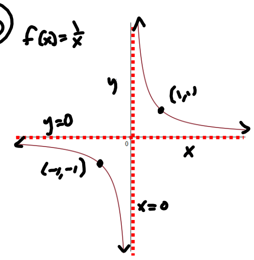
Solutions Using Method 2

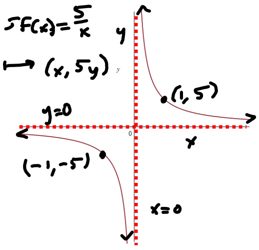
1. (5 pts) $g(x) = \frac{5}{3x-14} - 11$ (Use $(1,1)$, and $(-1,-1)$ as the 3 (x,y)'s in the 1st graph.). I hope and expect to see 2 asymptotes, clearly shown and labeled.

0. $f(x) = \frac{1}{x}$



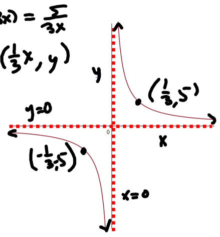
1. $5f(x) = \frac{5}{x}$

$(x,y) \longmapsto (x, 5y)$



2. $5f(3x) = \frac{5}{3x}$

$(x,y) \longmapsto (\frac{1}{3}x, y)$

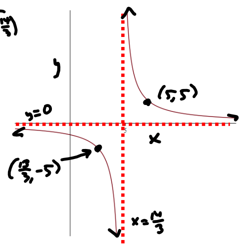


3. $5f(3(x-\frac{14}{3})) = \frac{5}{3(x-\frac{14}{3})}$

$(x,y) \longmapsto (x+\frac{14}{3}, y)$

$-\frac{1}{3}+\frac{14}{3} = \frac{13}{3}$

$+\frac{1}{3}+\frac{14}{3} = \frac{15}{3} = 5$

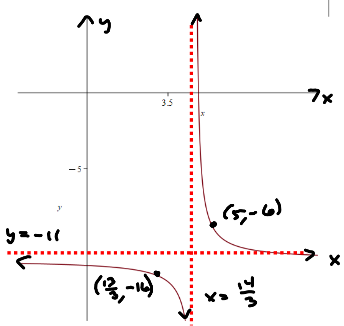


4. $g(x) = 5f(3(x-\frac{14}{3})) - 11$

$(x,y) \longmapsto (x, y-11)$

$5-11=-6$

$-5-11=-16$

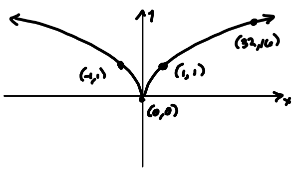


Method 2

2. (5 pts) $g(x) = 5(7x+21)^{4/5} - 8$ (Use $(0,0)$, $(1,1)$, and $(32,16)$ as the 3 points in the 1st graph.)

