Section 5.2 -- Systems of Linear Equations in Several Variables

#s 1 - 4: Click here.

Use back-substitution to solve the triangular system.

5
$$\begin{cases} 5x - 5y + z = 0 \\ \quad\quad y + 5z = 23 \\ \quad\quad\quad\quad \boxed{z = 5} \end{cases}$$

$\Rightarrow y + 5z = y + 5(5) = y + 25 = 23 \Rightarrow \boxed{y = -2}$

$\Rightarrow 5x - 5y + z = 5x - 5(-2) + 5 = 5x + 10 + 5 = 5x + 15 = 0$

$\Rightarrow 5x = -15$

$\Rightarrow \boxed{x = -3}$

$\boxed{(x, y, z) = (-3, -2, 5)}$

6 Find the complete solution of the linear system, or show that it is inconsistent. (If the system has infinitely many solutions, express your answer in terms of t, where $x = x(t)$, $y = y(t)$, and $z = t$. If there is no solution, enter NO SOLUTION.)

$$\begin{array}{l} E1 \\ E2 \\ E3 \end{array} \begin{cases} x - 2y + 3z = -16 \\ 3y + z = 4 \\ x + y - z = 13 \end{cases}$$

$$\begin{array}{ll} -E1 & -x + 2y - 3z = 16 \\ E3 & x + y - z = 13 \\ \hline & 3y - 4z = 29 \end{array}$$

$$\begin{array}{ll} E1 & x - 2y + 3z = -16 \\ E2 & 3y + z = 4 \\ -1E1 + E3 & 3y - 4z = 29 \end{array}$$

$$\begin{array}{ll} -E2 & -3y - z = -4 \\ E3 & 3y - 4z = 29 \\ \hline & -5z = 25 \\ & z = -5 \end{array}$$

$$\begin{array}{ll} E1 & x - 2y + 3z = -16 \\ E2 & 3y + z = 4 \\ -E2 + E3 & \boxed{z = -5} \end{array}$$

$E2 \Rightarrow 3y + (-5) = 4$

$3y - 5 = 4$

$3y = 9$

$\boxed{y = 3}$

$\Rightarrow E1 \Rightarrow x - 2(3) + 3(-5) = -16$

$\Rightarrow x - 6 - 15 = -16$

$x - 21 = -16$

$\boxed{x = 5}$

$\boxed{(x, y, z) = (5, 3, -5)}$

7 Find the complete solution of the linear system, or show that it is inconsistent. (If the system has infinitely many solutions, express your answer in terms of t, where $x = x(t)$, $y = y(t)$, and $z = t$. If there is no solution, enter NO SOLUTION.)

$$\begin{array}{l} E1 \\ E2 \\ E3 \end{array}\begin{cases} x + y + z = 10 \\ x + 3y + 3z = 20 \\ 2x + y - z = 13 \end{cases}$$

$$\begin{array}{lr} -E1 & -x - y - z = -10 \\ E2 & x + 3y + 3z = 20 \\ \hline & 2y + 2z = 10 \end{array}$$

E1 $x + y + z = 10$

$-E1 + E2$ $2y + 2z = 10$

$-2E1 + E3$ $-y - 3z = 3 \to -7$

$$\begin{array}{lr} -2E1 & -2x - 2y - 2z = -10 \to -20 \\ E3 & 2x + y - z = 13 \\ \hline & -y - 3z = 3 \to -7 \end{array}$$

E1 $x + y + z = 10$ E1

$-E3$ $y + 3z = -3 \to +7$ E2

E2 $2y + 2z = 10$ E3

$$\begin{array}{lr} -2E2 & -2y - 6z = 6 \to -14 \\ E3 & 2y + 2z = 10 \\ \hline & -4z = 16 \to -4 \end{array}$$

$\boxed{z = -4 \to 1}$

E1 $x + y + z = 10$

E2 $y + 3z = -3 \to 7$

E3 $z = -4$

$y + 3(-4 \to 1) = y + 12 \to 3 = -3 \to 7!$

$\boxed{y = 9 \to 4}$

$x + 9 \to 4 + (-4 \to 1) = 10$

$x + 5 = 10$

$\boxed{x = 5}$

$\boxed{(x, y, z) = (5, 9, -4)} \to (5, 4, 1)$

Find the complete solution of the linear system, or show that it is inconsistent. (If the system has infinitely many solutions, express your answer in terms of t, where $x = x(t)$, $y = y(t)$, and $z = t$. If there is no solution, enter NO SOLUTION.)

