$$f(x)=2x^5-9x^4-7x^3+135x^2-373x+252$$

1. (2 pts) Describe the end behavior of f with a simple graphic.

4x5

2. (2 pts) Use Descartes' Rule of Signs to determine the *possible* number of positive and negative zeros.

$$f(x)=2x^5-9x^4-7x^3+135x^2-373x+252$$

1 2 3 4

$\boxed{\text{4, 2 or 0 positive zeros}}$

$f(-x) = -2x^5 - 9x^4 + 7x^3 + 135x^2 + 373x + 252$

1

$\boxed{\text{Exactly 1 negative zero}}$

3. (2 pts) Use the Rational Zeros Theorem to determine the *possible* rational zeros (roots) of f.

$$f(x)=2x^5-9x^4-7x^3+135x^2-373x+252$$

$a_n = 2$, $a_0 = 252$

2 | 252
2 | 126
3 | 63
3 | 21
7

$168 = 3 \cdot 7 \cdot 2 \cdot 2 \cdot 2$
ugh!

If $\frac{p}{q}$ is a zero of f, then

p is a divisor of $a_0 = 252$ &

q is a divisor of $a_n = 2$

p's: $252 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7 \cdot 1$

q's: $2 \cdot 1$

$2 \cdot 28$
3
84

$\pm 1, \pm \frac{1}{2}, \pm 2, \pm \cancel{\frac{2}{2}}, \pm 3, \pm \frac{3}{2}, \pm 4, \pm \cancel{\frac{4}{2}}, \pm 6, \pm \cancel{\frac{6}{2}}, \pm 9, \pm \frac{9}{2}, \pm 7, \pm \frac{7}{2},$

$\pm 12, \pm \cancel{\frac{12}{2}}, \pm 14, \pm \cancel{\frac{14}{2}}, \pm 21, \pm \frac{21}{2}, \pm 28, \pm \cancel{\frac{28}{2}}, \pm 18, \pm \cancel{\frac{18}{2}},$

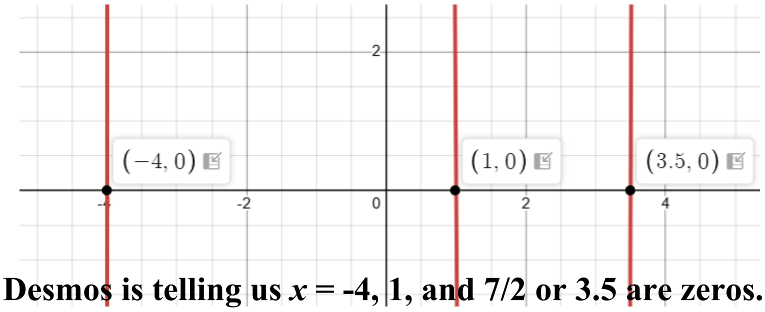
$\pm 36, \pm \cancel{\frac{36}{2}}, \pm 63, \pm \frac{63}{2}, \pm 42, \pm \cancel{\frac{42}{2}}, \pm 126, \pm \cancel{\frac{126}{2}}, \pm 84, \pm \cancel{\frac{84}{2}},$

$\pm 252, \pm \cancel{\frac{252}{2}}$ 24 of 'em!

The "cheat," here, is to narrow down your guesses with some kind of graphing utility. This will allow us to combine technology and our knowledge of rational zeros to break *f* down all the way (Split it into linear factors).

4. (2 pts) Using the information, above, find all real zeros of f. Finding all zeros includes finding the multiplicity of each. This means performing multiple synthetic divisions. Always check for multiplicity greater than 1 with another synthetic division, just in case.

$$f(x) = 2x^5 - 9x^4 - 7x^3 + 135x^2 - 373x + 252$$



Desmos is telling us x = -4, 1, and 7/2 or 3.5 are zeros. For our work, we will convert the 0.75 to 3/4. Now, we divide by (x + 4), (x - 1) and then (x - 7/2).

-4	2	-9	-7	135	-373	252	
		-8	68	-244	436	-252	
1	2	-17	61	-109	63	0	sweet!
		2	-15	46	-63		
$\frac{7}{2}$	2	-15	46	-63	0	sweet!	
		7	-28	63			
	2	-8	18	0	sweet!		