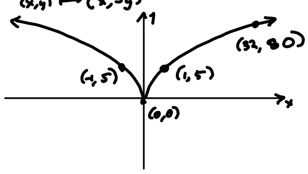
$7x+21 = 7(x+3)$

⓪ $f(x) = x^{\frac{4}{5}}$



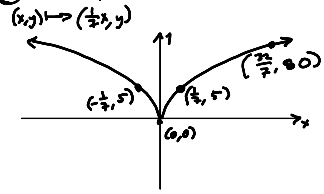
① $5f(x) = 5x^{\frac{4}{5}}$

$(x,y) \longmapsto (x, 5y)$



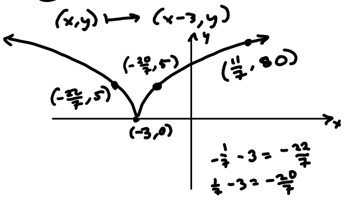
② $5f(7x) = 5(7x)^{\frac{4}{5}}$

$(x,y) \longmapsto (\frac{1}{7}x, y)$



③ $5f(7(x+3)) = 5(7(x+3))^{\frac{4}{5}}$

$(x,y) \longmapsto (x-3, y)$



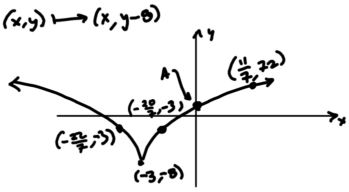
$-\frac{1}{7} - 3 = -\frac{22}{7}$

$\frac{1}{7} - 3 = -\frac{20}{7}$

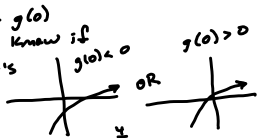
$\frac{32}{7} - 3 = \frac{11}{7}$

④ $5f(7(x+3)) - 8 = 5(7(x+3))^{\frac{4}{5}} - 8 = g(x)$

$(x,y) \longmapsto (x, y-8)$



check $g(0)$ to know if it's



$g(0) = 5(7(0)+21)^{\frac{4}{5}} - 8$

$= 5(21^{\frac{4}{5}}) - 8$

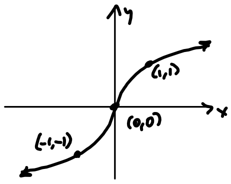
$\approx 49.11437650 > 0$

$\boxed{A \approx (0, 49.11437650)}$

Method 2

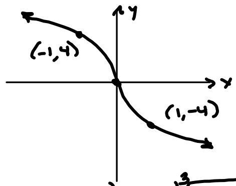
3. (5 pts) $g(x) = -4\sqrt[3]{4x-16}+7$

⓪ $f(x)=\sqrt[3]{x} = x^{\frac{1}{3}}$



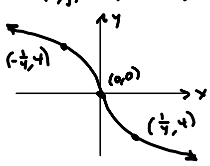
① $-4f(x) = -4\sqrt[3]{x}$

$(x,y) \mapsto (x,-4y)$



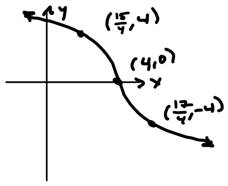
② $-4f(4x) = -4\sqrt[3]{4x}$

$(x,y) \mapsto (\frac{1}{4}x, y)$



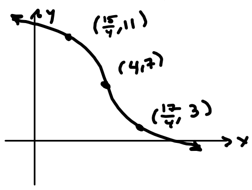
③ $-4f(4(x-4)) = -4\sqrt[3]{4(x-4)}$

$(x,y) \mapsto (x+4, y)$



④ $-4f(4(x-4)) + 7 = g(x) = -4\sqrt[3]{4(x-4)} + 7$

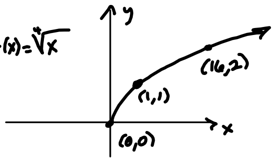
$(x,y) \mapsto (x, y+7)$



Method 2

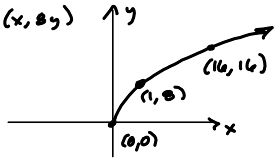
4. (5 pts) $g(x) = 8\sqrt[4]{7x+14} + 11$

⓪ $f(x) = \sqrt[4]{x}$



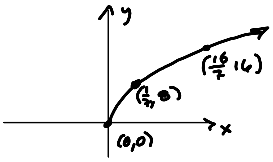
① $8f(x) = 8\sqrt[4]{x}$

$(x,y) \longmapsto (x, 8y)$



② $8f(7x) = 8\sqrt[4]{7x}$

$(x,y) \longmapsto (\frac{1}{7}x, y)$

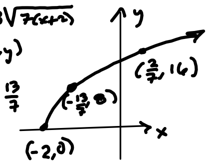


③ $8f(7(x+2)) = 8\sqrt[4]{7(x+2)}$

$(x,y) \longmapsto (x-2, y)$

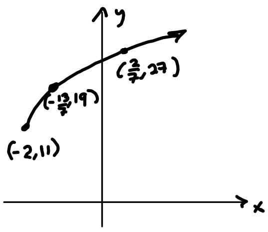
$\frac{1}{7} - 2 = \frac{1-14}{7} = -\frac{13}{7}$

$\frac{16}{7} - \frac{14}{7} = \frac{2}{7}$



④ $g(x) = 8\sqrt[4]{7(x+2)} + 11 = 8f(7(x+2)) + 11$

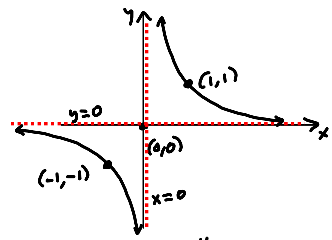
$(x,y) \longmapsto (x, y+11)$



Method 2

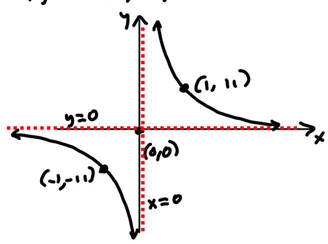
5. (5 pts) $g(x)=\dfrac{11}{(6x-42)^3}+8$

