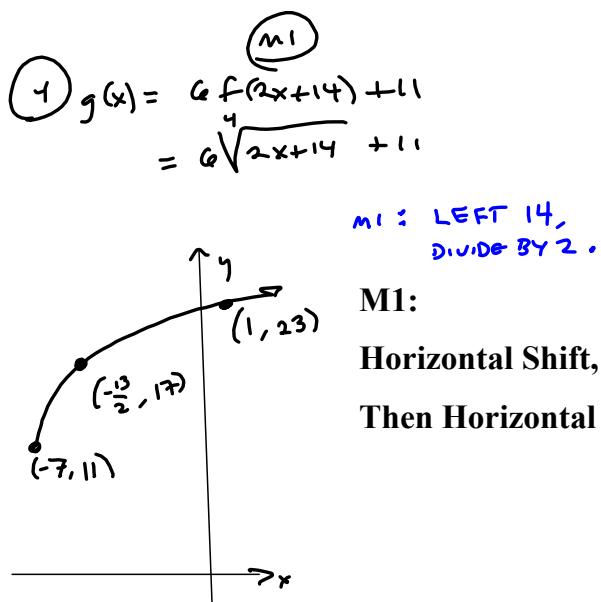
$$\begin{aligned} 5+7 &= 12 \\ 0+7 &= 7 \\ -5+7 &= 2 \end{aligned}$$





**M2:**  $6f(2(x+7))$   
 $= 6\sqrt[4]{2(x+7)}$   
 $x \mapsto x+7$

$0-7 = -7$   
 $\frac{1}{2}-7 = \frac{-14}{2} = -\frac{13}{2}$   
 $8-7 = 1$



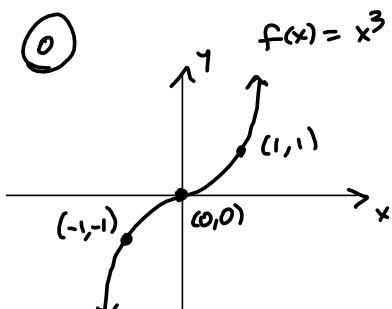
**M2:**  $6f(2(x+7)) + 11 =$   
 $= 6\sqrt[4]{2(x+7)} + 11$

$m_2: \text{Divide by } 2,$   
 $\text{Left } 7.$

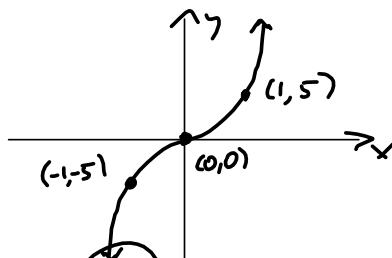
**M2:**  
Horizontal Shrink,  
Then Horizontal Shift.

$$5. \text{ (5 pts)} \quad g(x) = 5(6x - 42)^3 + 8 = 5(6(x-7))^3 + 8$$

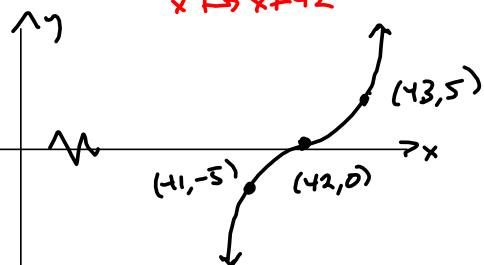
(0)



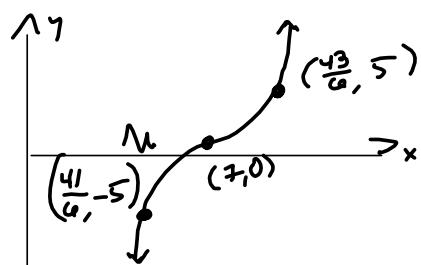
(1)



(2)



(3)



*m1*  
 $x \mapsto \frac{1}{6}x$

$$\begin{aligned} 41 &\mapsto \frac{41}{6} \\ 42 &\mapsto 7 \\ 43 &\mapsto \frac{43}{6} \end{aligned}$$