$18 \times 8 = 144$

$2x^2 - 8x + 18 = 0$

$a = 2, b = -8, c = 18$

$b^2 - 4ac = 8^2 - 4(2)(18) = 64 - 144 = -80 < 0 \Rightarrow$ No more real zeros

$\therefore$ $\boxed{x = -4, 1, \frac{7}{2} \text{ are the only real zeros}}$

5. (2 pts) From your work, above, factor f over the real numbers. This will involve an irreducible quadratic factor that your grapher has no way of helping you to see. without the synthetic divisions in #4, bringing you closer and closer, step by step, to the irreducible quadratic.

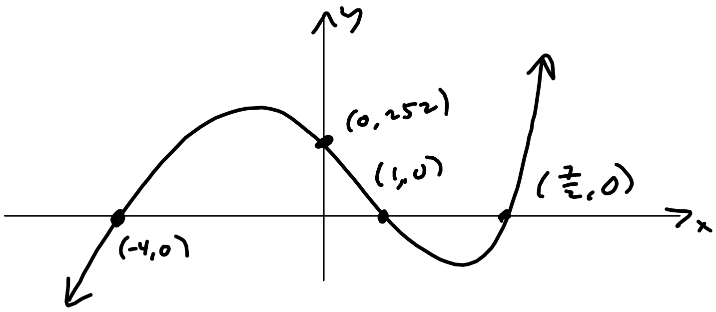
This work says

$\boxed{f(x) = (x+4)(x-1)(x-\frac{7}{2})(2x^2 - 8x - 18)}$

or $\boxed{f(x) = 2(x+4)(x-\frac{7}{2})(x^2-4x-9)}$

OR $\boxed{f(x) = (x+4)(2x-7)(x^2-4x-9)}$ is also ok

6. (2 pts) Give a rough sketch of f from all of the above information. This is an *art* whose essence is really only found in my videos. If you're too tied to your grapher's output, you'll not capture the real essence of what's going on, or the key features I'm always looking for. Your picture will be more "vertical" than it should be.



This isn't *exactly* what it looks like, but it captures the shape and the *x*- and *y*-intercepts. By the time you made it tall enough to include the *actual* maximum, the scale would be too large to see $x = 1$ and $x = 7/2$ as separate points, let alone any part of the graph that's below the *x*-axis, even though we *know* there's a tiny bit there under the *x*- axis between $x = 1$ and $x = 7/2$.

7. (2 pts)
Now do your quadratic equation thing to find the 2 nonreal roots. Finally, apply the Factor Theorem to *all* the above work, and represent f as a product of linear factors, $f(x) = a(x - r_1)^{m_1}(x - r_2)^{m_2} \cdots (x - r_w)^{m_w}$.

$x^2 - 4x + 9 = 0$

$x^2 - 4x = -9$

$x^2 - 4x + 2^2 = -9 + 4$

$(x-2)^2 = -5$

$\boxed{x = 2 \pm \sqrt{5}\, i}$

$\Rightarrow f(x) = 2(x+4)(x-1)(x-\frac{7}{2})(x-2+\sqrt{5}i)(x-2-\sqrt{5}i)$

8. (5 pts) Sketch the graph of $R(x)=\dfrac{2x^2+x-28}{3x^2-2x-120}$, showing all intercepts, asymptotes, and capturing the *essential features* of the shape of the graph. If you're a slave to your grapher, and oblivious to the features I'm looking for, it'll jump off the page at me (and be bad).

This one had a typo in it. I'll be working two versions:

Intended

$$R(x)=\frac{2x^2+x-28}{3x^2-2x-120}$$

$$=\frac{(2x-7)(x+4)}{(3x-20)(x+6)}$$

Domain: $\mathbb{R}\setminus\{\frac{20}{3},-6\}$

Vertical Asymptotes: $x=\frac{20}{3}$, $x=-6$

x-ints: $(\frac{7}{2},0)$, $(-4,0)$

$\frac{20}{3}$, -6, -4, $\frac{7}{2}$

-6, -4, $\frac{7}{2}$, $\frac{20}{3}$

$\cancel{A}$ $=0$ $=0$ $\cancel{A}$

End Behavior:

$\lim\limits_{|x|\to\infty} R(x)=\boxed{\frac{2}{3}=y=H.A.}$

$+$ | -6 ($\cancel{A}$) | $-$ | -4 ($=0$) | $+$ | $\frac{7}{2}$ ($=0$) | $-$ | $\frac{20}{3}$ ($\cancel{A}$) | $+$

y-int: $(0,\frac{28}{120})=(0,\frac{7}{30})$

If you worked the unrevised version.

