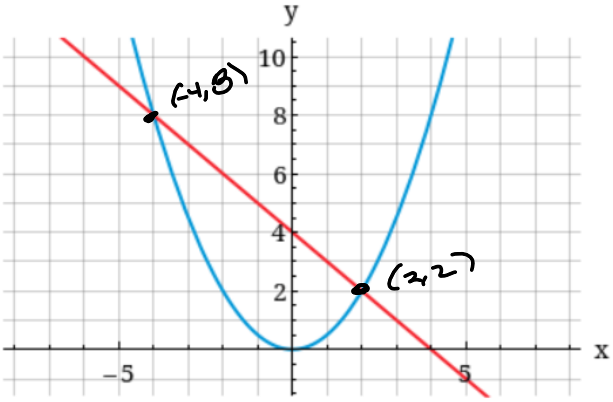
# 5.4 - Systems of Nonlinear Equations

1 The system of equations

$$\begin{cases} 2y - x^2 = 0 \\ y + x = 4 \end{cases}$$

is graphed below.



Use the graph to find the solutions of the system. (Assume that points of intersection fall on the grid lines.)

(-4,8), (2,2)

Use the substitution method to find all solutions of the system of equations.

2 $\begin{cases} y = x^2 \\ y = x + 72 \end{cases}$ $\Rightarrow$ $x^2 = x+72$

$x^2 - x - 72 = 0$

$(x-9)(x+8) = 0$

$\Rightarrow x = 9$ OR $x = -8$ $\Rightarrow$ $y = (-8)^2 = 64$

$y = -8+72 = 64$ ✓

$y = x^2 = 81$

OR $y = x + 72 = 9+72 = 81$ ✓

Sol'N SET = $\{(9, 81), (-8, 64)\}$

**4** Use the elimination method to find all solutions of the system of equations. (Order your answers from smallest to largest $x$, then from smallest to largest $y$.)

$$\begin{cases} x - y^2 + 6 = 0 \\ 2x^2 + y^2 - 7 = 0 \end{cases}$$

E1+E2 $2x^2+x-1=0 \Longrightarrow$

$(2x-1)(x+1)=0$

$\Longrightarrow x=\frac{1}{2}, -1$

$x=\frac{1}{2} \Longrightarrow x-y^2+6 = \frac{1}{2} - y^2+6 = -y^2+\frac{13}{2} = 0$

$\Longrightarrow -y^2 = -\frac{13}{2}$

$y = \pm\sqrt{\frac{13}{2}}$

oR $x=-1 \Longrightarrow x-y^2+6 = -1 -y^2+6 = -y^2+5 = 0$

$\Longrightarrow -y^2=-5$

$y^2=5$

$y=\pm\sqrt{5}$

$(x,y) = \left(\frac{1}{2}, \pm\sqrt{\frac{13}{2}}\right), \left(-1, \pm\sqrt{5}\right)$

Use the elimination method to find all solutions of the system of equations.

5

$$\begin{cases} x^2 - y^2 = 1 & E1 \\ 2x^2 - y^2 = x + 3 & E2 \end{cases}$$

Eliminate y.

$-E1+E2 \quad x^2 = -1+x+3 = x+2$

$\Rightarrow x^2 - x - 2 = (x-2)(x+1) = 0$

$x=-1$ or $x=2$

$E1 \rightarrow x^2 - y^2 = (-1)^2 - y^2 = 1$

$\Rightarrow -y^2 = 1$

$\Rightarrow$ No sol'n.

$2^2 - y^2 = 1$

$-y^2 = -3$

$y = \pm\sqrt{3} \rightsquigarrow (2, \pm\sqrt{3})$

$\boxed{(-1,0)}$ by inspection ?

$(-1)^2 - 0^2 = 1 = 1$ ✓

$2(-1)^2 - 0^2 \overset{?}{=} -1+3$

$2=2?$ ✓

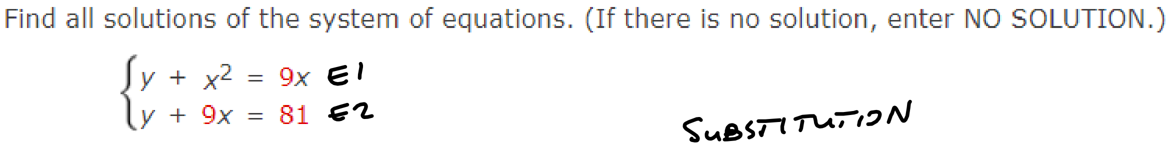
smallest y-value to the largest.