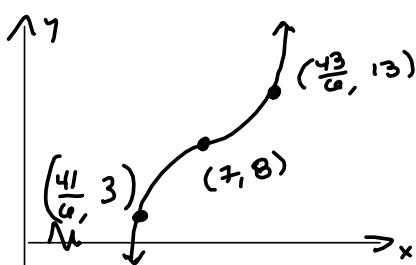
$$\begin{aligned} &= 5f(6(x-7)) \\ &= 5(6(x-7))^3 \\ &x \mapsto x+7 \end{aligned}$$

$$\begin{aligned} -\frac{1}{6} &\mapsto \frac{-1+42}{6} = \frac{41}{6} \\ 0 &\mapsto 7 \\ \frac{1}{6} &\mapsto \frac{1+42}{6} = \frac{43}{6} \end{aligned}$$

$$g(x) = 5f(6x-42) + 8 = 5(6(x-7))^3 + 8 \quad \text{or} \quad 5f(6(x-7)) + 8$$

*m1*

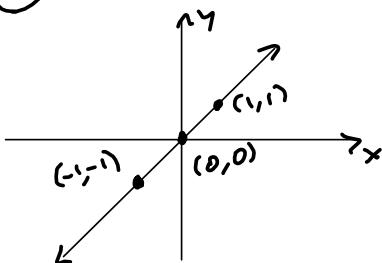
*m2*



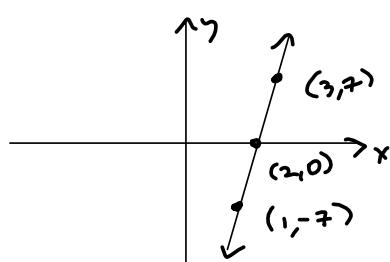
We treat lines and parabolas a little differently. They come up so often - plus the completing-the-square trick - we sidestep the whole  $f(bx)$  issue and just work with  $g(x) = a(x-h)^2 + k$  and  $g(x) = m(x-h) + k = m(x-x_1) + y_1$ .

6. (5 pts)  $g(x) = 7(x-2) + 6$

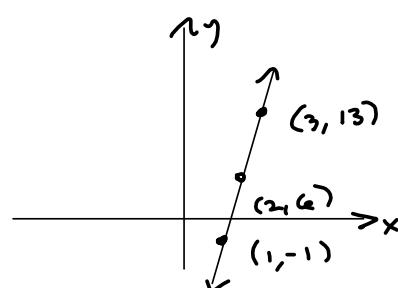
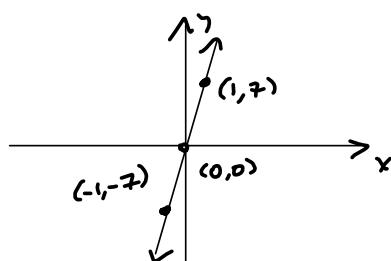
①  $f(x) = x$



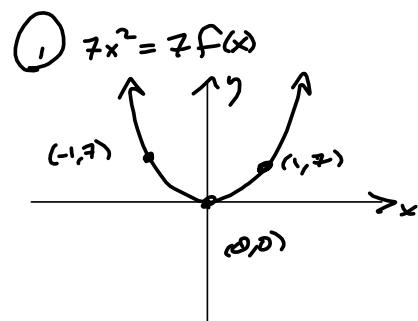
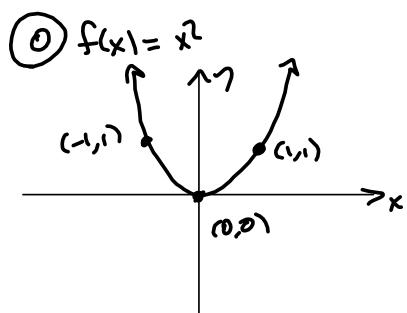
②  $y(x-2) = yf(x-2)$



