Section 8.5 - Combinations.

The General Idea

Permutation: Choose & arrange.

Combination: Choose!

$P(n,k) = k!\,C(n,k) \Rightarrow C(n,k) = \frac{P(n,k)}{k!}$

$k!$ is the # of ways to arrange the k things you chose.

$P(3,2) = \frac{3!}{(3-2)!} = \frac{3!}{1!} = 3! = 6.$

ab, ac, ba, bc, ca, cb. 6 things

$\{a,b,c\}$

$C(3,2) = \frac{3!}{(3-2)!\,2!} = \frac{3!}{1!\,2!}$

$\{a,b\}, \{a,c\}, \{b,c\}$

$= 3 = \frac{3}{2!} = \frac{P(3,2)}{2!}$

The Binomial Theorem

and

Pascal's Triangle

$$(x+y)^n = \sum_{k=0}^{n} C(n,k)\,x^{n-k}y^k$$

$$(x+y)^2 = \sum_{k=0}^{2} C(n,k)\,x^{n-k}y^k = C(2,0)x^2y^0 + C(2,1)x^1y^1 + C(2,2)x^0y^2$$
$$= 1x^2 + 2xy + 1y^2$$
$$= x^2+2xy+y^2$$

$$(x+y)^3 = \sum_{k=0}^{3} C(n,k)x^{n-k}y^k = C(3,0)x^3y^0 + C(3,1)x^2y^1 + C(3,2)x^1y^2 + C(3,3)x^0y^3$$
$$= x^3+3x^2y+3xy^2+y^3$$
1 3 3 1

Pascal's Triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

C(4,0) C(4,1) C(4,2) C(4,3) C(4,4)

$$(x+y)^4 = x^4+4x^3y+6x^2y^2+4xy^3+y^4$$

8.5.1 1

Fill in the blank. Combination

A _______ is a subset of a set of objects.

Sets: order doesn't matter.

8.5.5 2

Evaluate the expression below.

$$\frac{11!}{7!4!} = \frac{11!}{7!\,4!} = \frac{11\cdot \cancel{9}^{3}\cdot \cancel{8}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}} = \boxed{33}$$

$= C(11,7)$ or $C(11,4)$

8.5.9 3

Evaluate the expression C(16,1).

C(16,1) = ☐ (Type an integer or a simplified fraction.)

$$C(16,1) = \frac{16!}{15!\,1!} = 16$$

"16, choose 1" Pick one

= "16, choose 15" "Pick one that's not gonna be picked."

8.5.13 4

Evaluate the expression.

$$C(12,7) = \frac{12!}{5!\,7!} = \frac{\cancel{12}\cdot 11\cdot\cancel{10}\cdot 9\cdot 8}{\cancel{5}\cdot\cancel{4}\cdot 3\cdot\cancel{2}} = 99\cdot 8$$

$= 792$

$$\begin{array}{r} 799 \\ 8 \\ \hline 792 \end{array}$$

8.5.21 5

17 people are struggling to survive in the wilderness. In this week's episode, the producers will send 5 of the 17 back to civilization. In how many ways can the 5 be selected?

$$C(17,5) = \frac{17!}{12!\,5!} = \frac{17\cdot\cancel{16}^{2}\cdot 14\cdot 13\cdot\cancel{15}}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}} = 17\cdot 2\cdot 14\cdot 13 = \boxed{6188}$$

```
50))+sin(√(70)))
        16.76290818
17 nCr 5
               6188
17*2*13*14
               6188
```

8.5.21 (6)

17 people are struggling to survive in the wilderness. In this week's episode, the producers will send 5 of the 17 back to civilization. In how many ways can the 5 be selected?

Repeat!

8.5.27 (7)

How many five-card hands can be drawn from a deck of 52? 2,598,960, obviously.

$$C(52,5) = \frac{52!}{47!\,5!} = \frac{52\cdot 51\cdot \cancel{50}^{10}\cdot 49\cdot \cancel{48}^{2}}{\cancel{5\cdot 4\cdot 3\cdot 2}} = 52\cdot 51\cdot 10\cdot 49\cdot 2$$

```
                6188
17*2*13*14
                6188
52 nCr 5
             2598960
52*51*10*49*2
             2598960
```

8.5.31 (8)

Solve, using the idea of labeling. How many permutations are possible using the 6 letters in the word CANADA?

$$\frac{6!}{3!} = 6\cdot 5\cdot 4 = 120.$$

The number of possible permutations is ☐.

8.5.35 (9) Question Help

15 students volunteered to work in the governor's reelection campaign. 5 will be assigned to making phone calls, 3 will be assigned to stuffing envelopes, and 7 will be assigned to making signs. In how many ways can the assignments be made?

