1 The system of equations

$$\begin{cases} 2x + 4y = 20 \\ 4x - y = 13 \end{cases}$$

is a system of two equations in the two variables [---Select---] (x & y). To determine whether $(7, -3)$ is a solution of this system, we check whether $x = 7$ and $y = -3$ satisfy each [---Select---] (equation) in the system. Which of the following are solutions of this system? (Select all that apply.)

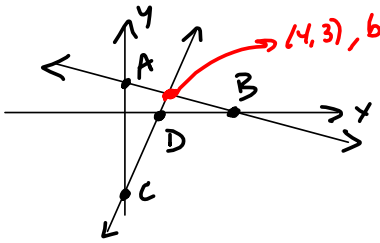
- ☐ $(7, -3)$ $2(7)+4(-3) = 14-12 = 5 \neq 20$ NO
- ☐ $(-3, 5)$ $2(-3)+4(5) = -6+20 = 14$ NO
- ☑ $(4, 3)$ $2(4)+4(3) = 8+12 = 20$ ✓ $4(4)-3 = 16-3 = 13$ ✓

$2x+4y=20$

x	y	
0	5	$(0,5)=A$
10	0	$(10,0)=B$

$4x-y=13$

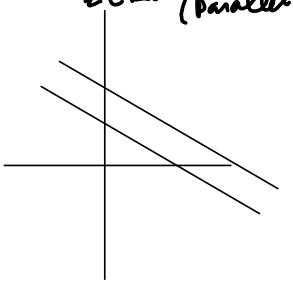
x	y	
0	-13	$(0,-13)=C$
$\frac{13}{4}$	0	$\rightsquigarrow (3.25,0)=D = (\frac{13}{4},0)$



$(4,3)$, by plugging in.

2 A system of two linear equations in two variables can have [---Select---] (zero, one, or infinitely-many solutions).

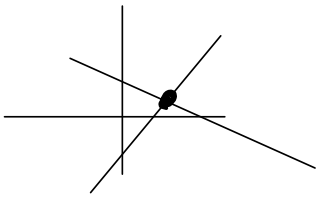
ZERO (parallel)



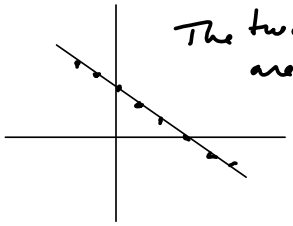
$x+y=10$

$5x+5y=12$

ONE



Infinitely Many



The two lines are one!

$2x+2y=20$

$-3x-3y=-30$

3 The following is a system of two linear equations in two variables.

$$\begin{cases} x + y = 5 \\ 2x + 2y = 10 \end{cases}$$

The graph of the first equation is the same as the graph of the second equation, so the system has [infinitely many solutions] . We express these solutions by writing

$x = t$

$y =$ [5-t]

$x+y = t+y = 5 \rightarrow y = 5-t$

where t is any real number. Some of the solutions of this system are

$(1,$ [4] $)$, $(-3,$ [8] $)$, and $(6,$ [-1] $)$.

General solution $\{(t, 5-t) \mid t \in \mathbb{R}\}$ Particular solutions

4 Use the substitution method to find all solutions of the system of equations.

$$\begin{cases} 2x + y = 13 \\ x + 2y = 5 \end{cases} \rightarrow y = 13-2x$$

$(x, y) =$ ( )

$x+2y = x+2(13-2x) = 5$

$x+26-4x = -3x+26 = 5$

$-3x = -21$

$x = 7 \rightarrow y = 13-2(7) = -1 = y$ Back-sub.

$(x, y) = (7, -1)$

5 Graph the linear system, either by hand or using a graphing device.

$$\begin{cases} x - y = 5 \\ 2x + y = 4 \end{cases}$$

Eliminate me!