13

$$\begin{cases} -x + 2y + 5z = 9 \\ x - 2z = 0 \\ 4x - 2y - 11z = 2 \end{cases} \Rightarrow \begin{array}{ll} E1 & x - 2z = 0 \\ E_2 & -x + 2y + 5z = 9 \\ E_3 & 4x - 2y - 11z = 2 \end{array}$$

We assume there is a solution.

$$\begin{array}{ll} E' & x - 2z = 0 \\ E1+E2 & 2y + 3z = 9 \\ -4E1+E3 & -2y - 3z = 2 \end{array}$$

$$\begin{array}{ll} -4E1 & -4x + 8z = 0 \\ E3 & 4x - 2y - 11z = 2 \\ \hline & -2y - 3z = 2 \end{array}$$

$$\begin{array}{ll} E1 & x - 2z = 0 \\ E2 & 2y + 3z = 9 \\ E2+E3 & 0 = 11 \text{ ?! Absurd.} \end{array}$$

We arrive at an absurdity; therefore, the assumption was false!

NO SOLUTION!

This is kind of the idea behind "Proof by Contradiction."

"*Reductio Ad Absurdum*" "Reduce to an absurdity."

14 Find the complete solution of the linear system, or show that it is inconsistent. (If the system has infinitely many solutions, express your answer in terms of t, where $x = x(t)$, $y = y(t)$, and $z = t$. If there is no solution, enter NO SOLUTION.)

$$\begin{array}{l} E1 \\ E2 \\ E3 \end{array} \begin{cases} 2x + 3y - z = 8 \\ x + 2y = 9 \\ x + 3y + z = 15 \end{cases}$$

Swap the order of the equations, if there's one whose coefficient of *x* is '1.' With a choice between 2 equations with a "leading 1," put the simplest one on top and the other one in the 2nd row. The "ugliest" equation goes on the bottom, so it doesn't mess with the other equations.

Re-write:

$$\begin{array}{lll} E2 & x + 2y = 9 & E1 \\ E3 & x + 3y + z = 15 & E2 \\ E1 & 2x + 3y - z = 8 & E3 \end{array}$$

$$\begin{array}{ll} -E1 & -x - 2y = -9 \\ E2 & x + 3y + z = 15 \\ \hline -E1+E2 & y + z = 6 \end{array}$$

$$\begin{array}{lll} E1 & x + 2y = 9 & E1 \\ -E1+E2 & y + z = 6 & E2 \\ -2E1+E3 & -y - z = -10 & E3 \end{array}$$

$$\begin{array}{ll} -2E1 & -2x - 4y = -18 \\ E3 & 2x + 3y - z = 8 \\ \hline -2E1+E3 & -y - z = -10 \end{array}$$

$$\begin{array}{ll} E1 & x + 2y = 9 \\ E2 & y + z = 6 \\ E2+E3 & 0 = -4 \end{array}$$

?! Absurd!

$\boxed{\text{No solution!}}$

15 Find the complete solution of the linear system, or show that it is inconsistent. (If the system has infinitely many solutions, express your answer in terms of t, where $x = x(t)$, $y = y(t)$, and $z = t$. If there is no solution, enter NO SOLUTION.)

$$\begin{array}{l} E1 \\ E2 \\ E3 \end{array}\begin{cases} x - 2y + z = 3 \\ 2x - 5y + 6z = 7 \\ 2x - 3y - 2z = 5 \end{cases}$$

$z = t$ is "free."

