MAT 1340 Writing Project #4 Spring '23

① We solve the system of linear equations

$$\begin{aligned} 2x+5y &= 20 \\ 3x-2y &= 18 \end{aligned}$$ in 3 ways:

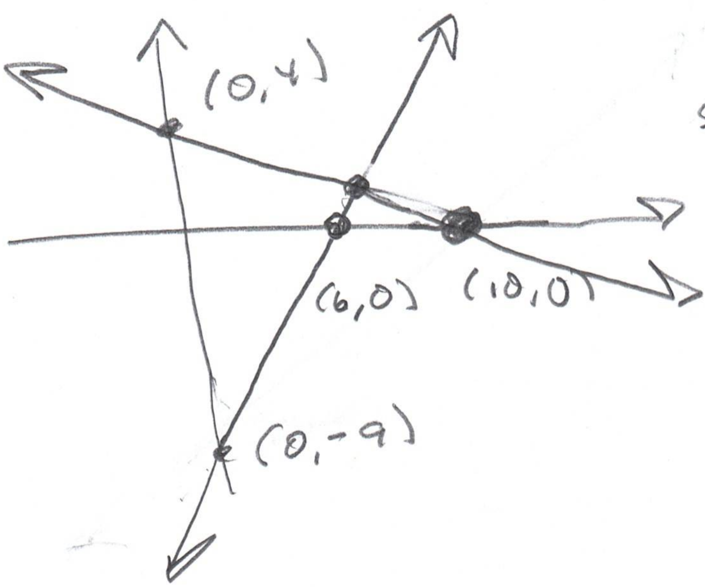
(a) (10 pts) Graphing:

$2x+5y=20$

x	y	
0	4	$\rightsquigarrow (0,4)$
10	0	$\rightsquigarrow (10,0)$

$3x-2y=18$

x	y	
0	-9	$\rightsquigarrow (0,-9)$
6	0	$\rightsquigarrow (6,0)$



Looks like the solution is roughly (a)

$$\boxed{(7,1)=(x,y)}$$

1b Substitution

$$2x+5y=20 \longrightarrow$$

$$2x=20-5y \longrightarrow$$

$$x=\frac{20-5y}{2} \longrightarrow$$

$$3x-2y=3\left(\frac{20-5y}{2}\right)-2y=18$$

$$\Rightarrow \frac{60-15y}{2}-2y=18$$

$$\Rightarrow 60-15y-4y=36$$

$$\Rightarrow 60-19y=36$$

$$\Rightarrow -19y=-24$$

$$\Rightarrow \boxed{y=\frac{24}{19}} \longrightarrow x=\frac{20-5y}{2}=\frac{20-5\left(\frac{24}{19}\right)}{2}$$

$$\frac{\frac{380-120}{19}}{2}=\frac{260}{38}=\boxed{\frac{130}{19}=x}$$

Check: $2\left(\frac{130}{19}\right)+5\left(\frac{24}{19}\right)=\frac{260+120}{19}=\frac{380}{19}=20$ ✓

$3\left(\frac{130}{19}\right)-2\left(\frac{24}{19}\right)=\frac{390-48}{19}=\frac{342}{19}=18$ ✓

1c 10 pts

$2x+5y=20$ E1

$3x-2y=18$ E2

-3E1 $-6x-15y=-60$

2E2 $6x-4y=36$

ADD $-19y=-24$

$\boxed{y=\frac{24}{19}} \Longrightarrow$

$2x+5y=2x+5\left(\frac{24}{19}\right)=2x+\frac{120}{19}=\left(\frac{20}{1}\right)\left(\frac{19}{19}\right)$

$\Longrightarrow 2x=\frac{380}{19}-\frac{120}{19}=\frac{260}{19}$

$\rightarrow \boxed{x=\frac{130}{19}}$ Already checked.

ALTERNATE ROUTE:

-E1 $-2x-5y=-20$

E2 $3x-2y=18$

ADD $x-7y=-2$

SO $x-7y=-2$

$3x-2y=18$

-3E1 $-3x+21y=6$

E2 $3x-2y=18$

$19y=24$, etc.

wp #4

2 We solve

$$\begin{aligned} 2x - 2y + z &= -13 \\ 3x - 5y - z &= -21 \\ x \quad\quad - 2z &= 7 \end{aligned}$$

Re-write

$$\begin{array}{ll} E1 & x \quad\quad - 2z = 7 \\ & 2x - 2y + z = -13 \\ & 3x - 5y - z = -21 \end{array}$$

$$\begin{array}{ll} -2E1 & -2x \quad\quad + 4z = -14 \\ E2 & 2x - 2y + z = -13 \\ \hline & -2y + 5z = -27 \end{array}$$

$$\begin{array}{ll} -3E1 & -3x \quad\quad + 6z = -21 \\ E3 & 3x - 5y - z = -21 \\ \hline & -5y + 5z = -42 \end{array}$$

