This exercise uses the exponential growth model.

1 A certain culture of the bacterium *Streptococcus A* initially has 8 bacteria and is observed to double every 1.5 hours.

(a) Find an exponential model $n(t) = n_0 2^{t/a}$ for the number of bacteria in the culture after t hours.

(b) Estimate the number of bacteria after 34 hours. (Round your answer to the nearest whole number.)

(c) After how many hours will the bacteria count reach 10,000? (Round your answer to one decimal place.)

The book treats these doubling and tripling time questions as separate, special cases, where the doubling time appears magically in the exponent.

I prefer one sledgehammer for all of these questions, so I'm demonstrating the book method and the "works for everything" method.

EXPONENTIAL GROWTH (DOUBLING TIME)

If the initial size of a population is n_0 and the doubling time is a, then the size of the population at time t is

$$n(t) = n_0 2^{t/a}$$

where a and t are measured in the same time units (minutes, hours, days, years, and so on).

I don't like this "rote" method for the special case.

Better to know how to build the general model and use any given form of info.

Book Way

(a) $a = 1.5$ hrs, $n_0 = 8$ bacteria (a)

$n(t) = n_0 \cdot 2^{\frac{t}{a}} = \boxed{8 \cdot 2^{\frac{t}{1.5}} = n(t)}$

(b)

(b) $n(34) = 8 \cdot 2^{\frac{34}{1.5}} \approx$ $\boxed{53{,}264{,}341 \text{ bacteria.}}$

(c) $n(t) = 8 \cdot 2^{\frac{t}{1.5}} = 10000$

$\Longrightarrow \quad 2^{\frac{t}{1.5}} = \frac{10000}{8} = \frac{5000}{4} = \frac{2500}{2} = 1250$

```
8*2^(34/1.5)
         53264340.66   (b)
1.5ln(1250)/ln(2
)
         15.43156857   (c)
```

i.e, $2^{\frac{t}{1.5}} = 1250$

$\log_2\left(2^{\frac{t}{1.5}}\right) = \log_2(1250)$

$\frac{t}{1.5} = \log_2(1250)$

$t = 1.5 \log_2(1250)$

(c)

$= \frac{1.5 \ln(1250)}{\ln(2)} \approx \boxed{t \approx 15.4 \text{ hrs}}$

General Way

EXPONENTIAL GROWTH (RELATIVE GROWTH RATE)

A population that experiences **exponential growth** increases according to the model

$$n(t) = n_0 e^{rt}$$

where $n(t)$ = population at time t

n_0 = initial size of the population

r = relative rate of growth (expressed as a proportion of the population)

t = time

$n_0 = 8$ & we know it doubles every 1.5 hours

$n_0 e^{rt} = 2n_0$ when $t = 1.5$

Solve $n_0 e^{1.5r} = 2n_0$ "n - nought"

$e^{1.5r} = 2$

$\ln(e^{1.5r}) = 1.5r = \ln(2)$

$r = \frac{\ln(2)}{1.5}$

(a) $n(t) = 8 \cdot e^{\frac{\ln(2)}{1.5}t} = 8\, e^{\ln(2)\left(\frac{t}{1.5}\right)} = 8\left(e^{\ln(2)}\right)^{\frac{t}{1.5}} = 8 \cdot 2^{\frac{t}{1.5}}$

(b) $8e^{34\left(\frac{\ln(2)}{1.5}\right)}$

(c) $8e^{\frac{\ln(2)}{1.5}t} = 10000$

$\Rightarrow e^{\frac{\ln(2)}{1.5}t} = \frac{10000}{8} = 1250$

$\Rightarrow \frac{\ln(2)t}{1.5} = \ln(1250)$

$\Rightarrow \boxed{t = \frac{1.5\ln(125)}{\ln(2)}}$

2 This exercise uses the exponential growth model.

A grey squirrel population was introduced in a certain county of Great Britain 35 years ago. Biologists observe that the population doubles every 7 years, and now the population is 78,400.

(a) What was the initial size of the squirrel population?

(b) Estimate the squirrel population 10 years from now. (Round your answer to the nearest whole number.)

(c) Sketch a graph of the squirrel population. (Assume $t = 0$ corresponds to the initial introduction.)

