MAT 1340 **Writing Project #1 Solutions** **Fall, 2024** **Mills**

#s 1 – 4 Use the quadratic formula to solve each of the following quadratic equations for x. Be sure to compute the discriminant, first, and separately. I'm looking for that on tests, as well, *whenever* you face a quadratic expression. It modularizes the work, and it tells you what you're getting into.

1. $x^2 + x - 42 = 0$

$a=1, b=1, c=-42$

$b^2-4ac = 1^2-4(1)(-42)$

$= 1+168 = 169 \Rightarrow \sqrt{169} = \sqrt{13^2} = 13$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm 13}{2(1)} = \frac{-1+13}{2} = \frac{12}{2} = 6$, $\frac{-1-13}{2} = \frac{-14}{2} = -7$

$\boxed{x \in \{-7, 6\}}$

2. $3.08x^2 - 3.17x - 7.35 = 0$ (Round your final answer to 4 decimal places.)

TIMES 100:

$308x^2 - 317x - 735 = 0$

$\Rightarrow a=308, b=-317, c=-735$

$\Rightarrow b^2-4ac = 317^2-4(308)(-735)$

$= 100489 + 905520$

$= 1006009$

$\sqrt{b^2-4ac} = \sqrt{1006009} = 1003$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{317 \pm 1003}{2(308)} = \frac{317 \pm 1003}{616}$

$\frac{15}{7} \approx 2.142857143$

$-\frac{49}{44} \approx -1.113636364$

Useful for the!

$\Rightarrow \boxed{x \in \{2.1429, -1.1136\}}$ to 4 places

Another way to write it: $\boxed{x \approx 2.1429, -1.1136}$ if you prefer.

I like you to see it as a solution set, to beef up your understanding of sets. Both sol'ns are OK. You didn't need to provide both.

3. $13x^2 - 12x + 17 = 0$ (Give an exact answer, in simplified radical form.

$a = 13, b = -12, c = 17$

$b^2 - 4ac = 12^2 - 4(13)(17)$

$= 144 - 884 = -740$

$2 \,|\, 740$, $2 \,|\, 370$, $5 \,|\, 185$, 37

$\sqrt{-740} = 2i\sqrt{185}$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{12 \pm 2i\sqrt{185}}{2(13)}$

$= \boxed{\frac{6 \pm i\sqrt{185}}{13} = x}$ (or $x \in \left\{ \frac{6 \pm i\sqrt{185}}{13} \right\}$

4. $12rx^2 - 15px - 11q = 0$ (Your answers will have letters in them. That's OK!)

$a = 12r, b = -15p, c = -11q$

$b^2 - 4ac = (15p)^2 - 4(12r)(-11q)$

$= 225p^2 + 528rq$

$\longrightarrow x = \frac{15p \pm \sqrt{225p^2 + 528rq}}{2(12r)}$

$= \boxed{\frac{15p \pm \sqrt{225p^2 + 528rq}}{24r} = x}$

48×11: $48 + 480 = 528$

$2 \,|\, 528$, $2 \,|\, 264$, $2 \,|\, 132$, $2 \,|\, 66$, $3 \,|\, 33$, 11

$3 \,|\, 225$, $3 \,|\, 75$, $5 \,|\, 25$, 5

$225 = 15^2$ duh

$= 3^2 \cdot 5^2$

$528 = 2^4 \cdot 3 \cdot 11$

No common factor.

#s 5, 6 Solve the following quadratic equations for x by factoring. You may use a sledgehammer, if you wish, but write the polynomial in factored form, after you find the solutions, to show you understand the connection between factors and solutions, frontwards and backwards! Give answers as integers or fractions, in lowest terms.

5. $x^2 + x - 42 = 0$ $ac = 1(-42) = -42 = -2(3)(7)$.

want factors of -42 whose sum is +1 for the 1x in the middle.

$-6+7$ works, so

$x^2 - 6x + 7x - 42 = x(x-6) + 7(x-6)$

$= (x-6)(x+7) \overset{\text{SET}}{=} 0$

$\Rightarrow x-6=0$ or $x+7=0$

$\Rightarrow \boxed{x \in \{-7, 6\}}$

6. $308x^2 - 317x - 735 = 0$

Method 1: Old-School Factoring:

$a=308, b=-317, c=-735$

$ac = (308)(-735) = -226380 =$ MAGIC #

See next page for Sledgehammer.

Factoring by the *ac* method, however you do it (My way's unconventional, but pretty systematic), gets really hard with bigger numbers.

We can look for factors of the "Magic Number" that add up to -317 *or* we can play Price Is Right!

I prefer "Price Is Right!" We look for numbers that add up to -317 whose product is 226380

	Product	
$-317 = -318 + 1$	-318	HIGHER!
$= -327 + 10$	-3270	HIGHER!
$= -417 + 100$	-41700	"
$= -1317 + 1000$	$-1,317,000$	Lower!
$= -917 + 600$	-550200	"
$= -717 + 400$	-286800	"
$= -667 + 350$	-233450	"
$= -647 + 330$	-213510	HIGHER!
$= -657 + 340$	223380	"
$= -662 + 345$	-228390	Lower
$= -660 + 343$	-226380!	Sweet! Magic!

-226380

One of the toughest factorizations by this method that I've ever seen.

But at least it's systematic, and you're just playing higher-lower (like Price Is Right!)

