1. Solve the system of linear equations $\begin{matrix} 2x+7y=30 \\ 3x-5y=8 \end{matrix}$ in 3 ways:

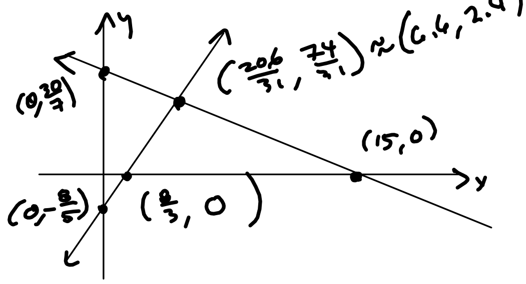
a. (10 pts) Find the general vicinity of the solution by graphing the system. This should give you a general idea. Don't worry about it being super-accurate, although the more care you take, the better the estimate will be. Just graph the two lines by the intercept method. Supply the exact answer after you work parts b and c, below. I care much more about ordered-pair labels (OPLs) than tickmarks. OPLs are required. *x*- and *y*-intercepts are required. Tickmarks are not. On a test, I'm always looking for the labels. The tickmarks are just busy work that slows you down, when you're on the clock, and slow *me* down counting tickmarks!

$2x+7y=30$

x	y
0	$\frac{30}{7}$
15	0

$3x-5y=8$

x	y
0	$-\frac{8}{5}$
$\frac{8}{3}$	0



b. (10 pts) Use the Substitution Method

$2x+7y=30 \Rightarrow 7y = 30-2x \Rightarrow y = \frac{30-2x}{7}$
$3x-5y=8$

$\Rightarrow 3x-5y = 3x-5\left(\frac{30-2x}{7}\right) = 8$

$\Rightarrow 21x-5(30-2x)=56$

$\Rightarrow 21x-150+10x=56$

$\Rightarrow 31x=206$

$\boxed{x=\frac{206}{31}} \Rightarrow y = \frac{30-2\left(\frac{206}{31}\right)}{7} = \frac{30-\frac{412}{31}}{7}$

$= \frac{\frac{930-412}{31}}{7} = \frac{518}{(7)(31)} = \frac{518}{217} = \frac{74\cdot 7}{31\cdot 7}$

$2\,|\underline{518}$
$7\,|\underline{259}$
$\quad 37$

$\Rightarrow \boxed{\frac{74}{31}=y}$

$(x,y) \approx (6.645161290, 2.387096774)$

c. (10 pts) Use the Elimination Method.

$2x+7y=30$
$3x-5y=8$

$-3E1 \quad -6x-21y=-90$
$2E2 \quad 6x-10y=16$

$-31y=-74$

$\boxed{y=\frac{74}{31}}$

$2x+7\left(\frac{74}{31}\right)=30$

$62x+7(74)=30(31)=930$

$62x=930-518$

$62x=412$

$x=\frac{412}{62}=\boxed{\frac{206}{31}=x}$

2. (10 pts) Use Elimination to solve the independent system of linear equations: $\begin{aligned} 5x+6y+27z&=2 \\ 4x+5y+21z&=5 \\ 2x-2y+11z&=-3 \end{aligned}$. Hint:

-1 Equation 2 + Equation 1 will put a nice '1' in the top-left corner, which makes the arithmetic a lot easier!

$$\left[\begin{array}{ccc|c} 5 & 6 & 27 & 2 \\ 4 & 5 & 21 & 5 \\ 2 & -2 & 11 & -3 \end{array}\right] \xrightarrow{-R2+R1} \left[\begin{array}{ccc|c} 1 & 1 & 6 & -3 \\ 4 & 5 & 21 & 5 \\ 2 & -2 & 11 & -3 \end{array}\right] \begin{array}{c} \\ -4R1+R2 \\ -2R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 6 & 3 \\ 0 & 1 & -3 & 17 \\ 0 & -4 & -1 & 3 \end{array}\right]$$

$$\begin{array}{c} \\ \\ 4R2+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 6 & 3 \\ 0 & 1 & -3 & 17 \\ 0 & 0 & -13 & 71 \end{array}\right]$$

$-13z=71$

$\boxed{z=-\frac{71}{13}}$

$y-3z=y-3\left(-\frac{71}{13}\right)=y+\frac{213}{13}=17$

$13y+213=221$

$13y=8$

$\boxed{y=\frac{8}{13}}$

$$\begin{array}{r} 170 \\ 51 \\ \hline 221 \end{array}$$

$$\begin{array}{r} 71 \\ 27 \\ \hline 497 \\ 1420 \\ \hline 1917 \end{array}$$

$5x+6y+27z=5x+6\left(\frac{8}{13}\right)+27\left(-\frac{71}{13}\right)=2$

$\Rightarrow 65x+48-1917=26$

$$\begin{array}{r} 1917 \\ -\ 48 \\ \hline 1869 \end{array}$$

$65x-1869=26$

$65x=1895$

$x=\frac{1895}{65}=\boxed{\frac{379}{13}=x}$

$\boxed{(x,y,z)=\left(\frac{379}{13},\frac{8}{13},-\frac{71}{13}\right)}$

$\approx(29.15384615, .615384615 4, 5.461538462)$

3

a

$x + 2y + -5z = 3$
$2x + 5y - 12z = 8$
$4x + 12y - 28z = 20$

$$\left[\begin{array}{ccc|c} 1 & 2 & -5 & 3 \\ 2 & 5 & -12 & 8 \\ 4 & 12 & -28 & 20 \end{array}\right] \begin{array}{l} -2R1+R2 \\ -4R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -5 & 3 \\ 0 & 1 & -2 & 2 \\ 0 & 4 & -8 & 8 \end{array}\right]$$

Note R3 = 4 R2 !

So $\left[\begin{array}{ccc|c} 1 & 2 & -5 & 3 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$, i.e., $y - 2z = 2 \Rightarrow \boxed{y = 2z+2}$

$\Rightarrow x+2y-5z = x+2(2z+2)-5z = x+4z+4-5z = x-z+4 = 3$

$\Rightarrow \boxed{x = z-1} \Rightarrow \boxed{(x,y,z) = (z-1,\ 2z+2,\ z)}$

b

z	(x,y,z)
0	(-1, 2, 0)
1	(0, 4, 1)
-1	(-2, 0, -1)

4

$$\begin{aligned} x + 2y + -5z &= 3 \\ 2x + 5y - 12z &= 8 \\ 4x + 12y - 28z &= 20 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -5 & 3 \\ 2 & 5 & -12 & 8 \\ 4 & 12 & -28 & 4 \end{array}\right] \begin{array}{l} -2R1+R2 \\ -4R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -5 & 3 \\ 0 & 1 & -2 & 2 \\ 0 & 4 & -8 & -8 \end{array}\right]$$

Note R3 = 4 R2, except for the -8

So
$$\begin{array}{r} \\ \\ -4R2+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -5 & 3 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & -16 \end{array}\right]$$

$$\begin{aligned} x + 2y - 5z &= 3 \\ y - 2z &= 2 \\ 0 &= -16 \; ?! \end{aligned}$$

That's absurd!

Assuming I've made no errors, this says that there is no solution

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