New system:

$$\begin{array}{ll} x \quad\quad -2z = 7 & E1 \\ -2y + 5z = -27 & E2 \\ -5y + 5z = -42 & E3 \end{array}$$

$$\begin{array}{ll} -5E2 & 10y - 25z = +135 \\ 2E3 & -10y + 10z = -84 \\ \hline & -15z = 51 \end{array}$$

$$z = \frac{51}{-15} = -\frac{17}{5} = \boxed{-3.4 = z}$$

1340 WP #4 5

5 2 cnt'd

$$\boxed{z = -\frac{17}{5}} \longrightarrow$$

$$-5y + 5z = -5y + 5\left(-\frac{17}{5}\right) = -5y - 17 = -42$$

$$\longrightarrow -5y = -25$$

$$\longrightarrow \boxed{y = 5} \longrightarrow$$

$$x - 2z = x - 2\left(-\frac{17}{5}\right) = x + \frac{34}{5} = 7$$

$$\Rightarrow x = \left(\frac{7}{1}\right)\left(\frac{5}{5}\right) - \frac{34}{5} = \frac{35-34}{5} = \boxed{\frac{1}{5} = x}$$

Check Via Matrix Mults:

$$\begin{bmatrix} 1 & 0 & -2 \\ 2 & -2 & 1 \\ 3 & -5 & -1 \end{bmatrix} \begin{bmatrix} 1/5 \\ 5 \\ -\frac{17}{5} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{5} - 2\left(-\frac{17}{5}\right) \\ \frac{2}{5} - 10 - \frac{17}{5} \\ \frac{3}{5} - 25 + \frac{17}{5} \end{bmatrix} = \begin{bmatrix} \frac{1}{5} + \frac{34}{5} \\ -\frac{15}{5} - 10 \\ \frac{20}{5} - 25 \end{bmatrix} = \begin{bmatrix} \frac{35}{5} \\ -13 \\ -21 \end{bmatrix} = \begin{bmatrix} 7 \\ -13 \\ -21 \end{bmatrix} \checkmark$$

3 We solve the DEPENDENT SYSTEM

$$7x + 17y + 27z = 30$$
$$2x + 5y + 8z = 8$$
$$x + 2y + 3z = 6$$

Re-write: $(E1 \leftrightarrow E3)$

$$x + 2y + 3z = 6 \quad E1$$
$$2x + 5y + 8z = 8 \quad E2$$
$$7x + 17y + 27z = 30 \quad E3$$

$\cancel{-7E1 \quad -7x - 14y - 21z = -42}$

$\cancel{E3}$

$$\begin{array}{ll} -2E1 & -2x - 4y - 6z = -12 \\ E2 & 2x + 5y + 8z = 8 \\ \hline & y + 2z = -4 \end{array}$$

$$\begin{array}{ll} -7E1 & -7x - 14y - 21z = -42 \\ E3 & 7x + 17y + 27z = 30 \\ \hline & 3y + 6z = -12 \end{array}$$

OR $\quad y + 2z = -4$

NEW SYSTEM:

$$x + 2y + 3z = 6$$
$$\left.\begin{array}{r} y + 2z = -4 \\ y + 2z = -4 \end{array}\right\} \text{SAME!}$$

New System is only 2 equations!

#3 cnt'd

(a) General Solution:

$x + 2y + 3z = 6$

$y + 2z = -4$

$\rightarrow \boxed{z \text{ is free}}$

$\rightarrow \boxed{y = -2z - 4}$ ✓

$x + 2y + 3z = x + 2(-2z - 4) + 3z$

$= x - 4z - 8 + 3z = x - z - 8 = 6$

$\rightarrow \boxed{x = z + 14}$

10 pts

Solution Set:

$\boxed{(x, y, z) \in \{ (z+14, -2z-4, z) \mid z = \text{any real \#} \}}$

(b) $z = 0$: $\boxed{(x, y, z) = (14, -4, 0)}$

$z = -1$: $(x, y, z) = (-1+14, -2(-1)-4, -1)$

$= \boxed{(13, -2, -1) = (x, y, z)}$

(c) $z = 1$: $(x, y, z) = (1+14, -2(1)-4, 1)$

$= \boxed{(15, -6, 1) = x, y, z}$

(4) We show the linear system has no solution.

$7x + 17y + 27z = 30$ E1

$2x + 5y + 8z = 3$ E2

$x + 2y + 3z = 6$ E3

E1 $\leftrightarrow$ E3

$x + 2y + 3z = 6$ E1

$2x + 5y + 8z = 3$ E2

$7x + 17y + 27z = 30$ E3

$$
\begin{array}{lr}
-2E1 & -2x - 4y - 6z = -6 \\
E2 & 2x + 5y + 8z = 3 \\
\hline
 & y + 2z = -3
\end{array}
$$

$$
\begin{array}{lr}
-7E1 & -7x - 14y - 21z = -42 \\
E3 & 7x + 17y + 27z = 30 \\
\hline
 & 3y + 6z = -12
\end{array}
$$

$\div 3$: $y + 2z = -6$

NEW SYSTEM:

$x + 2y + 3z = 6$ E1

$y + 2z = -3$ E2

$y + 2z = -6$ E3

We can already see what's going to happen, but we make it explicit.

$$\begin{array}{lr} -E2 & -y-2z=3 \\ E3 & y+2z=-6 \\ \hline \text{ADD:} & 0=-3 \end{array}$$

?!

Absurd!

$\therefore \not\exists$ solution

There is no

Therefore

10 pts

Must draw conclusion from the absurdity at which you arrive.