Typo

$$R(x)=\frac{2x^2+x+28}{3x^2-2x-120}$$

$$=\frac{2x^2+x+28}{(3x-20)(x+6)}$$

Domain: $\mathbb{R}\setminus\{\frac{20}{3},-6\}$

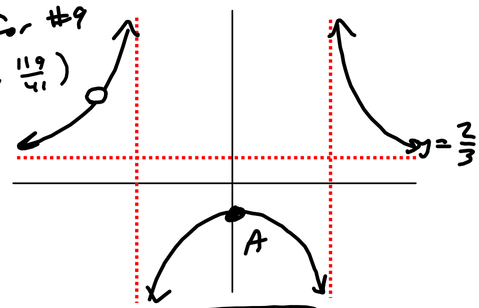
Vertical Asymptotes: $x=\frac{20}{3}$, $x=-6$

No x-ints

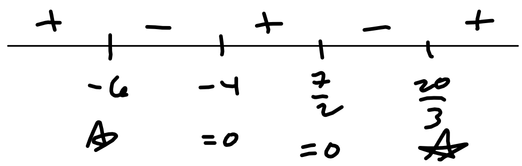
y-int: $(0,-\frac{7}{30})$

$+$ | -6 ($\cancel{A}$) | $-$ | $\frac{20}{3}$ ($\cancel{A}$) | $+$

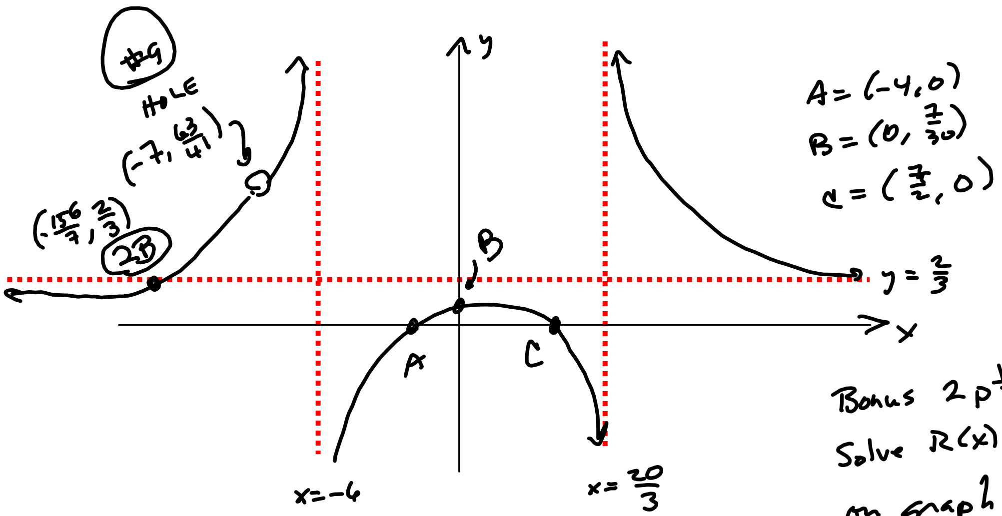
HOLE for #9 $(-7,\frac{119}{41})$



$A=(0,-\frac{7}{30})$



Combine this work:



Bonus 2 pts:
Solve $R(x) = \frac{2}{3}$ & show on graph

9. (2 pts) Sketch the graph of $Q(x)=\dfrac{2x^3+15x^2-21x-196}{3x^3+19x^2-134x-840}$. Q has exactly the same graph as R, *except* for the *hole* in the graph of Q, which I expect you to find and clearly label in your graph. I'll give you full credit for #8 and #9, if you show the hole in the graph of Q on your sketch for R in #8 above.

One way or another, this factors into

$$Q(x)=\frac{(2x-7)(x+4)(x+7)}{(3x-20)(x+6)(x+7)}$$

It's harder to get there, if you didn't work the revised ~~#8~~, but working from the denominator $3x^3+19x^2-134x-840 = (3x-20)(x+6)(x-c)$ allows you to find $x-c$!

$$\begin{array}{r|rrrr} -6 & 3 & 19 & -134 & -840 \\ & & -18 & -6 & 840 \\ \hline & 3 & 1 & -140 & 0 \end{array}$$

$$\begin{array}{r} 140 \\ 6 \\ \hline 840 \end{array}$$

$3x^2+x-140=0$

$(3x-20)(x+7)=0 \Rightarrow c=-7.$

Hole @ $x=-7$.