Need to "kill" the $2x$ with $-2x$

$-2E1 = -2x + 2y = -10$

$+ \quad E2 = 2x + y = 4$

$3y = -6$

$y = -2$

Back-Substitute: $x - y = x - (-2) = x + 2 = 5$

$x = 3$

$(x, y) = (3, -2)$

$x - y = 5$

x	y
0	-5
5	0

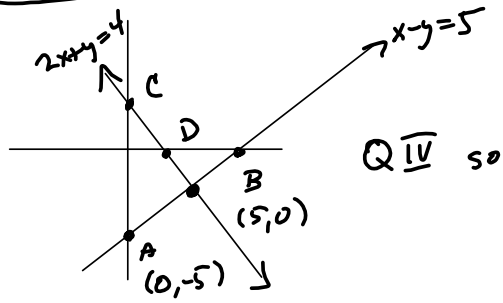
$A = (0, -5), (5, 0) = B$

$2x + y = 4$

x	y
0	4
2	0

$C = (0, 4)$

$D = (2, 0)$



QIV solution

(3, -2) from above looks roughly correct.

Use the elimination method to find all solutions of the system of equations.

6 $\begin{cases} 4x + 4y = 36 \\ 8x + y = 44 \end{cases}$

$(x, y) =$ ( )

$-2E1 \quad -8x - 8y = -72$

$E2 \quad 8x + y = 44$

$-2E1+E2 \quad -7y = -28$

$\Rightarrow y = \frac{-28}{-7} = 4 = y$

Back-Substitute:

$\Rightarrow 4x+4y = 4x+4(4) = 4x+16 = 36$

$\Rightarrow 4x = 20$

$\Rightarrow x = \frac{20}{4} = 5 = x$

$(x, y) = (5, 4)$

Graph the linear system, either by hand or using a graphing device.

8 $\begin{cases} 3x - y = 3 \\ 5x + y = 5 \end{cases}$

$3x-y=3$

x	y
0	-3
1	0

$A = (0, -3)$

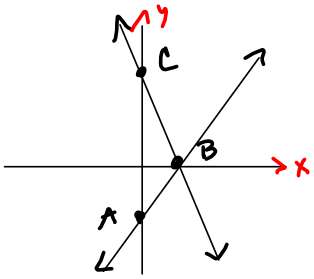
$B = (1, 0)$

$5x+y=5$

x	y
0	5
1	0

$C = (0, 5)$

$D = (1, 0) = B$!



9 Graph the linear system, either by hand or using a graphing device.

$$\begin{cases} 4x + 6y = 0 \\ -6x - 9y = 18 \end{cases}$$

$4x+6y=0$

x	y
0	0
3	2

A = (0,0)
B = (3,2)

Gives Nice integer.

$4(3)=12$
$6(2)=12$

Pick an x so that $4x$ is a multiple of 6. It's like finding the LCD for $\frac{1}{4}+\frac{1}{6}=$

$4 \cdot 6 = 24$ not the least

$\frac{1}{4}\cdot\frac{3}{3}+\frac{1}{6}\cdot\frac{2}{2}=$ etc.

Least Common Multiple of 4 & 6 is 12.

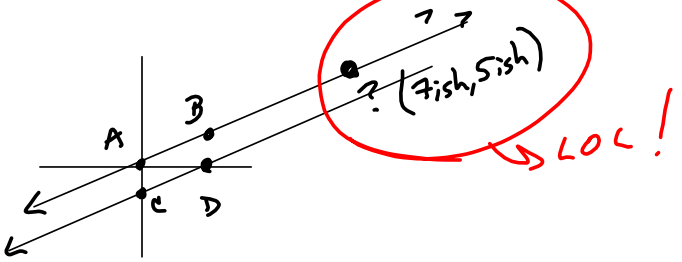
$-6x-9y=18$

x	y
0	-2
3	0

C = (0,-2)
D = (3,0)

$\frac{2y}{2}=\frac{-6}{3}$

LCD = 2·3



Graph the linear system, either by hand or using a graphing device.