(Choose 5 callers) (Choose 3 stuffers) (Choose 7 signers)

$$C(15,5)\cdot C(10,3)\cdot C(7,7) = \frac{15!}{10!\,5!}\cdot\frac{10!}{7!\,3!}\cdot 1$$

$$= \frac{\cancel{15}\cdot \cancel{14}^{7}\cdot 13\cdot \cancel{12}^{3}\cdot 11}{\cancel{5\cdot 4\cdot 3\cdot 2}}\cdot\frac{10\cdot \cancel{9}^{3}\cdot \cancel{8}^{4}}{\cancel{3\cdot 2}} = 7\cdot 13\cdot 3\cdot 11\cdot 10\cdot 3\cdot 4$$

$$\boxed{= 360{,}360}$$

```
7*13*3*11*10*3*4
              360360
(15 nCr 5)*(10 n
Cr 3)
              360360
```

8.5.39 10

A health inspector must visit 4 of 6 restaurants on Monday. In how many ways can she pick the 4 restaurants?

$$C(6,4) = \frac{6!}{4!2!} = \frac{6\cdot 5}{2} = \boxed{15}$$

8.5.41 11

From the 12 male and 10 female sales representatives for an insurance company, a team of 2 will be picked.

In how many ways can the team of 4 representatives be selected?

men and 2 women will be selected to attend a national conference on insurance fraud.

In how many ways can the team of 4 be selected? Messed-up the copy-paste.

$$C(12,2)\cdot C(10,2) = \frac{12\cdot 11}{2}\cdot\frac{10\cdot 9}{2} = 66\cdot 45 = 2970$$

8.5.43 12

In an experiment on social interaction, 6 people will sit in 6 seats in a row. In how many ways can this be done?

A bit ambiguous. Does the ordering of the 6 matter?
Apparently not.
So $C(6,6) = 1$

8.5.49 13

1
1 1
1 2 1

Write the complete binomial expansion for $(r+s)^2$. $= r^2 + 2rs + s^2$

8.5.57 14

132
$\underline{6}$
192

Expand.

$(2-h)^6 = (h-2)^6$ is easier for me.

$(2-h)^6 = ((-1)(h-2))^6$

$= (-1)^6 (h-2)^6 = (h-2)^6$

$= h^6 + 6(h)^5(-2)^1 + 15(h)^4(-2)^2 + 20(h)^3(-2)^3 + 15(h)^2(-2)^4 + 6(h)^1(-2)^5 + (-2)^6$

$= \boxed{h^6 - 12h^5 + 60h^4 - 160h^3 + 240h^2 - 192h + 64}$

8.5.59 15

Write the complete binomial expansion for the following power of a binomial.

$(x^3+2)^4$

$= (x^3)^4 + 4(x^3)^3(2)^1 + 6(x^3)^2(2^2)$

$+ 4(x^3)^1(2)^3 + 2^4$

$\boxed{= x^{12} + 8x^9 + 24x^6 + 32x^3 + 16}$

8.5.61

16

Write the complete binomial expansion for $(u-2)^5$.

$= u^5 + 5(u)^4(-2) + 10(u)^3(-2)^2 + 10(u)^2(-2)^3 + 5(u)(-2)^4 + (-2)^5$

$= \boxed{u^5 - 10u^4 + 40u^3 - 80u^2 + 80u - 32}$

8.5.67

17

Write the first three terms of the binomial expansion.

$(5x-y^5)^5 = 1(5x)^5(-y^5)^0 + 5(5x)^4(-y^5)^1 + 10(5x)^3(-y^5)^2$
$+ 10(5x)^2(-y^5)^3 + 5(5x)^1(-y^5)^4 + (-y^5)^5$

$= \boxed{5^5x^5 - 5(5^4)x^4(y^5) + 10(5^3)x^3y^{10}} - 10(5^2)x^2y^{15} + 5(5x)y^{20} - y^{25}$

$= \boxed{3125x^5 - 3125x^4y^5 + 1250x^3y} - 250x^2y^{15} + 25xy^{20} - y^{25}$

8.5.69

18

Write the first three terms in the binomial expansion of $(4a-0.5b)^3$.

$1(4a)^3(-.5b)^0 + 3(4a)^2(-.5b)^1 + 3(4a)^1(-.5b)^2$

$= \boxed{64a^3 - 24a^2b + 3ab^2}$

$(12)(.5)^2 = \frac{12}{4} = 3$

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8.5.73 19

What is the coefficient of w^2y^6 in the expansion of $(w + y)^8$?

28

$C(8,6)w^{8-6}y^6$

No. $2 = 8-6$

$C(8,6)w^{8-6}y^6$

↳ same as $C(8,2)$

8.5.85 20

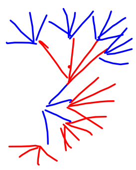
$w^8 \quad w^7 \quad w^6 \quad w^5 \quad w^4 \quad w^3 \quad w^2$

A multiple-choice test consists of 6 questions with each question having 4 possible answers.

I guess they want to know how many ways to answer the test.

$\underline{4} \cdot \underline{4} \cdot \underline{4} \cdot \underline{4} \cdot \underline{4} \cdot \underline{4}$

$= 4^6 = \boxed{4096 \text{ ways}}$



1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

8.5.89 21

Find a_5 if $a_1 = 7$ and $a_n = 1 - a_{n-1}$ for $n > 1$.

$a_1 = 7, \quad a_2 = 1 - a_1 = 1 - 7 = -6$

$a_3 = 1 - a_2 = 1 - (-6) = 7$

$a_4 = 1 - a_3 = 1 - 7 = -6$

$a_5 = 7$

$\boxed{a_5 = 7}$