$n(t) = n_0 e^{rt}$

$n(35) = n_0 e^{35r} = 78400$

2 unknowns.

we can find r from

"doubling time is 7 years."

$n_0 e^{7r} = 2n_0$

$e^{7r} = 2$

$7r = \ln(2)$

$r = \frac{\ln(2)}{7}$.

$$\boxed{n(t) = n_0 e^{\frac{\ln(2)}{7}t}}$$

(a) Initial Size of the Pop?

We do know $n(35) = n_0 e^{\frac{\ln(2)}{7}(35)} = n_0 e^{\frac{35\ln(2)}{7}} = 78400$

$\Rightarrow n_0 = \dfrac{78400}{e^{\frac{35\ln(2)}{7}}} = 78400 e^{-\frac{35\ln(2)}{7}} \approx n_0 \approx$

```
78400e^(-35ln(2)
/7)
                  2450
2450e^(45ln(2)/7
)
           211037.1502
```

$= n_0$

$\boxed{211037.1502 \approx n(45)}$

$\boxed{\approx 211037 \text{ squirrels}}$

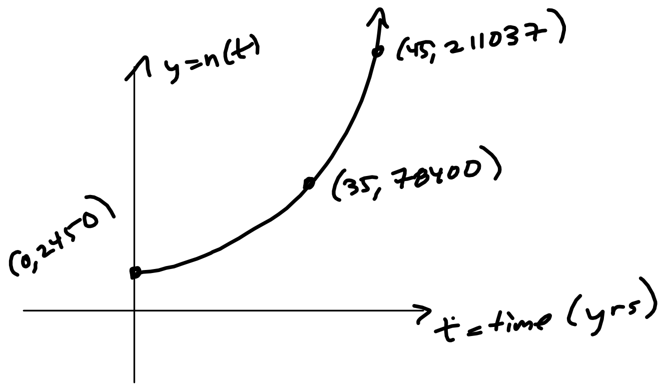
(c)

$n(t) = 2450 e^{\frac{\ln(2)}{7} t}$

Pop 10 years from now is

$t = 35 + 10 = 45$

$n(45) = 2450 e^{\frac{\ln(2)}{7}(45)}$



This exercise uses the radioactive decay model.

5 The half-life of radium-226 is 1,600 years. Suppose we have a 27-milligram sample.

(a) Find a function $m(t) = m_0 2^{-t/h}$ that models the mass remaining after t years.

(b) Find a function $m(t) = m_0 e^{-rt}$ that models the mass remaining after t years. (Round your r value to six decimal places.)

(c) How much of the sample (in mg) will remain after 5,000 years? (Round your answer to one decimal place.)

(d) After how many years will only 17 mg of the sample remain? (Round your answer to one decimal place.)

b?! $\boxed{m(t) = m_0 e^{-kt}}$, where k = relative decay rate

$\frac{1}{2}$-life is 1600 yrs $\Rightarrow$

$m(1600) = m_0 e^{-1600k} = \frac{1}{2} m_0$

$\Rightarrow e^{-1600k} = \frac{1}{2}$

$\Rightarrow -1600k = \ln(1/2) = -\ln(2)$

$\Rightarrow \boxed{k = \frac{-\ln(2)}{-1600}} = \frac{\ln(2)}{1600}$ w

& $m(t) = \boxed{m_0 e^{-\frac{\ln(2)}{1600}t}}$ want this to be rounded to 6 places in the exponent.

$m_0 = 27\text{mg} \Rightarrow$

a $m(t) = 27\cdot 2^{-\frac{t}{1600}}$ $\qquad m(t) = 27e^{-\frac{\ln(2)}{1600}t}$

c $m(5000) = 27e^{-\frac{\ln(2)}{1600}(5000)}$ Round to one decimal place.