E1 E2 $\begin{cases} 4x + 6y = 0 \\ -6x - 9y = 18 \end{cases}$ **These are parallel, silly!**

Simplify:

$\frac{1}{2}E1 \quad 2x+3y=0$
$-\frac{1}{3}E2 \quad 2x+3y=-6$

#10 $2x - y = 8$

x	y
0	-8
4	0

Solve the system, or show that it has no solution. (If there is no solution, enter NO SOLUTION. If there are an infinite number of solutions, enter the general solution in terms of x, where x is any real number.)

11

$$\begin{cases} \left(\frac{3}{4}x + \frac{1}{3}y = 5\right)(12) \\ \left(-\frac{1}{4}x - \frac{4}{3}y = 2\right)(12) \end{cases}$$

$\rightarrow 12\left(\frac{3}{4}\right)x + 12\left(\frac{1}{3}\right)y = 60 \Rightarrow 9x + 4y = 60$

$\rightarrow 12\left(-\frac{1}{4}x\right) + 12\left(-\frac{4}{3}\right)y = 24 \Rightarrow -3x - 16y = 24$

E1 $-3x - 16y = 60$

E2 $9x + 4y = 24$

3E1 $-9x - 48y = 180$

E2 $9x + 4y = 24$

3E1+E2 $-44y = 204$

$y = \frac{204}{-44} = \frac{102}{-22} = -\frac{51}{11} = y$

E1 $-3x - 16y = 24$

E2 $9x + 4y = 60$

3E1 $-9x - 48y = 72$

E2 $9x + 4y = 60$

3E1+E2 $-44y = 132$

$\Rightarrow y = -\frac{132}{44} = -\frac{66}{22} = -3 = y$

$\Rightarrow -3x - 16(-3) = -3x + 48 = 24$

$\Rightarrow -3x = -24$

$\Rightarrow x = \frac{-24}{-3} = 8 = x$

17 Use a graphing device to graph both lines in the same viewing rectangle. (Note that you must solve for y in terms of x before graphing if you are using a graphing calculator.) Solve the system either by zooming in and using TRACE or by using `Intersect`. Round your answers to two decimals.

$$\begin{cases} 18.62x - 14.57y = 10.89 \\ 6.63x - 12.5y = 16.16 \end{cases}$$ Times 100!

$$1862x - 1457y = 1089$$
$$663x - 1250y = 1616$$

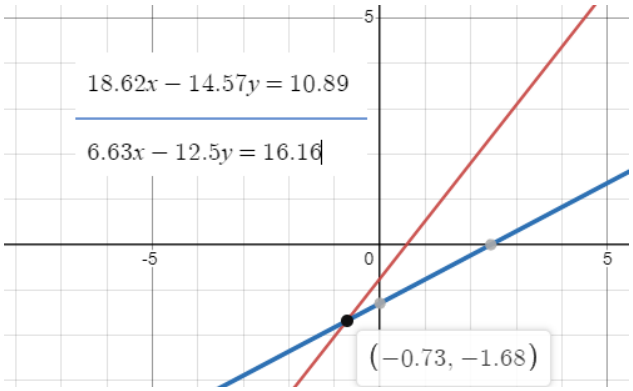
$$-663E1 \quad -1234506x + 965991y = -722007$$
$$1862E2 \quad 1234506x - 2327500y = 3008992$$
$$-663E1 + 1862E2 \quad -1361509y = 2286985$$

$$\Rightarrow y = \frac{2286985}{-1361509y} \approx -1.679742844 \approx -1.68 \approx y$$

$$\Rightarrow 18.62x - 14.57(-1.679742844) = 10.89$$

$$\Rightarrow 18.62x \approx -13.58385324$$

$$\Rightarrow x \approx \frac{-13.58385324}{18.62} \approx -.7295302\,49 \approx -.73 \approx x$$

