Section 5.3 - Partial Fractions

1 Write the form of the partial fraction decomposition of the function (as in Example 4). Do not determine the numerical values of the coefficients.

$$\frac{1}{(x-4)(x+3)} = \frac{A}{x-4} + \frac{B}{x+3}$$, where A & B are real #s

We find A & B by reverse-engineering the $\frac{1}{(x-4)(x+3)}$.

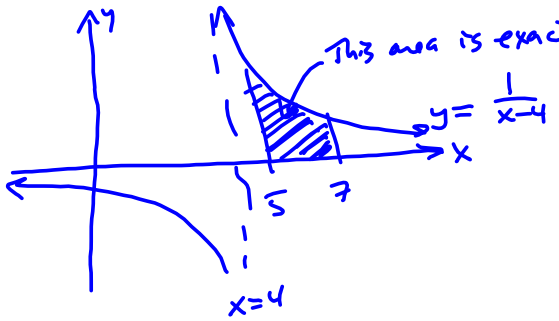
$\frac{A}{x-4}$ is nice for Calculus.

$$\int \frac{dx}{x-4} = \ln|x-4| + C$$

$$\int_5^7 \frac{dx}{x-4} = \ln|7-4| - \ln|5-4|$$

$$= \ln(3).$$

This area is exactly



Write the form of the partial fraction decomposition of the function (as in Example 4). Do not determine the numerical values of the coefficients.

2 $\frac{x}{x^2+6x-7} = \frac{x}{(x+7)(x-1)} = \frac{A}{x+7} + \frac{B}{x-1}$

3 Write the form of the partial fraction decomposition of the function (as in Example 4). Do not determine the numerical values of the coefficients.

$\frac{x^2-3x+5}{(x-3)^2(x+1)} = \frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{C}{x+1}$

Linear factor of $x-3$ raised to a power of 2.

4 Write the form of the partial fraction decomposition of the function (as in Example 4). Do not determine the numerical values of the coefficients.

$\frac{7}{x^4-x^3} = \frac{7}{x^3(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-1}$

$x^4 - x^3 = x^3(x-1)$

5 Write the form of the partial fraction decomposition of the function (as in Example 4). Do not determine the numerical values of the coefficients.

$\frac{x^2}{(x-8)(x^2+4)} = \frac{A}{x-8} + \frac{Bx+C}{x^2+4}$

Keep it real!

x^2+4 is irreducible over the reals.

6 Write the form of the partial fraction decomposition of the function (as in Example 4). Do not determine the numerical values of the coefficients.

$\frac{1}{x^4-625} = \frac{A}{x-5} + \frac{B}{x+5} + \frac{Cx+D}{x^2+25}$

$x^4-625 = x^4 - 25^2 = (x^2)^2 - 25^2 = (x^2-25)(x^2+25)$

$= (x-5)(x+5)(x^2+25)$

7 Write the form of the partial fraction decomposition of the function (as in Example 4). Do not determine the numerical values of the coefficients.

$$\frac{x^3 - 4x^2 + 2}{(x^2 + 25)(x^2 + 8)} = \frac{Ax+B}{x^2+25} + \frac{Cx+D}{x^2+8}$$

10 Write the form of the partial fraction decomposition of the function (as in Example 4). Do not determine the numerical values of the coefficients.

$$\frac{1}{(x^3 - 125)(x^2 - 25)} = \frac{1}{(x-5)(x^2+5x+25)(x-5)(x+5)}$$

$$x^3 - 5^3 = (x-5)\underbrace{(x^2+5x+25)}_{\text{irreducible}}$$

$$= \frac{1}{(x-5)^2(x+5)(x^2+5x+25)} = \frac{A}{x-5} + \frac{B}{(x-5)^2} + \frac{C}{x+5} + \frac{Dx+E}{x^2+5x+25}$$

Find the partial fraction decomposition of the rational function.

11 $\frac{4}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} \Rightarrow$

$4 = A(x+1) + B(x-1) \Rightarrow$

$4 = Ax + A + Bx - B \Rightarrow$

$Ax + Bx = 0$ $\quad$ $A - B = 4$

$x(A+B) = 0 \Rightarrow$ $\quad$ $A = B + 4$

$x = 0$ or $A + B = 0$

$\Rightarrow A = -B$ $\quad$ $\Rightarrow -B = B + 4$

$-2B = +4$

$\boxed{B = -2}$ &

$A = -B \Rightarrow \boxed{A = +2}$

-1

$\frac{-2}{x-1} + \frac{2}{x+1}$

$= \frac{-2(x+1) + 2(x-1)}{(x-1)(x+1)} = \frac{-2x - 2 + 2x - 2}{(x-1)(x+1)} = \frac{-4}{(x-1)(x+1)}$

14 Find the partial fraction decomposition of the rational function.

$$\frac{9x^2 - 14x + 16}{2x^3 - x^2 - 8x + 4} = \frac{A}{2x-1} + \frac{B}{x-2} + \frac{C}{x+2}$$

$x^2(2x-1) - 4(2x-1)$

$= (2x-1)(x^2-4) = (2x-1)(x-2)(x+2)$

$9x^2 - 14x + 16 = A(x-2)(x+2) + B(2x-1)(x+2) + C(2x-1)(x-2)$

$= A(x^2-4) + B(2x^2+3x-2) + C(2x^2-5x+2)$

$\Rightarrow 9x^2 = Ax^2 + 2Bx^2 + 2Cx^2$ $\quad -14x = 3Bx - 5Cx$

$9 = A + 2B + 2C$ $\quad -14 = 3B - 5C$

$16 = -4A - 2B + 2C$

$A + 2B + 2C = 9$ E1
$3B - 5C = -14$ E2
$-4A - 2B + 2C = 16$ E3

E1 $A + 2B + 2C = 9$ E1
E2 $3B - 5C = -14$ E2
$4E1 + E3$ $6B + 10C = 52$ E3

E1 $A + 2B + 2C = 9$
E2 $3B - 5C = -14$
$-2E2 + E3$ $20C = 80$

$\Rightarrow \boxed{C = 4}$

$\Rightarrow E2 \Rightarrow 3B - 5(4) = 3B - 20 = -14$

$\Rightarrow 3B = 6$

$\Rightarrow \boxed{B = 2}$

$\Rightarrow E1 \Rightarrow A + 2(2) + 2(4) = A + 4 + 8 = A + 12 = 9$

$A = -3$

$$= \frac{A}{2x-1} + \frac{B}{x-2} + \frac{C}{x+2} = \frac{-3}{2x-1} + \frac{2}{x-2} + \frac{4}{x+2}$$

16 Find the partial fraction decomposition of the rational function.

$$\frac{x^3 - 4x^2 - 3x + 5}{x^4} = \frac{x^3}{x^4} - \frac{4x^2}{x^4} - \frac{3x}{x^4} + \frac{5}{x^4}$$

$$= \frac{1}{x} - \frac{4}{x^2} - \frac{3}{x^3} + \frac{5}{x^4}$$