Plug $x=-7$ into $R(x)$

Revised:

$$R(x)=\frac{(2x-7)(x+4)}{(3x-20)(x+6)} \Longrightarrow$$

$$R(-7)=\frac{(2(-7)-7)(-7+4)}{(3(-7)-20)(-7+6)}$$

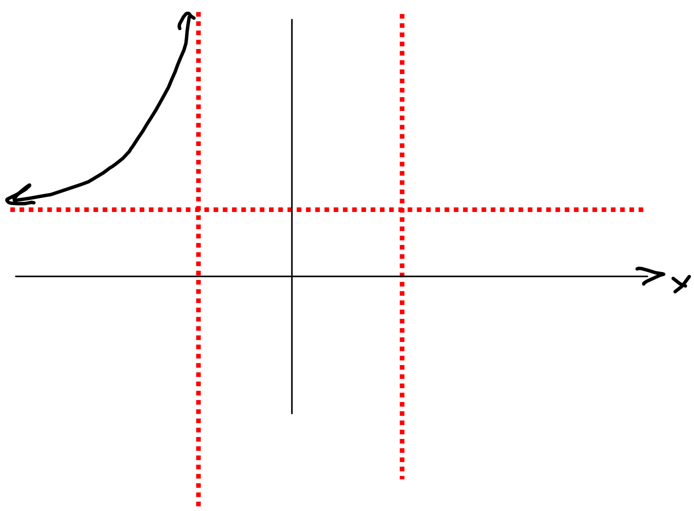
$$=\frac{(-14-7)(-3)}{(-21-20)(-1)}=\frac{(-21)(-3)}{41}=\frac{63}{41}$$

Before Revised

$$R(x)=\frac{2x^2+x+28}{3x^2-2x-120} \Rightarrow R(-7)=\frac{2(-7)^2+(-7)+28}{41}$$

$$=\frac{2(49)-7+28}{41}=\frac{98+21}{41}=\frac{119}{41}$$

HOLE: $\left(-7, \frac{119}{41}\right)$



10. (5 pts) Sketch the graph of $T(x)=\dfrac{2x^3+19x^2-19x-252}{3x^2-2x-120}$, showing all intercepts and asymptotes. This was also built off #8, so use the zeros you found for the numerator in #8 to help you find the 3rd zero of this new numerator.

$T(x)=\dfrac{(x+9)(2x-7)(x+4)}{(x+6)(3x-20)}$

$\boxed{x=-6,\ x=\frac{20}{3}\ \text{V.A.}}$

$D=\mathbb{R}\setminus\{-6,\frac{20}{3}\}$

$\boxed{\text{y-int:}}$ $\frac{252}{120}=\frac{126}{60}=\frac{63}{30}=\frac{21}{10}$

$\boxed{(0,\frac{21}{10})}$

$\boxed{\text{x-int: } (-9,0), (-4,0), (\frac{7}{2},0)}$

$y=\frac{2}{3}$

Slant Asymptote:

$$\begin{array}{r|l} & \frac{2}{3}x+\frac{61}{9} \quad r \\ 3x^2-2x-120 & 2x^3+19x^2-19x-252 \\ & -(2x^3-\frac{4}{3}x^2-80x) \\ & \frac{61}{3}x^2 \\ & -(\frac{61}{3}x^2) \end{array}$$

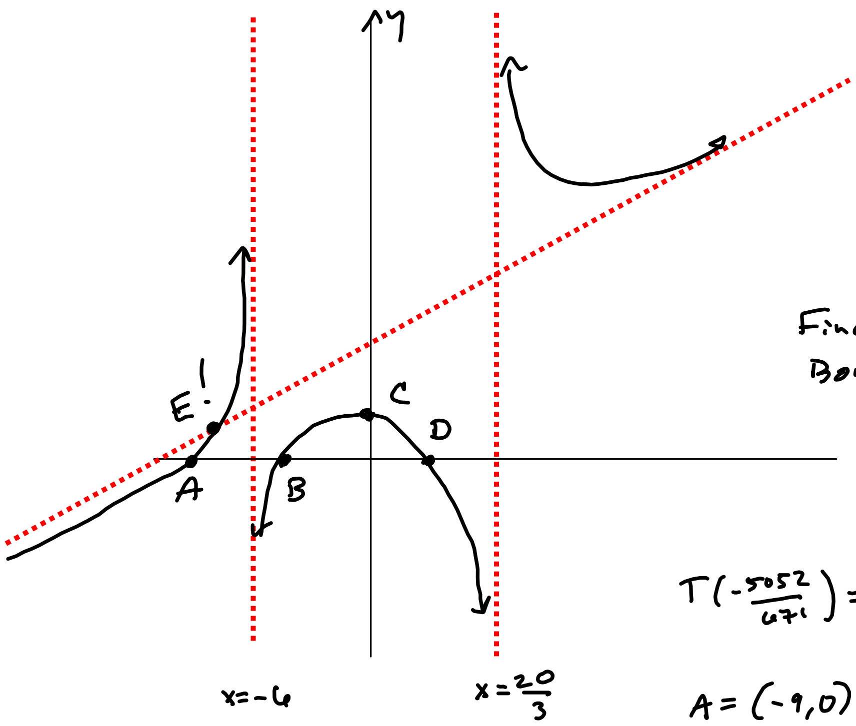
$19+\frac{4}{3}=\frac{19\cdot 3+4}{3}=\frac{57+4}{3}=\frac{61}{3}$