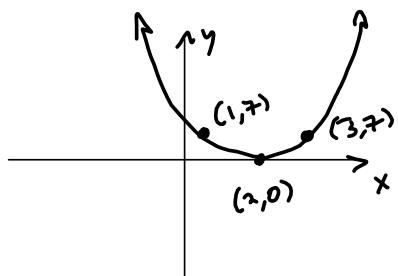
③  $y(x-2) + 6$



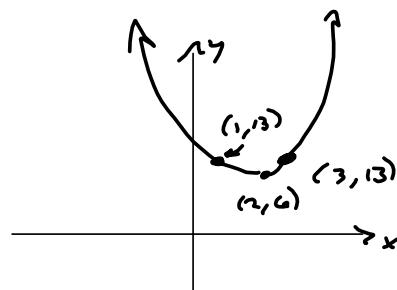
7. (5 pts)  $g(x) = 7(x - 2)^2 + 6$



②  $7(x - 2)^2 = 7f(x - 2)$

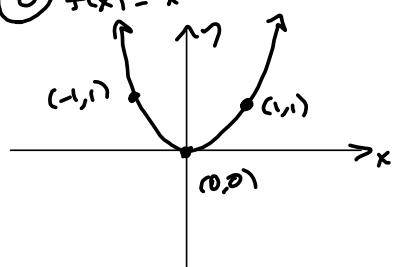


③  $7(x - 2)^2 + 6 = 7f(x - 2) + 6$   
 $= g(x)$

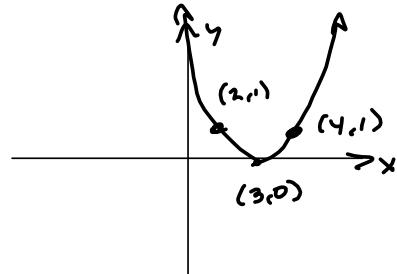


8. (5 pts)  $g(x) = x^2 - 6x - 27 = x^2 - 6x + 9 - 9 - 27 = (x-3)^2 - 36$

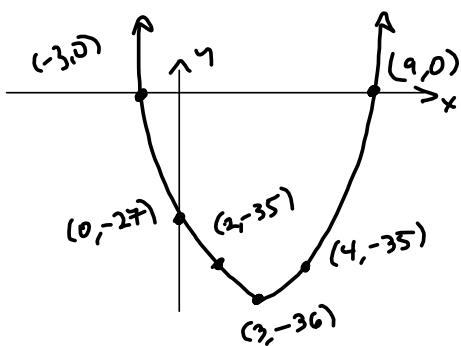
①  $f(x) = x^2$



②  $(x-3)^2 = f(x-3)$



③  $g(x) = (x-3)^2 - 36 = f(x-3) - 36$



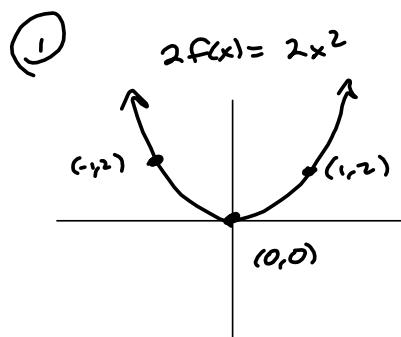
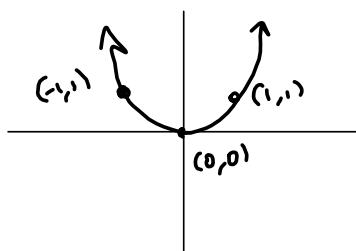
9. (5 pts)  $g(x) = 2x^2 - 4x + 20$

$\rightarrow 10$  pts

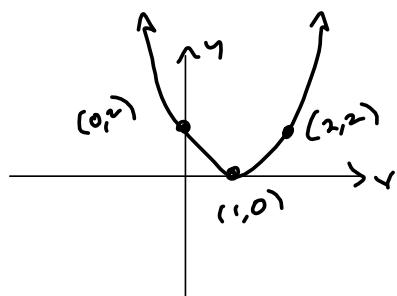
$$= 2(x^2 - 2x + 1^2) + 20 - 2(1)$$

$$= 2(x-1)^2 + 18$$

(0)  $f(x) = x^2$



(2)  $2(x-1)^2 = 2f(x-1)$



(3)  $2(x-1)^2 + 18 = 2f(x-1) + 18$   
 $= g(x)$