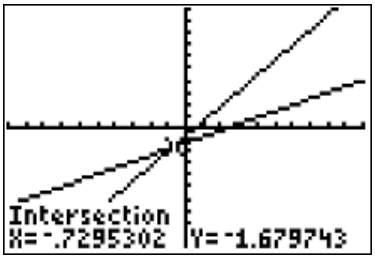


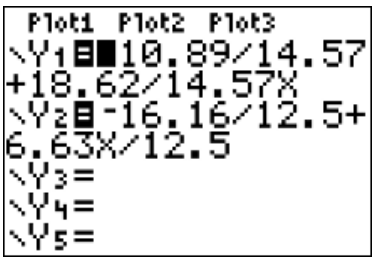
Use a graphing device to graph both lines in the same viewing rectangle. (Note that you must solve for y in terms of x before graphing if you are using a graphing calculator.) Solve the system either by zooming in and using TRACE or by using `Intersect`. Round your answers to two decimals.

$$\begin{cases} 18.62x - 14.57y = 10.89 \\ 6.63x - 12.5y = 16.16 \end{cases}$$

$$y = \frac{16.16}{-12.5} + \frac{6.63x}{12.5}$$

$$y = \frac{10.89}{-14.57} + \frac{18.62x}{14.57}$$





19 The sum of two numbers is twice their difference. The larger number is 7 more than twice the smaller. Find the numbers.

$x+y=2(x-y)$

$x=2y+7$

Let x = Larger #

& y = Smaller #

$2y+7+y = 2x-2y$

$3y+7 = 2(2y+7)-2y$

$3y+7 = 4y+14-2y = 2y+14$

$y=7$ $\Longrightarrow$ $x=2(7)+7 = 21=x$

$21+7=28$

$28=2(21-7)$?

Yes!

21 The admission fee at an amusement park is $1.50 for children and $4.00 for adults. On a certain day, 2,900 people entered the park, and the admission fees that were collected totaled $6,100. How many children and how many adults were admitted?

Let x = the # of children admitted

& y = " " " adults admitted

E1 Revenue $1.50x + 4y = 6100$ $

E2 Total People $x + y = 2900$ People

E2 $\Rightarrow$ $y = 2900 - x$

E1 $1.5x + 4y = 1.5x + 4(2900 - x) = 1.5x + 11600 - 4x$

$= -2.5x + 11600 = 6100$

$-2.5x = -5500$

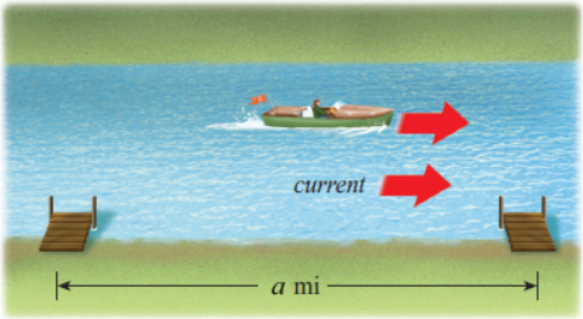
$\Rightarrow x = \frac{5500}{2.5} = \frac{11000}{5} = 2200 = x$

-11600
$+6100$
-5500

& $y = 2900 - x = 700 = y$

22 A boat on a river travels downstream between two points, 20 mi apart, in 1 h. The return trip against the current takes $2\frac{1}{2}$ h. What is the boat's speed (in still water)?

How fast does the current in the river flow?



2.5 hr or $\frac{5}{2}$ hr.

	D	r	t
up	20	$r_1 - r_2$	2.5
Down	20	$r_1 + r_2$	1

Let r_1 = speed of boat in still water - $(\frac{mi}{hr})$

r_2 = speed of current $(\frac{mi}{hr})$

$D = rt$, where

D = distance, in miles (mi)

r = rate, in miles per hour $(\frac{mi}{hr})$

t = time, in hours (hr)

