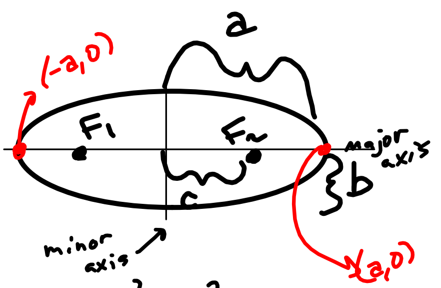
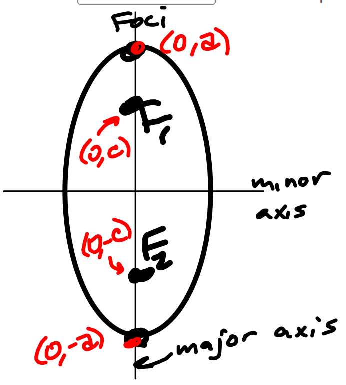
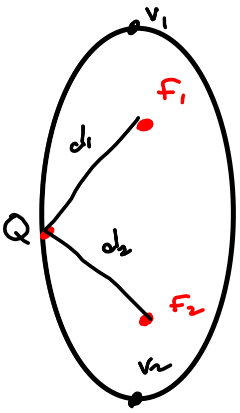
1 An ellipse is the set of all points in the plane for which the [---Select---] sum of the distances from two fixed points F_1 and F_2 is constant. The points F_1 and F_2 are called the [---Select---] of the ellipse.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$



$d_1 + d_2 =$ constant

$V_1, V_2 =$ vertices

Let c = focal length
= distance from center to the foci.

$c^2 = a^2 - b^2$

$e =$ eccentricity $= \frac{c}{a}$

A perfect circle has eccentricity = 0.
Also c=0.

2 The graph of the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a > b > 0$ is an ellipse with vertices $(x, y) = ($ $\pm a, 0$ $)$ and $(x, y) = (-a, 0)$ and foci $(\pm c, 0)$, where $c =$ $\sqrt{a^2 - b^2}$. So the graph of $\frac{x^2}{10^2} + \frac{y^2}{8^2} = 1$ is an ellipse with vertices

$(x, y) = ($ 10, 0 $)$ (larger x-value) and $(x, y) = ($ -10, 0 $)$ (smaller x-value) and foci

$(x, y) = ($ 6, 0 $)$ (larger x-value) and $(x, y) = ($ -6, 0 $)$ (smaller x-value).

$c^2 = a^2 - b^2 = 100 - 64 = 36 = c^2 \Rightarrow c = +\sqrt{36} = \boxed{6 = c}$

3 The graph of the equation $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ with $a > b > 0$ is an ellipse with vertices $(x, y) = ($ $)$ and $(x, y) = (0, -a)$ and foci $(0, \pm c)$, where $c =$ $\sqrt{a^2 - b^2}$. So the graph of $\frac{x^2}{4^2} + \frac{y^2}{5^2} = 1$ is an ellipse with vertices

$(x, y) = ($ 0, 5 $)$ (larger y-value) and $(x, y) = ($ 0, -5 $)$ (smaller y-value) and foci

$(x, y) = ($ 0, 3 $)$ (larger y-value) and $(x, y) = ($ 0, -3 $)$ (smaller y-value).

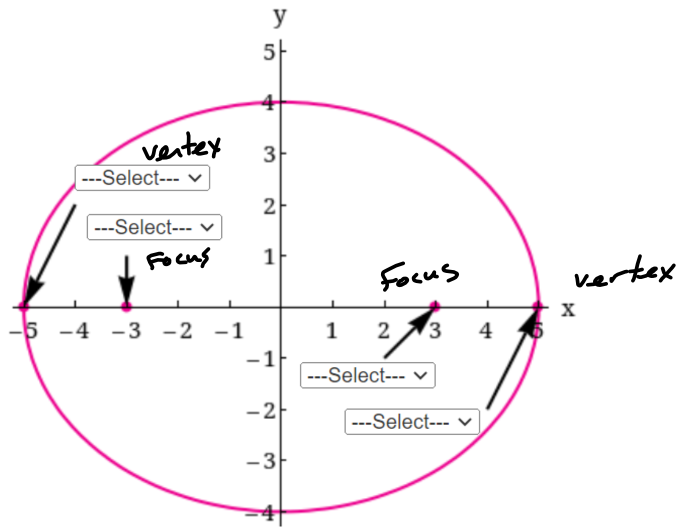
$c^2 = a^2 - b^2 = 25 - 16 = 9 = 3^2 \rightarrow \boxed{c = 3}$