Scratch

$-2E1 \quad -2x + 4y - 2z = -6$
$E2 \quad 2x - 5y + 6z = 7$
$-y + 4z = 1$

$-2E1 \quad -2x + 4y - 2z = -6$
$E3 \quad 2x - 3y - 2z = 5$
$y - 4z = -1$

$E1 \quad x - 2y + z = 3$
$-2E1 + E2 \quad y - 4z = -1$
$-2E1 + E3 \quad y - 4z = -1$

MIS-COPY!

$E1 \quad x - 3y + z = 3$
$E2 \quad y - 4z = -1$
$E3 \quad 0 = 0$

$x - 2y + z = 3$
$y - 4z = -1$
$0 = 0$

$\Rightarrow E2 \quad \boxed{y = 4z - 1}$

NEWP!

$\Rightarrow E1 \quad ~~x - 3(4z-1) + z = 3~~$
$\Rightarrow ~~x - 12z + 3 + z = 3~~$
$\Rightarrow ~~x - 11z + 3 = 3~~$
$~~\boxed{x = 11z}~~$

~~my way:~~ $~~(x, y, z) = (11z, 4z-1, z)~~$
~~WebAssign way:~~ $~~(x, y, z) = (11t, 4t-1, t)~~$

$\Rightarrow \boxed{y = 4z - 1}$
$x - 2y + z = x - 2(4z-1) + z =$
$= x - 8z + 2 + z = x - 7z + 2 = 3$
$\Rightarrow \boxed{x = 7z + 1}$

my way
$(x, y, z) = (7z+1, 4z-1, z)$

use parameter t:
$= \boxed{(7t+1, 4t-1, t)}$

FINAL ANS

A graded #17 with few errors

Very clearly done. The only problem I found was mis-copy of an E2 about halfway in.

Find the complete solution of the linear system, or show that it is inconsistent. (If the system has infinitely many solutions, express your answer in terms of t, where $x = x(t)$, $y = y(t)$, $z = z(t)$, and $w = t$. If there is no solution, enter NO SOLUTION.)

$\frac{9}{10}$ is your score.

17

$$\begin{cases} x + z + 2w = 7 & E1 \\ y - 2z = -2 & E2 \\ x + 2y - z = 1 & E3 \\ 2x + y + 3z - 2w = 0 & E4 \end{cases}$$

$-E1$: $-x - z - 2w = -7$
$E3$: $x + 2y - z = 1$
$-E1+E3$: $2y - 2z - 2w = -6$

$-2E1$: $-2x - 2z - 4w = -14$
$E4$: $2x + y + 3z - 2w = 0$
$-2E1+E4$: $y + z - 6w = -14$

E1: $x + z + 2w = 7$ E1
E2: $y - 2z = -2$ E2
$-E1+E3$: $2y - 2z - 2w = -6$ E3
$-2E1+E4$: $y + z - 4w = -14$ E4

THIS is pretty much where you went wrong. 0 4

$-2E2$: $-2y + 4z + 4w = 12$
$E3$: $2y - 2z - 2w = -6$
$-2E2+E3$: $2z + 2w = 6$ -2 -2 $\Rightarrow z + w = 3$

$z - w = -1$

E1: $x + z + 2w = 7$ E1
E2: $y - 2z - 2w = -6$ E2
$-2E2+E3$: $z + w = 3$ E3
$-E2+E4$: $z - 2w = -4$ E4

$z - w = -1$

$-E2$: $-y + 2z = 2$
$E4$: $y + z - 6w = -14$
$E2+E4$: $3z - 6w = -12$
$z - 2w = -4$

FACTOR OUT The "3"

E1: $x + z + 2w = 7$ E1
E2: $y - 2z - 2w = -6$ E2 OK
E3: $z + w = 3$ E3
$-E3+E4$: $w = \frac{7}{3}$ E4

$-E3$: $-z - w = -3$
$E4$: $z - 2w = -4$
$-E3+E4$: $-3w = -7$ OK
$\boxed{w = \frac{7}{3}}$

