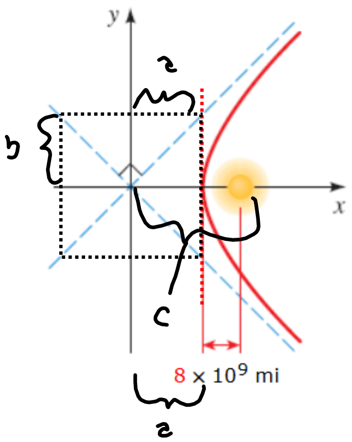
7.3 #18

Some comets, such as Halley's comet, are a permanent part of the solar system, traveling in elliptical orbits around the sun. Other comets pass through the solar system only once, following a hyperbolic path with the sun at a focus. The figure shows the path of such a comet.



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

They must be equal, b/c the diagonals are @ 90°, so the "box" is a square.

$$\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$$

Find an equation for the path, assuming that the closest the comet comes to the sun is 8×10^9 mi and that the path the comet was taking before it neared the solar system is at a right angle to the path it continues on after leaving the solar system. (Round your answer to two decimal places.)

Distance from focus to vertex is 8×10^9

c = distance from focus to center &

$$c^2 = a^2 + b^2 = 2a^2 = (a + 8\times10^9)^2$$

$$2a^2 = a^2 + 2a(8\times10^9) + (8\times10^9)^2$$

$$-2(8\times10^9)a = -2(8\times10^9)a$$

$$a^2 - 2(8\times10^9)a = (8\times10^9)^2$$

$$a^2 - 2(8\times10^9)a + (8\times10^9)^2 = (8\times10^9)^2 + (8\times10^9)^2 = 2(8\times10^9)^2$$

$$(a - 8\times10^9)^2 = 2(8\times10^9)^2$$

$$a - 8\times10^9 = \pm\sqrt{2(8\times10^9)^2} = \pm 8\times10^9\sqrt{2}$$

Drop the "–"

$$\Rightarrow a = 8\times10^9 + 8\times10^9\sqrt{2} = 8\times10^9(1+\sqrt{2})$$

$$\Rightarrow a^2 = 3.7301933598 \times 10^{20}$$

$$\approx 3.73 \times 10^{20} \approx a^2$$

$$\Rightarrow \frac{x^2}{3.73\times10^{20}} - \frac{y^2}{3.73\times10^{20}} = 1$$

OR $x^2 - y^2 = 3.73\times10^{20}$