$\boxed{m(5000) \approx 3.1 \text{ mg}}$

d Solve $m(t) = 17$ Round to one decimal place.

```
)
    26.98830567
ln(2)/1600
  4.332169878E-4
27e^(-ln(2)/1600
*5000)
    3.094888646
```

$= .0004332169878 \approx K$ $\qquad \approx .000433$ to 6 places

≈ 3.1 mg c

$$27e^{-\frac{\ln(2)}{1600}t} = 17$$

$$e^{-\frac{\ln(2)}{1600}t} = \frac{17}{27}$$

```
1600ln(17/27)/ln
(2)
    -1067.879457
```

$$-\frac{\ln(2)t}{1600} = \ln\left(\frac{17}{27}\right)$$

$$t = -\frac{1600\ln(17/27)}{\ln(2)}$$

c

$\approx$ 1067.9 yrs $\approx$ when $m(t) = 17$ mg.

This exercise uses the radioactive decay model.

7 A wooden artifact from an ancient tomb contains 70% of the carbon-14 that is present in living trees. How long ago (in yr) was the artifact made? (The half-life of carbon-14 is 5,730 years. Round your decay rate, r, to 6 decimal places. Then round your answer to the nearest whole number.)

How old is the wooden artifact if 70% of C-14 remains.

$m_0 e^{-rt} = m(t)$. $\frac{1}{2}$-life is 5730 yrs.

$$m(5730) = m_0 e^{-5730r} = \frac{1}{2} m_0$$

$$e^{-5730r} = \frac{1}{2}$$

$$-5730r = \ln\left(\frac{1}{2}\right) = -\ln(2)$$

$$r = \frac{\ln(2)}{5730}$$

$$\boxed{m(t) = m_0 e^{-\frac{\ln(2)}{5730}t}}$$

70% of original C-14 remains

$$m(t) = m_0 e^{-\frac{\ln(2)}{5730}t} = .7m_0$$

$$-\frac{\ln(2)}{5730}t = \ln(.7)$$

$$t = \frac{5730\ln(.7)}{-\ln(2)}$$

```
5730ln(.7)/-ln(2
)
        2948.50428
```

$\approx \boxed{2949 \text{ yrs old}}$

10 - Newton's Law of Cooling

NEWTON'S LAW OF COOLING

If D_0 is the initial temperature difference between an object and its surroundings, and if its surroundings have temperature T_s, then the temperature of the object at time t is modeled by the function

$$T(t) = T_s + D_0 e^{-kt}$$

where k is a positive constant that depends on the type of object.

A graphing device is recommended. $T(t) = T_s + (T_0 - T_s)e^{-kt}$

This exercise uses Newton's Law of Cooling.

$\to T(29)$, not $T(14)$

A kettle full of water is brought to a boil in a room with temperature 20°C. After 15 minutes the temperature of the water has decreased from 100° to 75°C. Find the temperature (in °C) after another 14 minutes. (Round your answer to one decimal place.)

$T_s = 20$ $\quad T(15) = T_s + (T_0 - T_s)e^{-k(15)} = 75$ is given

°C $\quad T_0 = 100$

Illustrate by graphing the temperature function. (Assume temperature is in °C and time in minutes. Round your k value to five decimal places. Select Update Graph to see your response plotted on the screen. Select the Submit button to grade your response.)

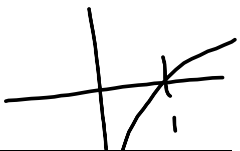
$$T(15) = 20 + (100 - 20)e^{-15k} = 20 + 80e^{-15k} = 75$$

$$80e^{-15k} = 55$$

$$e^{-15k} = \frac{55}{80} = \frac{11}{16}$$

$$\ln(e^{-15k}) = -15k = \ln\left(\frac{11}{16}\right)$$

$$k = \frac{\ln(11/16)}{-15} \approx \boxed{.0249795633 \approx k}$$




```
ln(11/16)/-15
         .0249795633
```

$T(t) = 20 + 80e^{-.0249795633t}$

```
ln(11/16)/-15
         .0249795633
20+80e^(-Ans*14)
         76.39117917
```

→ No

$T(14) \approx 76.39117917$

$\approx 76.4°C$, but THEY'RE ASKING FOR $T(15+14) = T(29)$, not $T(14)$.

```
         58.76893568
ln(11/16)/-15
         .0249795633
20+80e^(-Ans*29)
         58.76893568
```

→ Yes.

$\approx T(29) \approx \boxed{58.8° \approx T(29)}$