E4 $\Rightarrow$ $z + w = z + \frac{7}{3} = 3$
$z = \frac{3}{1} \cdot \frac{3}{3} - \frac{7}{3} = \frac{9-7}{3} = \boxed{\frac{2}{3} = z}$

Looks pretty solid. OK

E3 $\Rightarrow$ $y - 2z - 2w = y - 2\left(\frac{2}{3}\right) - 2\left(\frac{7}{3}\right)$
$= y - \frac{4}{3} - \frac{14}{3} = y - \frac{18}{3} = y - 6 = -6$
$\Rightarrow \boxed{y = 0}$

E2 $\Rightarrow$ $x + z + 2w = x + \frac{2}{3} + 2\left(\frac{7}{3}\right) = x + \frac{16}{3} = \frac{7}{1} \cdot \frac{3}{3} = \frac{21}{3}$
$\Rightarrow x = \frac{-16+21}{3} = \boxed{\frac{5}{3} = x}$

$(x, y, z, w) = \left(\frac{5}{3}, 0, \frac{2}{3}, \frac{7}{3}\right)$ Nope! But OK, based on previous work!

#17 done correctly

Find the complete solution of the linear system, or show that it is inconsistent. (If the system has infinitely many solutions, express your answer in terms of t, where $x = x(t)$, $y = y(t)$, $z = z(t)$, and $w = t$. If there is no solution, enter NO SOLUTION.)

17

$$\begin{cases} x + z + 2w = 7 & E1 \\ y - 2z = -2 & E2 \\ x + 2y - z = 1 & E3 \\ 2x + y + 3z - 2w = 0 & E4 \end{cases}$$

E1: $x + z + 2w = 7$ E1
E2: $y - 2z = -2$ E2
$-E1+E3$: $2y - 2z - 2w = -6$ E3
$2E1+E4$: $y + z - 6w = -14$ E4

$-E1$: $-x - z - 2w = -7$
E3: $x + 2y - z = 1$
$-E1+E3$: $2y - 2z - 2w = -6$

$-2E1$: $-2x - 2z - 4w = -14$
E4: $2x + y + 3z - 2w = 0$
$-2E1+E4$: $y + z - 6w = -14$

E1: $x + z + 2w = 7$ E1
E2: $y - 2z = -2$ E2
$-2E2+E3$: $z - w = -1$ E3
$-E2+E4$: $z - 2w = -4$ E4

$-2E2$: $-2y + 4z = 4$
E3: $2y - 2z - 2w = -6$
$-2E2+E3$: $2z - 2w = -2 \Rightarrow z - w = -1$

Factor out the "2."

$-E2$: $-y + 2z = 2$
E4: $y + z - 6w = -14$
$-E2+E4$: $3z - 6w = -12$
$z - 2w = -4$

FACTOR OUT The "3"

E1: $x + z + 2w = 7$ E1
E2: $y - 2z = -2$ E2
E3: $z - w = -1$ E3
$-E3+E4$: $w = 3$ E4

$-E3$: $-z + w = 1$
E4: $z - 2w = -4$
$-E3+E4$: $-w = -3$
$\boxed{w = 3}$

$\Rightarrow E3 \Rightarrow z - w = z - 3 = -1$
$\Rightarrow \boxed{z = 2}$

$\Rightarrow E2 \Rightarrow y - 2z = y - 2(2) = y - 4 = -2$
$\Rightarrow \boxed{y = 2}$

$z=2$
$w=3$

$\Rightarrow E1 \Rightarrow x + z + 2w = x + 2 + 2(3) = x + 8 = 7$
$\Rightarrow \boxed{x = -1}$

$\Rightarrow \boxed{(x, y, z, w) = (-1, 2, 2, 3)}$

18 A biologist is performing an experiment on the effects of various combinations of vitamins. She wishes to feed each of her laboratory rabbits a diet that contains exactly 22 mg of niacin, 23 mg of thiamin, and 40 mg of riboflavin. She has available three different types of commercial rabbit pellets; their vitamin content (per ounce) is given in the table.