(0) $f(x)=\dfrac{1}{x^3}$



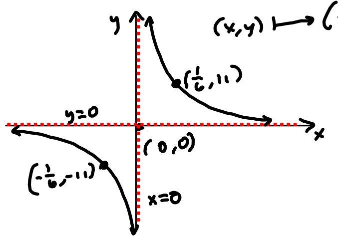
(1) $11f(x)=\dfrac{11}{x^3}$

$(x,y)\longmapsto(x,11y)$



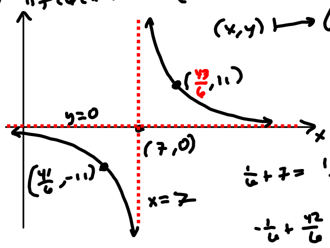
(2) $11f(6x)=\dfrac{4}{(6x)^3}$

$(x,y)\longmapsto(\frac{1}{6}x,y)$



(3) $11f(6(x-7))=\dfrac{11}{(6(x-7))^3}$

$(x,y)\longmapsto(x+7,y)$

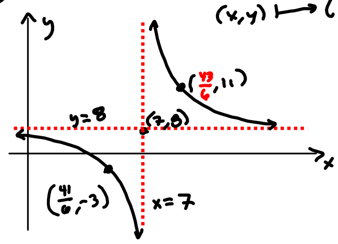


$\frac{1}{6}+7=\frac{1+42}{6}$

$-\frac{1}{6}+\frac{42}{6}=\frac{41}{6}$

(4) $11f(6(x-7))+8=\dfrac{11}{(6(x-7))^3}+8$

$(x,y)\longmapsto(x,y+8)$

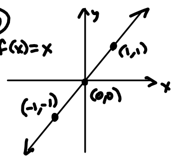


Method 2

We treat lines and parabolas a little differently. They come up so often - plus the completing-the-square trick – we sidestep the whole $f(bx)$ issue and just work with $g(x) = a(x-h)^2 + k$ and $g(x) = m(x-h) + k = m(x-x_1) + y_1$.

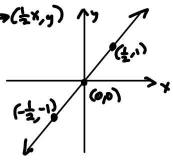
6. (5 pts) $g(x) = 2(x-7)+6$

0 $f(x) = x$



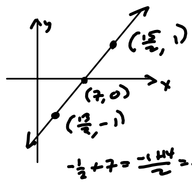
1 $2f(x) = 2x$

$(x,y) \mapsto (\frac{1}{2}x, y)$



2 $2f(x-7) = 2(x-7)$

$(x,y) \mapsto (x+7, y)$

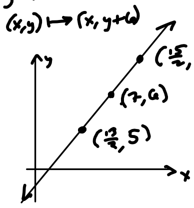


$-\frac{1}{2} + 7 = \frac{-1+14}{2} = \frac{13}{2}$

$\frac{1}{2} + \frac{14}{2} = \frac{15}{2}$

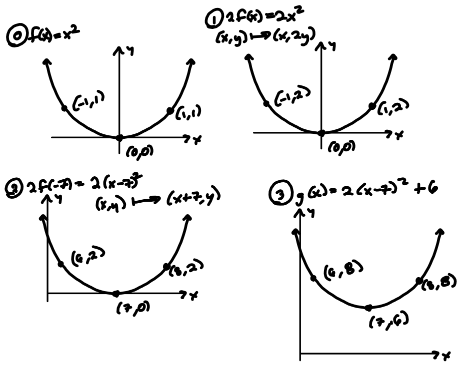
3 $g(x) = 2(x-7)+6$

$(x,y) \mapsto (x, y+6)$



Method 2

7. (5 pts) $g(x) = 2(x-7)^2 + 6$

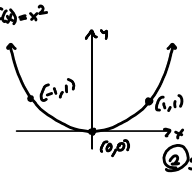


Method 2

8. (5 pts) $g(x) = x^2 - 4x - 27 = x^2 - 4x + 2^2 - 27 - 4 = (x-2)^2 - 31$

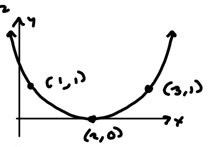
$\frac{4}{2} = 2 \to 2^2 = 4$

⓪ $f(x) = x^2$

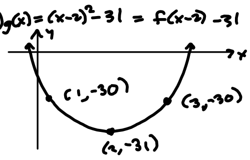


① $f(x-2) = (x-2)^2$

$x \mapsto x+2$

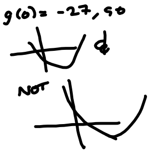


② $g(x) = (x-2)^2 - 31 = f(x-2) - 31$



Check

$g(0) = -27$, so



Method 2

9. (5 pts) $g(x) = 2x^2 - 5x + 20$

$= 2(x^2 - \frac{5}{2}x \quad) + 20$

$\frac{5}{2} \div 2 = \frac{5}{4} \rightarrow (\frac{5}{4})^2 = \frac{25}{16}$

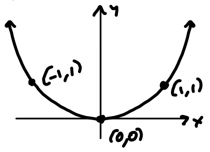
$= 2(x^2 - \frac{5}{2}x + (\frac{5}{4})^2) + 20 - 2(\frac{25}{16})$