$(2,-\sqrt{3})$

$(-1,0)$

$(2,\sqrt{3})$

7



Find all solutions of the system of equations. (If there is no solution, enter NO SOLUTION.)

$$\begin{cases} y + x^2 = 9x & E1 \\ y + 9x = 81 & E2 \end{cases}$$

**Elimination**

-E1+E2 $\quad -x^2+9x = -9x+81$

$\Rightarrow -x^2+18x-81=0$

$\Rightarrow x^2-18x+81=0$

$\Rightarrow x^2-2(9)x+9^2=0$

$= (x-9)^2=0$

$\boxed{x=9} \Rightarrow E1 \Rightarrow y+9^2=9(9)$

$\Rightarrow \boxed{y=0}$

SUBSTITUTION

$E1 \Rightarrow y = -x^2+9x \Rightarrow$

$E2 \Rightarrow (-x^2+9x)+9x=81$

$\Rightarrow -x^2+18x=81$

$\Rightarrow \ldots \Rightarrow x^2-18x+81=0$, etc.

Find all solutions of the system of equations. (If there is no solution, enter NO SOLUTION.)

**8**

$$\begin{cases} x^2y = 25 \\ x^2 + 4y + 20 = 0 \end{cases}$$

~~Elimination~~

Subs:

2choices:

E1 $\rightarrow$ $y = \frac{25}{x^2}$

$\Rightarrow$ E2 $\rightarrow$ $x^2 + 4\left(\frac{25}{x^2}\right) + 20 = 0$

$\Rightarrow x^2 + \frac{100}{x^2} + 20 = 0$

LCD $= x^2$ $\Rightarrow$

$x^4 + 100 + 20x^2 = 0$

$\Rightarrow x^4 + 20x^2 + 100 = 0$

$\Rightarrow u^2 + 20u + 100 = 0$, where $u = x^2$.

$\rightarrow (u+10)^2 = 0 \Rightarrow$

$\Rightarrow u = -10 \Rightarrow$

$x^2 = -10 \Rightarrow$

No Solution ?!

E2 $\Rightarrow$ $4y = -x^2 - 20$

$y = \frac{-x^2-20}{4} = -\frac{x^2+20}{4}$

$\Rightarrow x^2\left(\frac{-x^2-20}{4}\right) = 25$

$\Rightarrow \frac{-x^4-20x^2}{4} = 25 \Rightarrow$

$-x^4 - 20x^2 = 100$

$\Rightarrow -(x^4 + 20x^2) = 100$

$\Rightarrow 0 \leq x^4 + 20x^2 = -100 < 0$

$0 < 0$ ?!

No Sol'N.

10 Find all solutions of the system of equations. (If there is no solution, enter NO SOLUTION.)

$$\begin{cases} \frac{2}{x} - \frac{3}{y} = 1 \\ -\frac{4}{x} + \frac{7}{y} = 1 \end{cases}$$

2E1 + E2 $\quad \frac{1}{y} = 3$

$\Rightarrow \boxed{y = \frac{1}{3}}$

$\rightarrow$ E1 $\Rightarrow \frac{2}{x} - \frac{3}{\left(\frac{1}{3}\right)} = 1$

$\longrightarrow \frac{2}{x} - 9 = 1$

$\Rightarrow \frac{2}{x} = 10$

$\Rightarrow 2 = 10x$

$x = \frac{2}{10} = \boxed{\frac{1}{5} = x}$

A graphing calculator is recommended.

**11** Use the graphical method to find all solutions of the system of equations, rounded to two decimal places.

$$\begin{cases} \frac{x^2}{9} + \frac{y^2}{18} = 1 \\ y = -x^2 + 6x - 4 \end{cases} \Longrightarrow \frac{x^2}{9} + \frac{(-x^2+6x-4)^2}{18} = 1$$

$$\Longrightarrow 2x^2 + (x^2-6x+4)^2 = 18 \quad E1$$

Scratch:

