

Graphing Functions by Transforming Basic Functions

**Discussion**

FORMATTING: This is semi-formal writing, here. That means show some professionalism. You don't have to type it out, but you do need to be very clear. See [Course Schedule](#) for due dates. There are 3: Early-Bird, On-Time, and Late.

SUBMISSIONS: Upload your PDF file (of several pages) to the drop box in the Assignments.

FORMATTING:

1. Work should be written on Plain white paper or electronic medium with a plain white background. There shouldn't be any rulings on the page. "College-ruled" paper is for high school. Standard letter-size pages.
2. Leave margins around the edges of your paper. 1/2-inch, all around, should do it.
3. To do this assignment properly, each problem should take up most or all of a page.
4. Do NOT try to squeeze things in! Be LAVISH in your use of space. Leave room for proper labels *and* your teacher's annotations.
5. Early Birds receive the best service. On-Time's didn't get as good of service as I wished. But most of the class were Early Birds, and I was POOPED on Monday and Tuesday!
6. I generally fit an average of 2 graphs, side by side. All of the problems require 5 graphs, except for a few at the end.

Main Resources: [Chapter 2 Videos \(and notes\)](#), [Writing Project 2 Videos \(and notes\)](#), and a selection of [Old Writing Projects](#).

Students may use either of the following 2 methods. For full credit, I need to see 5 graphs for each problem, 1 point each. I expect to see you arrive at the graph of  $g$  by steps, applying each move, one at a time. There are 4 moves:

1. Replace  $y$  by  $ay$ :  $af(x): y \rightarrow ay$
2. Replace  $x$  by  $bx$ :  $f(bx): x \rightarrow \frac{1}{b}x$
3. Replace  $x$  by  $x+c$ :  $f(x+c): x \rightarrow x-c$
4. Replace  $y$  by  $y+d$ :  $f(x)+d: y \rightarrow y+d$

These 4 moves, in a proper sequence can get you from the graph of a basic function,  $f(x)$ , to the given function,

$$g(x) = af(bx+c)+d$$

There are two generally accepts ways of going about stringing the moves together. The two methods are identical, except that steps 2 and 3 are executed in reverse order. The reason I like Method 2 so much is that's how you want to think about functions in Trig and Calculus. Factoring out the coefficient of  $x$  inside the function allows you to *see* the phase shift in a trig function, at a glance. So it's very good for your mathematical intuition. Method 1 avoids having to add fractions, which some college students fear. You need only use one method. I will display both in the solutions. An upwardly mobile student should be able to do both.

**Method 1:** This method does the horizontal shift before the horizontal stretch/shrink. Students seem to like it, but they will see more **Method 2** in their math futures.

$$0. f(x) \Rightarrow 1. 3f(x) \Rightarrow 2. 3f(x+2) \Rightarrow 3. 3f(5x+2) \Rightarrow 4. 3f(5x+2)+7 = g(x)$$

$$1. (x, y) \mapsto (x, 3y) \quad 2. (x, y) \mapsto (x - 2, y) \quad 3. (x, y) \mapsto \left(\frac{1}{5}x, y\right) \quad 4. (x, y) \mapsto (x, y + 7)$$

**Method 2:** This method does the horizontal shrink/stretch before the horizontal shift.

$$0. f(x) \Rightarrow 1. 3f(x) \Rightarrow 2. 3f(5x) \Rightarrow 3. 3f\left(5\left(x + \frac{2}{5}\right)\right) \Rightarrow 4. 3f\left(5\left(x + \frac{2}{5}\right)\right) + 7 = g(x)$$

$$1. (x, y) \mapsto (x, 3y) \quad 2. (x, y) \mapsto \left(\frac{1}{5}x, y\right) \quad 3. (x, y) \mapsto \left(x - \frac{2}{5}, y\right) \quad 4. (x, y) \mapsto (x, y + 7)$$

### Problem Set

Graph the function  $g(x)$  by transforming the graph of a basic function,  $f(x)$ . Start with a basic function graph, with at least 2 – and preferably 3 – points labeled. Then track where each of those points is moved to at each step. Using the same points I always use is usually the easiest, because THEY are the easiest ones to obtain in your basic function graph that is always the first graph in these sequences.

$$1. g(x) = \frac{2}{5x+15} + 7 \text{ (Use } (1,1), \text{ and } (-1,-1) \text{ as the 3 } (x,y)'s \text{ in the 1}^{\text{st}} \text{ graph.) This graph has 2 asymptotes.}$$

$$2. g(x) = 5(7x+21)^{2/3} - 8 \text{ (Use } (0,0), (1,1), \text{ and } (8,4) \text{ as the 3 points in the 1}^{\text{st}} \text{ graph.)}$$

$$3. g(x) = \frac{7}{(9x+18)^3} - 5 \text{ (Asymptotes!)}$$

$$4. g(x) = -5\sqrt[3]{11x-55} + 7$$

$$5. g(x) = 6\sqrt[4]{2x+14} + 11$$

$$6. g(x) = 5(6x-42)^3 + 8$$

We treat lines and parabolas a little differently. They come up so often - plus the completing-the-square trick – we sidestep the whole  $f(bx)$  issue and just work with  $g(x) = a(x-h)^2 + k$  and  $g(x) = m(x-h) + k = m(x-x_1) + y_1$ .

$$7. g(x) = -7(x+2) + 5$$

$$8. g(x) = -7(x+2)^2 + 5$$

$$9. g(x) = x^2 - 6x - 8$$

$$10. g(x) = 2x^2 - 4x + 20$$

Completing-the-Square Cheat:

$$g(x) = ax^2 + bx + c = a(x-h)^2 + k = a\left(x - \frac{-b}{2a}\right)^2 + g\left(-\frac{b}{2a}\right)$$