MAT 1340

#s 1 – 3 4 Find all real (or imaginary) solutions of the following quadratic equations using the quadratic formula. Be sure to compute the discriminant, first, and separately. I'm looking for that on tests, as well, *whenever* you face a quadratic expression. It modularizes the work, and it tells you what you're getting into.

1. $x^2 + 7x - 18 = 0$ $\Rightarrow$ $a = 1, b = 7, c = -18$

$\Rightarrow b^2 - 4ac = 7^2 - 4(1)(-18) = 49 + 72 = 121 = 11^2 \rightarrow$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-7 \pm \sqrt{121}}{2(1)} = \dfrac{-7 \pm 11}{2}$ $\frac{4}{2} = 2$, $\frac{-18}{2} = -9$ $\rightarrow$

$\boxed{x \in \{-9, 2\}}$

2. $17.71x^2 - 3.72x - 2.88 = 0$ (Round your final answer to 4 decimal places.)

METHOD 1:

$a = 17.71, b = -3.72, c = -2.88 \rightarrow$

$b^2 - 4ac = (-3.72)^2 - 4(17.71)(-2.88)$

$= 13.8384 + 204.0192 = 217.8576 \rightarrow$ $\sqrt{217.8576} = 14.76$, exactly. RARE.

You don't expect this to be "=" here, but it actually IS exact.

$x = \dfrac{3.72 \pm \sqrt{217.8576}}{2(17.71)} \approx \dfrac{3.72 \pm 14.76}{35.42}$

$\dfrac{18.48}{35.42} \approx .5217391305$

$\dfrac{-11.04}{35.42} \approx -.3116883117$

$\Rightarrow \boxed{x \in \{-.3117, .5217\}}$ to 4 places

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3. $25x^2 - 20x + 7 = 0$ (Give an exact answer, in simplified radical form.

$a = 25,\ b = -20,\ c = 7 \longrightarrow$

$b^2 - 4ac = 20^2 - 4(25)(7) = 400 - 700 = -300$

$\sqrt{-300} = 2 \cdot 5 \cdot i\sqrt{3} = 10\sqrt{3}\, i \longrightarrow$

$x = \dfrac{20 \pm 10i\sqrt{3}}{2(25)} = \dfrac{10(2 \pm i\sqrt{3})}{50} = \dfrac{2 \pm i\sqrt{3}}{5}$

$$\boxed{x \in \left\{ \frac{2 \pm i\sqrt{3}}{5} \right\}}$$

$$\begin{array}{r|l} 2 & 300 \\ 2 & 150 \\ 3 & 75 \\ 5 & 25 \\ & 5 \end{array}$$

4. $12wx^2 - 5c\,x - 3q = 0$ (Your answers will have letters in them. That's OK!)

$a = 12w,\ b = 5c,\ c = -3q$

$b^2 - 4ac = (5c)^2 - 4(12w)(-3q) = 25c^2 + 144wq \longrightarrow$

$x = \dfrac{-5c \pm \sqrt{25c^2 + 144wq}}{2(12w)}$

$$= \boxed{\frac{-5c \pm \sqrt{25c^2 + 144wq}}{24w} = x}$$

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#s 5, 6 Solve the following by factoring. You may use a sledgehammer, if you wish, but write the polynomial in factored form, after you find the solutions, to show you understand the connection between factors and solutions, frontwards and backwards! Give answers as integers or fractions, in lowest terms.

5. $x^2 + 6x - 55 = 0 \Longrightarrow (x+11)(x-5) = 0 \Longrightarrow$

$\boxed{x \in \{-11, 5\}}$

6. $1771x^2 - 372x - 288 = 0$

M2 for big factoring questions: SLEDGEHAMMER

$a = 1771,\ b = -372,\ c = -288 \Longrightarrow$

$b^2 - 4ac = 372^2 - 4(1771)(-288)$

$= 138384 + 2040192$

$= 2178576 \Longrightarrow \sqrt{2178576} = 1476$ w/ tech.

$x = \dfrac{372 \pm 1476}{2(1771)} = \begin{cases} \dfrac{12}{23} \\ -\dfrac{24}{77} \end{cases} \rightsquigarrow \boxed{x \in \left\{-\dfrac{24}{77}, \dfrac{12}{23}\right\}}$

FINAL ANSWER

OLD-SCHOOL:

$$\begin{array}{r|l} 2 & 2178576 \\ 2 & 1089288 \\ 2 & 544644 \\ 2 & 272322 \\ 3 & 136161 \\ 3 & 45387 \\ 3 & 15129 \\ 3 & 5043 \\ 41 & 1681 \\ & 41 \end{array}$$

$\sqrt{2178576} = \sqrt{2^4 \cdot 3^4 \cdot 41^2} = 2^2 \cdot 3^2 \cdot 41 = 1476$

This page is a bit cramped, above.

The problem said "use factoring" and I just cheated it. Now to make the connection between the solution and the factored form:

The "Price Is Right!" method for factoring quadratic expressions whose discriminants are perfect squares (so they factor over the Rational Number).