(SCRATCH: $20 - 2(\frac{25}{16}) = 20 - \frac{25}{8} = \frac{160-25}{8} = \frac{135}{8} = 16 + \frac{7}{8}$)

$= 2(x - \frac{5}{4})^2 + \frac{135}{8}$

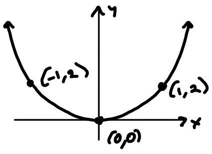
$= 2(x - \frac{5}{4})^2 + \frac{135}{8}$

⓪ $f(x) = x^2$



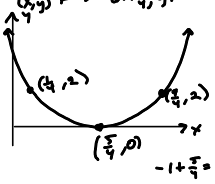
① $2f(x) = 2x^2$

$(x, y) \mapsto (x, 2y)$



② $2f(x - \frac{5}{4}) = 2(x - \frac{5}{4})^2$

$(x, y) \mapsto (x + \frac{5}{4}, y)$



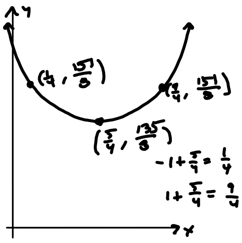
$-1 + \frac{5}{4} = \frac{1}{4}$

$1 + \frac{5}{4} = \frac{9}{4}$

③ $g(x) = 2f(x - \frac{5}{4}) + \frac{135}{8}$

$2 + \frac{135}{8} = \frac{16 + 135}{8}$

$= \frac{151}{8}$



$-1 + \frac{5}{4} = \frac{1}{4}$

$1 + \frac{5}{4} = \frac{9}{4}$

Method 2

Method 1

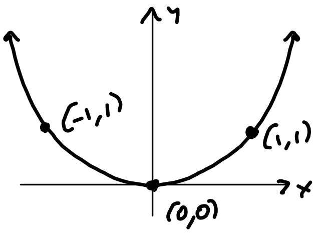
$$-20-\frac{49}{8}=\frac{-160-49}{8}=\frac{-209}{8}$$

10. (5 pts) $g(x)=2x^2-7x-20$

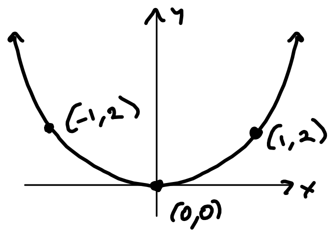
$$=2\left(x^2-\frac{7}{2}x+\left(\frac{7}{4}\right)^2\right)-20-2\left(\frac{49}{16}\right)$$

$$=2\left(x-\frac{7}{4}\right)^2-\frac{209}{8}$$

(0) $f(x)=x^2$

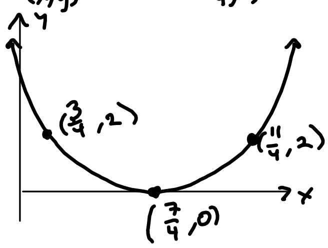


(1) $2f(x)=2x^2$ $\quad (x,y)\mapsto(x,2y)$



(2) $2f\left(x-\frac{7}{4}\right)=2\left(x-\frac{7}{4}\right)^2$

$(x,y)\mapsto\left(x+\frac{7}{4},y\right)$



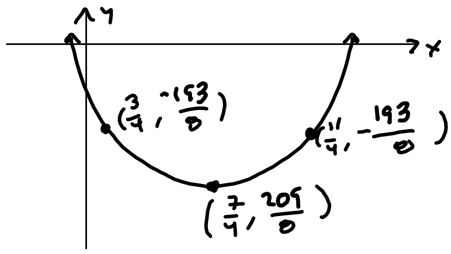
$-1+\frac{7}{4}=\frac{-4+7}{4}=\frac{3}{4}$

$\frac{4+7}{4}=\frac{11}{4}$

(3) $g(x)=2f\left(x-\frac{7}{4}\right)-\frac{209}{8}$

$=2\left(x-\frac{7}{4}\right)^2-\frac{209}{8}$

$2+\frac{135}{8}=\frac{16+135}{8}=\frac{151}{8}$



$$2-\frac{209}{8}=\frac{16-209}{8}=\frac{-193}{8}$$