	Type A	Type B	Type C
Niacin (mg)	2	3	1
Thiamin (mg)	3	1	3
Riboflavin (mg)	8	5	7

How many ounces of each type of food should each rabbit be given daily to satisfy the experiment requirements? (If there is no solution, enter NO SOLUTION.)

Let x = the # of ounces of Type A Pellets,
y = " " " " " " B " ,
z = " " " " " " C " .

Requirements:

Niacin $2x+3y+z=22$ E1 $-$E1 + E2 $x-2y+2z=1$ E1
Thiamin $3x+1y+3z=23$ E2 E2 $3x+y+3z=23$ E2
Riboflavin $8x+5y+7z=40$ E3 E3 $8x+5y+7z=40$ E3

E1 $x-2y+2z=1$ E1
-3E1+E2 $7y-3z=20$ E2
-8E1+E3 $21y-9z=32$ E3

-3E1 $-3x+6y-6z=-3$
E2 $3x+y+3z=23$
-3E1+E2 $7y-3z=20$

-8E1 $-8x+16y-16z=-8$
E3 $8x+5y+7z=40$
$21y-9z=32$

oops!
-3E2+E3: $0+0=-60+32$
$0=-28$
$0=1$?!
No Sol'n!

19 Kitchen Korner produces refrigerators, dishwashers, and stoves at three different factories. The table gives the number of each product produced at each factory per day. Kitchen Korner receives an order for 166 refrigerators, 210 dishwashers, and 172 ovens. How many days should each plant be scheduled to fill this order?

Appliance	Factory A	Factory B	Factory C
Refrigerators	8	10	14
Dishwashers	16	12	10
Stoves	10	18	6

Let x = the # of days Factory A is scheduled, (in "Factory A DAYS")
y = " " " " " " B " " , and (" B ")
z = " " " " " " C " " . (" C ")

$$\left(8\ \frac{\text{REFRIGERATORS}}{1\ \cancel{\text{FACTORY}}\ \#\ \text{DAYS}}\right)\left(x\ \cancel{\text{FACTORY A}}\ \text{DAYS}\right) = 8x\ \text{REFRIGERATORS}$$

# of fridges from factory A.

Refrigerators $8x+10y+14z=166$
Dishwashers $16x+12y+10z=210$
Stoves $10x+18y+6z=172$

$8x+10y+14z=166$ E1 $\quad \frac{1}{2}$E1 $\quad 4x+5y+7z=83$ E1
$16x+12y+10z=210$ E2 $\quad \frac{1}{2}$E2 $\quad 8x+6y+5z=105$ E2
$10x+18y+6z=172$ E3 $\quad \frac{1}{2}$E3 $\quad 5x+9y+3z=86$ E3

E1 $\quad 4x+5y+7z=83$ E1
-2E1+E2 $\quad -4y-9z=-61$ E2
-5E1+4E3 $\quad 11y-23z=-71$ E3

-2E1 $\quad -8x-10y-14z=-166$
E2 $\quad 8x+6y+5z=105$
$-4y-9z=-61$

-5E1 $\quad -20x-25y-35z=-415$
4E3 $\quad 20x+36y+12z=344$
$11y-23z=-71$

$23 \times 4 = 92$

E1 $\quad 4x+5y+7z=83$ E1
$-$E2 $\quad 4y+9z=61$ E2
11E2+4E3 $\quad -191z=-955$ E3

$z=\frac{-955}{-191}=5$

$\boxed{z=5}$

11E2 $\quad -44y-99z=-671$
4E3 $\quad 44y-92z=-284$
11E2+4E3 $\quad -191z=+955$

$\Rightarrow$ E2 $\Rightarrow 4y+9(5)=4y+45=61$
$\Rightarrow 4y=16$
$\boxed{y=4}$

$83-55=28$

$\Rightarrow$ E1 $\Rightarrow 4x+5(4)+7(5)=4x+20+35=4x+55=83$
$\Rightarrow 4x=28$
$\Rightarrow \boxed{x=7}$

$\Rightarrow \boxed{(x,y,z)=(7,4,5)}$