$b^2 = a^2 - c^2$

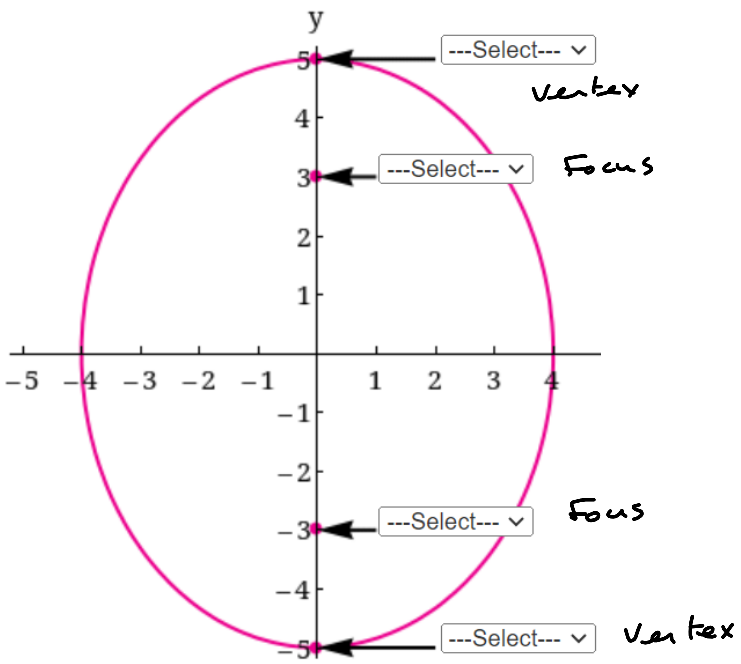
Label all the vertices and foci on the graphs given for the ellipses in Exercises 2 and 3.

4

(a) $\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$



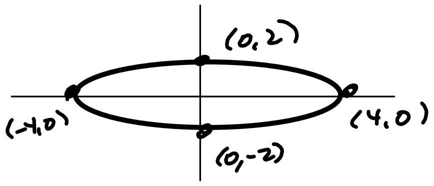
(b) $\frac{x^2}{4^2} + \frac{y^2}{5^2} = 1$



5 Match the equation with the graph.

$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$

$a=4, b=2$



An equation of an ellipse is given.

7 $$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

(a) Find the vertices, foci, and eccentricity of the ellipse.

vertex	$(x, y) =$	(−5, 0)	(smaller x-value)
vertex	$(x, y) =$	(5, 0)	(larger x-value)
focus	$(x, y) =$	(−3, 0)	(smaller x-value)
focus	$(x, y) =$	(3, 0)	(larger x-value)
eccentricity		$\frac{4}{5}$	

(b) Determine the length of the major axis.

$2a=$ 10

Determine the length of the minor axis.

$2b=$ 6

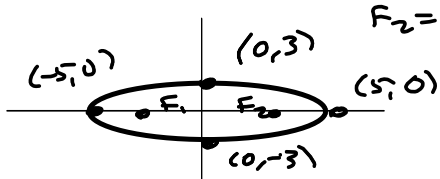
(c) Sketch a graph of the ellipse.

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

$a=5, b=3$

$F_1 = (-4,0)$

$F_2 = (4,0)$



$c^2 = a^2 - b^2 = 25 - 9 = 16$

$\Longrightarrow c = 4$

$e = \frac{c}{a} = \frac{4}{5}$

9 An equation of an ellipse is given.

$$16x^2 + 25y^2 = 400$$

$\frac{16x^2}{400} + \frac{25y^2}{400} = 1 \Rightarrow \frac{x^2}{25} + \frac{y^2}{16} = 1$

etc.

$c^2 = 25 - 16 = 9$

$\Rightarrow c = 3$

(a) Find the vertices, foci, and eccentricity of the ellipse.

vertex $(x, y) = ($ ___ $)$ (smaller x-value)

vertex $(x, y) = ($ ___ $)$ (larger x-value)

focus $(x, y) = ($ ___ $)$ (smaller x-value)

focus $(x, y) = ($ ___ $)$ (larger x-value)

eccentricity ___

(b) Determine the length of the major axis.