$\Rightarrow 308x^2 - 317x - 735 = 308x^2 - 660x + 343x - 735$

$\frac{308}{660} \overset{\div 2}{=} \frac{154}{330} \overset{\div 2}{=} \frac{77}{165} \overset{\div 11}{=} \frac{7}{15}$ GCF = 44

$\frac{343}{735} \overset{\div 7}{=} \frac{49}{105} \overset{\div 7}{=} \frac{7}{15}$ GCF = 49

$= 44x(15x-7) + 49(15x-7) = (15x-7)(44x+49) = 0$

$\Rightarrow \boxed{x \in \left\{\frac{7}{15}, -\frac{49}{44}\right\}}$

SLEDGEHAMMER

#6 $308x^2 - 317x - 735 = 0$

We used the Quadratic Formula on #2, which is what we have, here, in #6, when we multiply #2 through by 100, so,

$\Rightarrow a = 308, b = -317, c = -735$

$\Rightarrow b^2 - 4ac = 317^2 - 4(308)(-735)$

$= 100489 + 905520$

$= 1006009$

$\sqrt{b^2-4ac} = \sqrt{1006009} = 1003$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{317 \pm 1003}{2(308)} = \dfrac{317 \pm 1003}{604}$

$\nearrow \frac{15}{7} \approx x$

$\searrow -\frac{49}{44} \approx x$

NOW, CHEAT/REVERSE-ENGINEER THE FACTORED FORM, USING C3 concepts:

$x = -\frac{49}{44}, \frac{15}{7}$ are zeros $\Rightarrow$

$x - (-\frac{49}{44}) = x + \frac{49}{44}$ & $x - \frac{15}{7}$ are factors of

$\Rightarrow 308x^2 - 317x - 735 = 308\left(x + \frac{49}{44}\right)\left(x - \frac{15}{7}\right)$

$= (44)(7)\left(x + \frac{49}{44}\right)\left(x - \frac{15}{7}\right)$

$= 44\left(x + \frac{49}{44}\right)(7)\left(x - \frac{15}{7}\right)$

$= \boxed{(44x + 49)(7x - 15)}$

For full credit, I needed to see the solution in rational (fraction) form, and the factored form of the quadratic function.

By recognizing the connection between zeros of polynomials and factored form of polynomials from the *reverse* direction, we obtain a factored form of the quadratic polynomial.

You were trained to use the zero product principle and factoring top find zeros.

Now you know (and hopefully see) that not only do factors give us zeros, but that zeros give us factors!

#s 7 – 10 Solve the following quadratic equations for x by completing the square. Do not use decimals; rather, use *fractions*, as needed, to complete the square. For example, use $\left(\frac{7}{2}\right)^2$, instead of $(3.5)^2$ for #7.

7. $x^2 + 8x - 55 = 0$

$x^2 + 8x = 55$

$x^2 + 8x + 4^2 = 55 + 16 = 71$

$\frac{8}{2} = 4 \rightsquigarrow 4^2 = 16$

$(x+4)^2 = 71$

$x + 4 = \pm\sqrt{71}$

$\boxed{x = -4 \pm \sqrt{71}}$

8. $x^2 - 20x - 25 = 0$

$x^2 - 20x = 25$

$x^2 - 20x + 10^2 = 25 + 100 = 125$

$\frac{20}{2} = 10 \rightsquigarrow 10^2 = 100$

$(x-10)^2 = 125$

$x - 10 = \pm\sqrt{125} = \pm 5\sqrt{5}$

$\boxed{x = 10 \pm 5\sqrt{5}}$

5 | 125
5 | 25
5

$\sqrt{125} = 5\sqrt{5}$

9. $3x^2 + 7x + 13 = 0$

$x^2 + \frac{7}{3}x + \frac{13}{3} = 0$

$x^2 + \frac{7}{3}x + \left(\frac{7}{6}\right)^2 = -\frac{13}{3} + \frac{49}{36} = -\frac{13}{3}\cdot\frac{12}{12} + \frac{49}{36} = \frac{-156+49}{36} = -\frac{107}{36}$

$\frac{\frac{7}{3}}{2} = \frac{7}{3}\cdot\frac{1}{2} = \frac{7}{6} \rightarrow \left(\frac{7}{6}\right)^2 = \frac{49}{36}$

$\left(x+\frac{7}{6}\right)^2 = -\frac{107}{36}$

$x + \frac{7}{6} = \pm\sqrt{\frac{-107}{36}} = \frac{\pm\sqrt{107}\,i}{6}$

$x = -\frac{7}{6} \pm \frac{i\sqrt{107}}{6}$ or $\boxed{\frac{-7 \pm i\sqrt{107}}{6} = x}$

10. $3x^2 - 14x + 8 = 0 \rightarrow$

$x^2 - \frac{14}{3}x + \frac{8}{3} = 0 \rightarrow$

$x^2 - \frac{14}{3}x = -\frac{8}{3}$

$\frac{\frac{14}{3}}{2} = \frac{7}{3} \rightarrow \left(\frac{7}{3}\right)^2 = \frac{49}{9}$

$x^2 - \frac{14}{3}x + \left(\frac{7}{3}\right)^2 = -\frac{8}{3} + \frac{49}{9} = -\frac{8}{3}\cdot\frac{3}{3} + \frac{49}{9} = \frac{-24+49}{9} = \frac{25}{9}$

$\left(x - \frac{7}{3}\right)^2 = \frac{25}{9}$

$x - \frac{7}{3} = \pm\sqrt{\frac{25}{9}} = \pm\frac{5}{3} \rightarrow$

$x = \frac{7 \pm 5}{3}$ $\frac{12}{3} = 4$, $\frac{2}{3} = \frac{2}{3}$ $\boxed{x \in \left\{\frac{2}{3}, 4\right\}}$