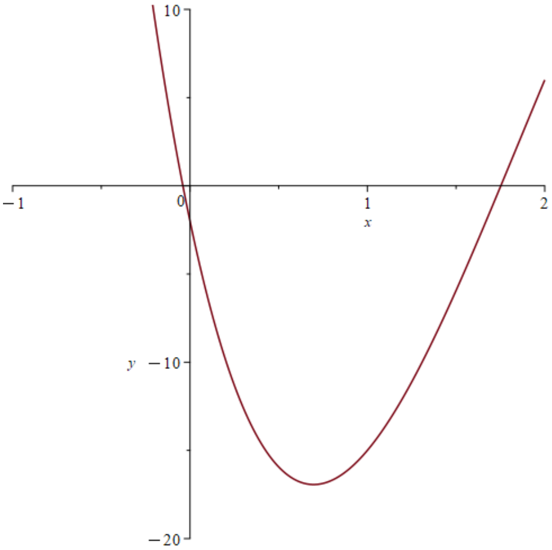
$$\left( \begin{aligned} (x^2-6x+4)(x^2-6x+4) = x^4 - 6x^3 &+ 4x^2 \\ -6x^3 &+ 36x^2 - 24x \\ &+ 4x^2 - 24x + 16 \\ \hline x^4 - 12x^3 &+ 44x^2 - 48x + 16 \end{aligned} \right)$$

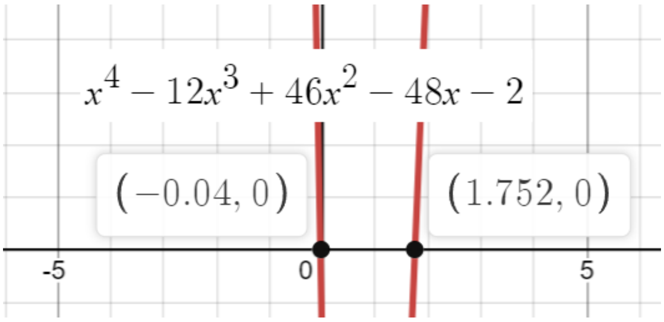
$$\Longrightarrow E1 \Longrightarrow 2x^2 + x^4 - 12x^3 + 44x^2 - 48x + 16 = 18$$

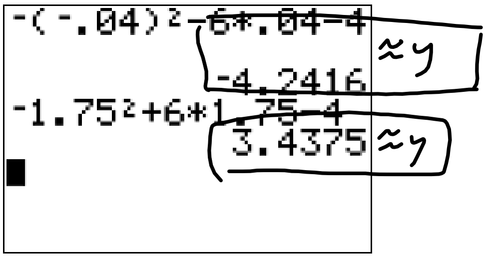
$$\Longrightarrow x^4 - 12x^3 + 46x^2 - 48x - 2 = 0$$

**Right away, Rational Zeros Theorem is no help.**

$\pm 1, \pm 2$ are clearly not x-intercepts







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-(-.04)²-6*.04-4
              -4.2416  ≈y
-1.75²+6*1.75-4
               3.4375  ≈y
█
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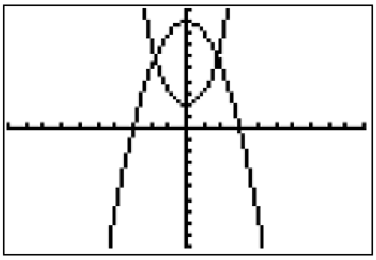
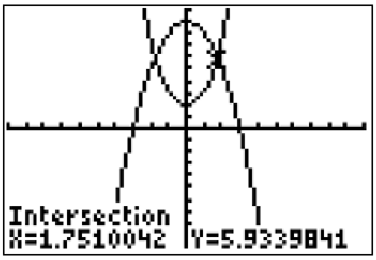
12 A graphing calculator is recommended.

Use the graphical method to find all solutions of the system of equations, rounded to two decimal places.

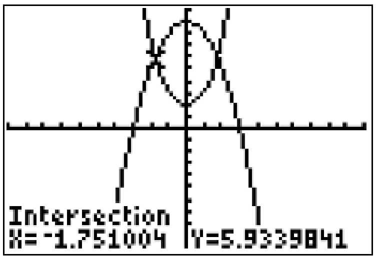
$$\left\{\begin{array}{l} y = e^x + e^{-x} \\ y = 9 - x^2 \end{array}\right\} \quad \text{Solve} \quad 9 - x^2 = e^x + e^{-x}$$

**We want *x*- and *y*-values, so getting '0' on one side is not as advantageous as finding the intersection.**





Intersection
X=1.7510042 Y=5.9339841



Intersection
X=-1.751004 Y=5.9339841

**Silly! If you'd been paying attention, you'd've seen that these are both even functions, and used symmetry to get the second solution.**