	D	r	t
up	20	$r_1 - r_2$	2.5
Down	20	$r_1 + r_2$	1

$20 = (r_1 - r_2)(2.5) = 2.5r_1 - 2.5r_2 = 20$

$20 = (r_1 + r_2)(1) = r_1 + r_2 = 20$

$\Rightarrow r_1 = 20 - r_2$

$\Rightarrow 2.5(20 - r_2) - 2.5r_2 = 20$

$\Rightarrow 50 - 2.5r_2 - 2.5r_2 = 20$

$50 - 5r_2 = 20$

$-5r_2 = -30$

$r_2 = \frac{-30}{-5} = 6 \text{ mph} = r_2$

$\Rightarrow r_1 = 20 - r_2 = 20 - 6 = 14 \text{ mph} = r_1$

23 A chemist has two large containers of sulfuric acid solution, with different concentrations of acid in each container. Blending 200 mL of the first solution and 700 mL of the second gives a mixture that is 14% acid, whereas blending 100 mL of the first mixed with 300 mL of the second gives a $14\frac{1}{2}$% acid mixture. What are the concentrations of sulfuric acid in the original containers?

14.5%

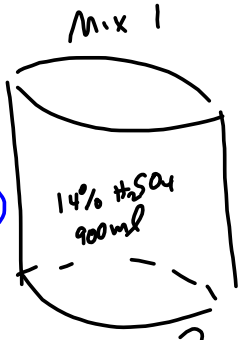
Amt of pure sulfuric = Amt of pure sulfuric

Amt of pure H_2SO_4 is concentration · volume

Let x = concentration of H_2SO_4 in 1st container.

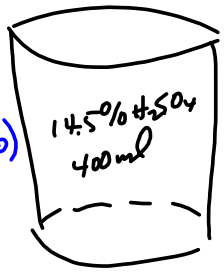
y = " " " " 2nd " "

Then $200x + 700y = .14(900)$



Mix 2

That's the setup!



$100x + 300y = .145(400)$

x is in units of .145 ml H_2SO_4 / ml of soln in 2nd container

is (200 ml in Mix)(x ml H_2SO_4 / ml 2nd)

Dimensional Analysis.

200x (200 ml of 1st soln)(x ml pure H_2SO_4 / 1 ml of 1st soln) = 200 ml pure H_2SO_4.

24 A biologist has two brine solutions, one containing 8% salt and another containing 32% salt. How many milliliters of each solution should she mix to obtain 1 L of a solution that contains 22.4% salt?

Tricky!

Amt pure salt = Amt of pure salt of

Amt of pure salt = (concentration)(volume)

Let x = concentration of pure salt in 1st mixture

y = " " " " 2nd mixture

Want 1L of 22.4% salt

Let x = volume of the 8% mixture in liters!

y = " " " 32% " in liters!

They want ml in the final ANSWER!

Then $x + y = 1 \Rightarrow y = 1 - x$

& $.08x + .32y = .224(1) = .224$

This is in Liters!

Times 1000.

$80x + 320y = 224$

$y = 1 - x \rightarrow$

$80x + 320(1-x) = 224$

$80x + 320 - 320x = 224$

$-240x = -96$

$x = \frac{96}{240} = \frac{48}{120} = \frac{24}{60} = \frac{12}{30} = \frac{6}{15} = \frac{2}{5} = .4 = x$

$x = .4\,L$

$-320 + 224 = -96$

$\Rightarrow y = 1 - x = 1 - .4 = .6\,L = y$

Check:

$.08(.4) + .32(.6)$

$= .032 + .192$

$= .224$

25 A woman invests a total of $20,000 in two accounts, one paying 3.5% and the other paying 7% simple interest per year. Her annual interest is $910. How much did she invest at each rate?

Let x = the amount invested at 3.5% APR (in $)

y = " " " " " 7% " "

SIMPLE INTEREST

TOTAL $x + y = 20000 \Rightarrow y = 20000 - x$

EARNINGS $.035x + .07y = 910$

$\Rightarrow .035x + .07(20000 - x) = 910$

$.035x + 1400 - .07x =$

$= -.035x + 1400 = 910$

$\Rightarrow -.035x = -490$

$x = \frac{-490}{-.035} = \frac{490000}{35} = \frac{14000}{1} = \$14{,}000 = x$

$20000 \times .07 = 1400$

$y = 20000 - x = 20000 - 14000 = \$6000 = y$