___

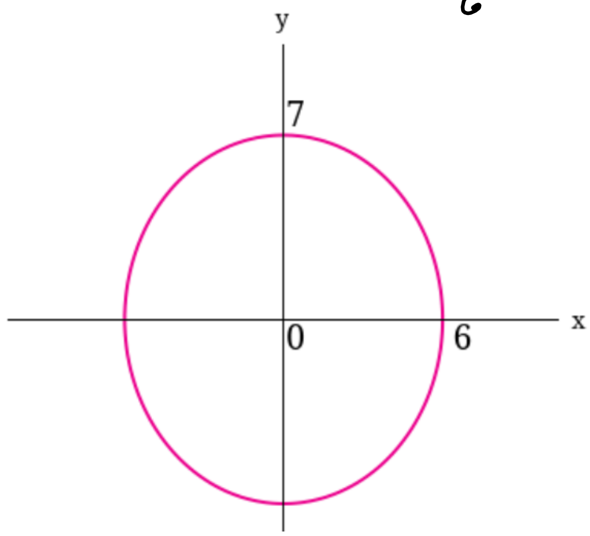
Determine the length of the minor axis.

___

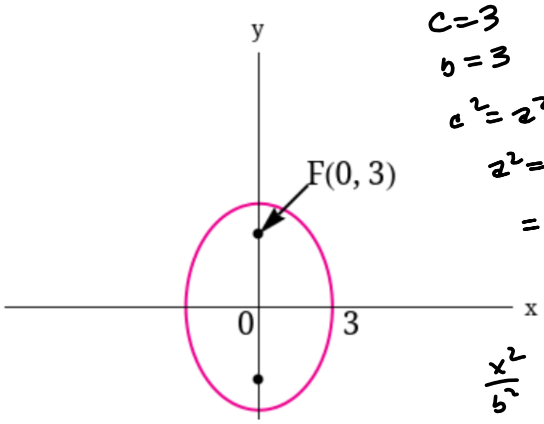
(c) Sketch a graph of the ellipse.

11 Find an equation for the ellipse whose graph is shown.

$$\frac{x^2}{6^2} + \frac{y^2}{7^2} = 1$$



12 Find an equation for the ellipse whose graph is shown.



$c = 3$

$b = 3$

$c^2 = a^2 - b^2 \rightarrow$

$a^2 = b^2 + c^2$

$= 3^2 + 3^2 = 2 \cdot 3^2 = 18$

$\rightarrow \cancel{a = 3\sqrt{2}}$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = \frac{x^2}{3^2} + \frac{y^2}{18} = 1$$

13 Find an equation for the ellipse that satisfies the given conditions.

Foci: $(\pm 4, 0)$, vertices: $(\pm 5, 0)$

$c=4$ $a=5$

$c^2=a^2-b^2 \Rightarrow$

$b^2=a^2-c^2 = 5^2-4^2 = 25-16 = \boxed{9=b^2}$

$\frac{x^2}{25}+\frac{y^2}{9}=1$

15 Find an equation for the ellipse that satisfies the given conditions.

Eccentricity: $\sqrt{3}/2$, foci on x-axis, length of major axis: 8, centered at the origin

$\underbrace{e=\frac{c}{a} \qquad 2a=8 \Rightarrow a=4}$

$\frac{\sqrt{3}}{2}=\frac{c}{4} \Rightarrow \boxed{c=2\sqrt{3}}$

$c^2=a^2-b^2$

$\Rightarrow b^2=a^2-c^2 = 4^2-(2\sqrt{3})^2 = 16-4\cdot 3 = 4 = b^2 \Rightarrow \boxed{2=b}$

$\frac{x^2}{4^2}+\frac{y^2}{2^2}=1$

16

Find the intersection points of the pair of ellipses.

$$\begin{cases} 9x^2 + y^2 = 9 & E1 \\ 9x^2 + 16y^2 = 144 & E2 \end{cases}$$

$(x, y) = ($ $)$ (smaller y-value)

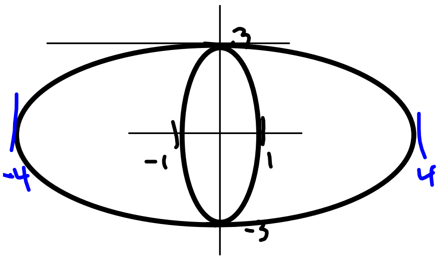
$(x, y) = ($ $)$ (larger y-value)

$$-E1 \quad -9x^2 - y^2 = -9$$
$$E2 \quad 9x^2 + 16y^2 = 144$$
$$-E1+E2 \quad 15y^2 = 135$$
$$y^2 = \frac{135}{15} = \frac{45}{5} = 9$$
$$\Rightarrow \boxed{y = \pm 3} \Rightarrow$$

Sketch the graphs of the pair of equations on the same coordinate axes.

