MAT 1340 Take-Home Test 3 – Fall, 2024 Name_________________
30 Points WRITING PROJECT #3

FORMATTING: See Writing Project #0 for instructions on formatting and submitting your work in PDF form in the drop box in Assignments on D2L

Main Resources: Homework (Chapter 3) Notes and Videos, Writing Project 3 Videos (and notes), and a selection of Old Writing Projects.

Upload your finished project as a multi-page, single-file PDF in one of the Writing Project #3 Drop-Box on D2L.

BEGIN TEST: **Do your own work!!!**

We will be working with $f(x) = 2x^5 - 9x^4 - 7x^3 + 135x^2 - 373x + 252$ for most of this test. We'll say everything about this polynomial that's worth saying.

1. (2 pts) Describe the end behavior of f with a simple graphic.
2. (2 pts) Use Descartes' Rule of Signs to determine the *possible* number of positive and negative zeros of f.
3. (2 pts) Use the Rational Zeros Theorem to determine the *possible* rational zeros (roots) of f.
4. (2 pts) Using the information, above, find all real zeros of f. Finding all zeros includes finding the multiplicity of each. This means performing multiple synthetic divisions. Always check for multiplicity greater than 1 with another synthetic division, just in case.
5. (2 pts) From your work, above, factor f over the real numbers. This will involve an irreducible quadratic factor that your grapher has no way of helping you to see. without the synthetic divisions in #4, bringing you closer and closer, step by step, to the irreducible quadratic.
6. (2 pts) Give a rough sketch of f from all of the above information. This is an *art* whose essence is really only found in my videos. If you're too tied to your grapher's output, you'll not capture the real essence of what's going on, or the key features I'm always looking for. Your picture will be more "vertical" than it should be.
7. (2 pts) Now we've covered everything *real* about f. Let's use that work to find *all* the roots of f and *split* f *into linear factors*. 5 roots (counting repetitions) are *guaranteed by the Fundamental Theorem of Algebra,* and we have found the 3 real ones. The other 2 are nonreal, hiding inside the irreducible quadratic polynomial that remains as the last, very very depressed piece that can't be factored with (or over) the reals.

   Now do your quadratic equation thing to find the 2 nonreal roots. Finally, apply the Factor Theorem to *all* the above work, and represent f as a product of linear factors, $f(x) = a(x - r_1)^{m_1}(x - r_2)^{m_2} \cdots (x - r_w)^{m_w}$.
   Don't forget the leading coefficient, *a*.
   This wrings (almost) every useful drop of the Theorems on Polynomials out of f, so now on to Rational Functions, which are *quotients* of polynomials!

8. (5 pts) Sketch the graph of $R(x)=\dfrac{2x^2+x-28}{3x^2-2x-120}$, showing all intercepts, asymptotes, and capturing the *essential features* of the shape of the graph. If you're a slave to your grapher, and oblivious to the features I'm looking for, it'll jump off the page at me (and be bad).

Note: There *is* a subtle feature to this graph that I downplay on tests, but you should pick up on with a take-home, namely, the horizontal asymptote *does* intersect the graph of the function.

I'm willing to part with **5 bonus points** if you can find the point of intersection of $R(x)$ with its horizontal asymptote and label it with an ordered-pair label. I'm also looking for its effect on the graph. There's a little wiggle to this graph in the 1^{st} quadrant.

9. (2 pts) Sketch the graph of $Q(x)=\dfrac{2x^3+15x^2-21x-196}{3x^3+19x^2-134x-840}$. *Q* has exactly the same graph as *R*, *except* for the *hole* in the graph of *Q*, which I expect you to find and clearly label in your graph. I'll give you full credit for #8 and #9, if you show the hole in the graph of *Q* on your sketch for *R* in #8 above.

10. (5 pts) Sketch the graph of $T(x)=\dfrac{2x^3+19x^2-19x-252}{3x^2-2x-120}$, showing all intercepts and asymptotes. This was also built off #8, so use the zeros you found for the numerator in #8 to help you find the 3^{rd} zero of this new numerator.

These are two great examples of polynomial and rational inequalities.

11. (2 pts) What is the domain of $W(x)=\sqrt{(x-5)(x-4)^2(x+5)^3(x-6)^2}$?

12. (2 pts) What is the domain of $\mathrm{K}(x)=\sqrt{\dfrac{(x-5)(x-4)^2}{(x+5)^3(x-6)^2}}$?