MAT 1340   Writing Project #4 – Fall, 2024   Name_____
100 Points

FORMATTING: See Writing Project #0 for instructions on formatting and submitting your work in PDF form in the drop box in Assignments on D2L

Main Resources: Homework (Chapter 4) Notes and Videos, Writing Project 4 Videos (and notes), and a selection of Old Writing Projects.

Upload your finished project as a multi-page, single-file PDF in the appropriate Writing Project #4 Drop-Box on D2L.

1. Solve the system of linear equations $\begin{array}{l} 2x + 7y = 30 \\ 3x - 5y = 8 \end{array}$ in 3 ways:

   a. (10 pts) Find the general vicinity of the solution by graphing the system. This should give you a general idea. Don't worry about it being super-accurate, although the more care you take, the better the estimate will be. Just graph the two lines by the intercept method. Supply the exact answer after you work parts b and c, below. I care much more about ordered-pair labels (OPLs) than tickmarks. OPLs are required. x- and y-intercepts are required. Tickmarks are not. On a test, I'm always looking for the labels. The tickmarks are just busy work that slows you down, when you're on the clock, and slow *me* down counting tickmarks!

   b. (10 pts) Use the Substitution Method

   c. (10 pts) Use the Elimination Method.

2. (10 pts) Use Elimination to solve the independent system of linear equations: $\begin{array}{l} 5x + 6y + 27z = 2 \\ 4x + 5y + 21z = 5 \\ 2x - 2y + 11z = -3 \end{array}$. Hint:

   -1 Equation 2 + Equation 1 will put a nice '1' in the top-left corner, which makes the arithmetic a lot easier!

3. Consider the dependent system of linear equations: $\begin{array}{l} x + 2y + -5z = 3 \\ 2x + 5y - 12z = 8 \\ 4x + 12y - 28z = 20 \end{array}$.

   a. (10 pts) Use Elimination to obtain the general solution. Be kind to your teacher and let z be free! That means, find an expression for x and y in terms of the variable z. (WebAssign always wants you to use $z = t$, but I'm fine with just using the z as the parameter.

   b. (10 pts) Give the particular solutions corresponding to $z = 0$, $z = 1$ and $z = -1$.

4. **The Underlying Assumption:** *All* of the techniques we learn for solving systems of linear equations are based on the *assumption* that the systems *have* solutions. So when we arrive at a false (*absurd!*) statement after a few elimination steps, the only explanation is that there was no solution in the first place*. Our incorrect assumption* led to something absurd, like $0 = 10$ or $0 = -5$.

*... or you made a mechanical error and should check your work, just to make sure. Stay organized and always check your work.

Higher Learning: In higher mathematics, this is the most basic method of proving something is false: "Assume it's true and conclude something absurd (like '$0 = 1$')." It's important that you realize what's happening when you arrive at those absurdities at the end of a perfectly logical and legal sequence of moves. That said, let me *finally* get to the question:

(10 pts) **Your Task:** Show that the dependent system of linear equations
$$\begin{aligned} x + 2y + {-5z} &= 3 \\ 2x + 5y - 12z &= 8 \\ 4x + 12y - 28z &= 4 \end{aligned}$$
has no solution. I expect to see the word "absurd" in your discussion.