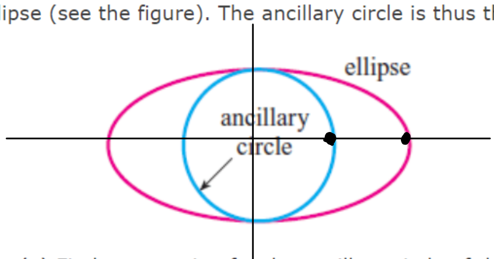
$$E1 \quad 9x^2 + y^2 = 9x^2 + \underset{(-3)^2}{3^2} = 9$$
$$9x^2 = 0$$
$$x = 0$$
$$\{(0,-3), (0,3)\}$$

$$9x^2 + y^2 = 9 \qquad 9x^2 + 16y^2 = 144$$
$$x^2 + \frac{y^2}{9} = 1 \qquad \frac{x^2}{16} + \frac{y^2}{9} = 1$$



18

The **ancillary circle** of an ellipse is the circle with radius equal to half the length of the minor axis and center the same as the ellipse (see the figure). The ancillary circle is thus the largest circle that can fit within an ellipse.



$$\frac{x^2}{64} + \frac{y^2}{4} = 1 \quad \Rightarrow b = r = 2$$

(a) Find an equation for the ancillary circle of the ellipse $x^2 + 16y^2 = 64$.

$\boxed{x^2 + y^2 = 2^2}$

(b) For the ellipse and ancillary circle of part (a), show that if (s, t) is a point on the ancillary circle, then $(4s, t)$ is a point on the ellipse.

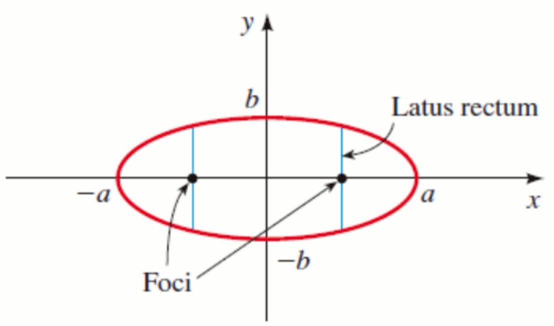
$$x^2 + y^2 = 4 \qquad \frac{x^2}{64} + \frac{y^2}{4} = 1$$

we show that if

$$s^2 + t^2 = 4, \text{ then } \frac{(4s)^2}{64} + \frac{t^2}{4} = 1$$

$$\Rightarrow s^2 = 4 - t^2$$
$$\Rightarrow s = \pm\sqrt{4 - t^2} \quad \text{OR} \quad t = \pm\sqrt{4 - s^2}$$
$$\Rightarrow \frac{16s^2}{64} + \frac{\left(\pm\sqrt{4-s^2}\right)^2}{4} = \frac{s^2}{4} + \frac{4-s^2}{4}$$
$$= \frac{s^2}{4} + \frac{4}{4} - \frac{s^2}{4} = 1,$$ i.e., $(4s, t)$ is on the ellipse.

19 A *latus rectum* for an ellipse is a line segment perpendicular to the major axis at a focus, with endpoints on the ellipse, as shown in the figure.



Show that the length of a latus rectum is $2b^2/a$ for the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad a > b.$$

The foci of the given ellipse are $(\pm$? $, 0)$, where $c^2 = a^2 - b^2$. If the endpoints of one latus rectum are the points $(c, \pm k)$, then the length of one latus rectum is ______ .