$\dfrac{\frac{61}{3}}{3}=\frac{61}{9}$

$\therefore$ $y=\frac{2}{3}x+\frac{61}{9}$ is slant asymptote.

y-int: $\left(0, \frac{21}{10}\right)$

x-int: $(-9,0), (-4,0), \left(\frac{7}{2}, 0\right)$



$y = \frac{2}{3}x + \frac{61}{9}$ SET $= 0$ $\longrightarrow$

$\frac{2}{3}x = -\frac{61}{9}$ $\longrightarrow$

$x = -\frac{61}{9} \cdot \frac{3}{2} = -\frac{61}{6}$

Find E for 5 points Bonus!

$\frac{2}{3}x + \frac{61}{9} = T(x)$ $\longrightarrow$

$x = -\frac{5052}{671} \approx -7.529061103$

$T\left(-\frac{5052}{671}\right) = \frac{10619}{6039} \approx 1.758403709$

$A = (-9, 0)$

$B = (-4, 0)$

$C = \left(0, \frac{21}{10}\right)$

$D = \left(\frac{7}{2}, 0\right)$

$E = \left(-\frac{5052}{671}, \frac{10619}{6039}\right)$ $\boxed{\text{5 pts Bonus}}$

$\approx (-7.529, 1.758)$

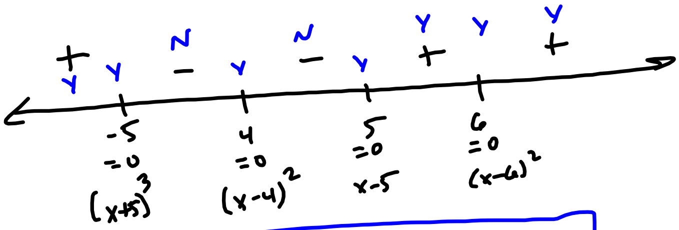
11. (2 pts) What is the domain of $W(x)=\sqrt{(x-5)(x-4)^2(x+5)^3(x-6)^2}$?

$W(x)=\sqrt{f(x)}$. Need $f(x)\geq 0$

$x=-5,\ 4,\ 5,\ 6$

x^8

$1+2+3+2=8$

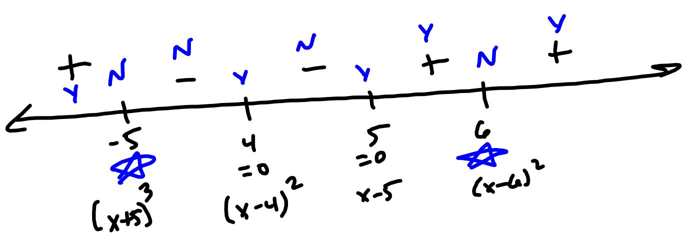


$\boxed{\mathcal{D}(W)=(-\infty,-5]\cup\{4\}\cup[5,\infty)}$

12. (2 pts) What is the domain of $K(x)=\sqrt{\dfrac{(x-5)(x-4)^2}{(x+5)^3(x-6)^2}}$?

$K(x)=\sqrt{f(x)}$

Need $f(x)\geq 0$ AND $(x+5)(x+3)^4\neq 0$



Almost the same as #11, except $x=-5$ & $x=6$ are blowups (not in domain)

$\boxed{\mathcal{D}(K)=(-\infty,-5)\cup\{4\}\cup[5,6)\cup(6,\infty)}$