$1771x^2 - 372x - 288 = 0$

$(1771)(-288) = -510048$ = MAGIC NUMBER

middle term is -372

$-372 = -472 + 100$ $\quad -47{,}200$ higher

$= -472 + 500$ $\quad -436{,}000$ higher

$= -472 + 600$ $\quad -583{,}200$ Lower (GETTING CLOSE)

$= -922 + 550$ $\quad -507{,}100$ higher

$= -923 + 551$ $\quad$ Doesn't end in 8

$= -924 + 552$ $\quad -510048$ Magic!

$1771x^2 - 924x + 552x - 288 = 0$

$$\begin{array}{r|l} 7 & 1771 \\ 11 & 253 \\ & 23 \end{array} \qquad \begin{array}{r|l} 2 & 924 \\ 2 & 462 \\ 3 & 231 \\ 7 & 77 \\ & 11 \end{array} \qquad \begin{array}{r|l} 2 & 552 \\ 2 & 276 \\ 2 & 138 \\ 3 & 69 \\ & 23 \end{array} \qquad \begin{array}{r|l} 2 & 288 \\ 2 & 144 \\ 2 & 72 \\ 2 & 36 \\ 2 & 18 \\ 3 & 9 \\ & 3 \end{array}$$

GCF $= 11 \cdot 7$

GCF $= 2^3 \cdot 3$
$= 24$

$77x(23x - 12) + 24(23x - 12)$

$= (23x - 12)(77x + 24) = 0$

By the above work,

$$1771x^2 - 372x - 288 = 1771\left(x - \left(-\frac{24}{77}\right)\right)\left(x - \frac{12}{23}\right) = *$$

Now, pretty it up:

SCRATCH 1771 out front. 77 & 23 the denominators.

$$\frac{1771}{77} = 23$$

Check: $\begin{array}{r} 77 \\ 23 \\ \hline 231 \\ 1540 \\ \hline 1771 \end{array}$ ✓

, so we have...

$$\ldots * = (77)(23)\left(x + \frac{24}{77}\right)\left(x - \frac{12}{23}\right)$$

$$= \boxed{(77x + 24)(23x - 12)}$$

is the factored form that's required for a "solve by factoring" question

You can always reverse-engineer the factored form from the correct answer(s)!

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#s 7 – 10 Solve the following by completing the square. Do not use decimals; rather, use *fractions*, as needed, to complete the square. For example, use $\left(\frac{7}{2}\right)^2$, instead of $(3.5)^2$ for #7.

7. $x^2 + 6x - 55 = 0$

$x^2 + 6x = 55$

$x^2 + 6x + 3^2 = 55 + 9$

$(x+3)^2 = 64$

$x + 3 = \pm\sqrt{64} = \pm 8$

$x = 3 \pm 8$ → 11, -5

$\boxed{x \in \{-11, 5\}}$

8. $x^2 - 18x - 25 = 0$

$x^2 - 18x = 25$

$x^2 - 18x + 9^2 = 25 + 81$

$2 \overline{)106}$
53

$(x-9)^2 = 10$

$x - 9 = \pm\sqrt{106}$

$\boxed{x = 9 \pm \sqrt{106}}$ or $\boxed{x \in \{9 \pm \sqrt{106}\}}$

9. $5x^2 + 6x + 23 = 0$ $\Rightarrow$

$5x^2 + 6x = -23$ $\Rightarrow$

$5(x^2 + \frac{6}{5}x) = -23$ $\Rightarrow$

$5(x^2 + \frac{6}{5}x + (\frac{3}{5})^2) = -23 + 5(\frac{9}{25}) = -23 + \frac{9}{5} = \frac{-115+9}{5}$

$5(x + \frac{3}{5})^2 = \frac{-106}{5}$ $\Rightarrow$

$2 \,|\, 106$
53

$(x + \frac{3}{5})^2 = \frac{-106}{25}$ $\Rightarrow$

$x + \frac{3}{5} = \pm \frac{i\sqrt{106}}{5}$ $\Rightarrow$ $\boxed{x = \frac{-3 \pm \sqrt{106}\, i}{5}}$

10. $2x^2 - 13x + 8 = 0$ $\Rightarrow$

$\Rightarrow 2x^2 - 13x = -8$

$\Rightarrow 2(x^2 - \frac{13}{2}x) = -8$

$2(x^2 - \frac{13}{2}x + (\frac{13}{4})^2) = -8 + 2(\frac{169}{16}) = -8 + \frac{169}{8} = \frac{-64+169}{8}$

$\Rightarrow 2(x - \frac{13}{4})^2 = \frac{105}{8}$ $\Rightarrow$

$3 \,|\, 105$
$5 \,|\, 35$
7

$(x - \frac{13}{4})^2 = \frac{105}{16}$ $\Rightarrow$

$x - \frac{13}{4} = \pm \frac{\sqrt{105}}{4}$ $\Rightarrow$

$\Rightarrow \boxed{x = \frac{13 \pm \sqrt{105}}{4}}$