Substitute one of the endpoints into the given equation for the ellipse and solve for k.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{\boxed{}}{a^2} + \frac{k^2}{b^2} = 1$$

$$\frac{k^2}{b^2} = 1 - \left(\boxed{}\right)$$

$$\frac{k^2}{b^2} = \frac{\boxed{}}{a^2}$$

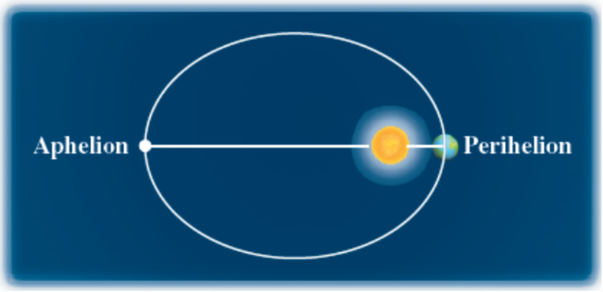
$$k^2 = \frac{b^2\left(\boxed{}\right)}{a^2}$$

$$k^2 = \frac{\boxed{}}{a^2}$$ Substitute b^2 for $a^2 - c^2$.

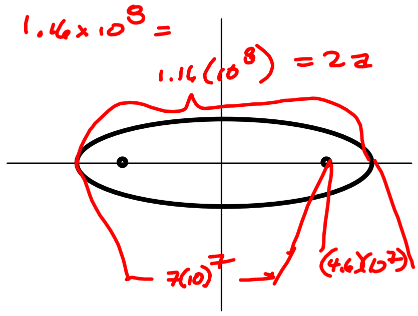
$$k = \boxed{}$$

Thus, the length of a latus rectum for the given ellipse is $2k =$ ______ .

20 The planets move around the sun in elliptical orbits with the sun at one focus. The point in the orbit at which the planet is closest to the sun is called **perihelion**, and the point at which it is farthest is called **aphelion**. These points are the vertices of the orbit. A planet's distance from the sun is 46,000,000 km at perihelion and 70,000,000 km at aphelion. Find an equation for the planet's orbit. (Place the origin at the center of the orbit with the sun on the *x*-axis. Use the following as necessary: *x*, *y*.)



$7\times10^7 + 4.6\times10^7 = 11.6\times10^7 = 1.16\times10^8 =$



$46{,}000{,}000 =$
$46\times10^6 = 4.6\times10^7$

$\Rightarrow a = \dfrac{1.16\times10^8}{2} = \boxed{5.8\times10^7 = a}$

$c = a - 4.6\times10^7 = \boxed{1.2\times10^7 = c}$

Ellipse $c^2 = a^2 - b^2 \Rightarrow$

$b^2 = a^2 - c^2 = (5.8\times10^7)^2 - (1.2\times10^7)^2$

$= 5.8^2\times10^{14} - 1.2^2\times10^{14}$

$= (5.8^2 - 1.2^2)(10^{14}) = 3.22\times10^{15}$

$\Rightarrow b = +\sqrt{3.22\times10^{15}} \approx 56745043.84$

$= \boxed{5.674504384 \times 10^7 = b}$

```
58000000
Ans-4.6*10^7
12000000
5.8²-1.2²
32.2
Ans*10^14
3.22E15
```

```
12000000
5.8²-1.2²
32.2
Ans*10^14
3.22E15
Ans^.5
56745043.84
```

$\boxed{3.22E15} = b^2$

$$\frac{x^2}{(5.8\times10^7)^2} + \frac{y^2}{3.22\times10^5} = 1$$

3.364×10^{15}

21 A carpenter wishes to construct an elliptical table top from a 4 ft by 8 ft sheet of plywood. He will trace out the ellipse using the "thumbtack and string" method illustrated in Figures I and II. What length of string should he use, and how far apart should the tacks be located, if the ellipse is to be the largest possible that can be cut out of the plywood sheet? (Round your answers to two decimal places.)

string 8 ft long

tacks 6.93 ft apart

$$\frac{x^2}{4^2} + \frac{y^2}{2^2} = 1$$

$$c^2 = 16 - 4 = 12$$

$$c = 2\sqrt{3} \approx 3.464101616$$

$c + a + a - c = 2a =$ length of string $= 2(4) = 8$ ft

Tacks $2c = 2(2\sqrt{3})$
$= 4\sqrt{3} \approx$

$\approx 6.928203232 \approx \boxed{6.93 \text{ ft} \approx \text{Tacks}}$

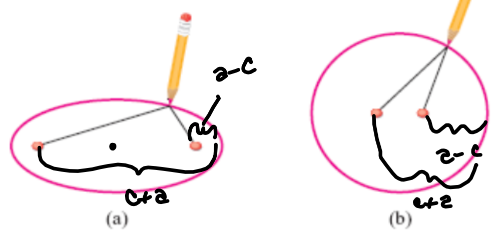


Figure I