1 Let's solve the exponential equation $4e^x = 80$. $\Rightarrow$ $e^x = \frac{80}{4} = 20$

(a) First, we isolate e^x to get the equivalent equation

$\ln(e^x) = \ln(20)$

$x = \ln(20)$

(b) Next, we take ln of each side to get the equivalent equation .

(c) Now we use a calculator to find $x \approx$ 2.996 . (Round your answer to three decimal places.)

```
ln(20
          2.995732274
```

2 Let's solve the logarithmic equation

$\log(3) + \log(x - 4) = \log(x)$.

Sum of the logs is the log of the product.

(a) First, we combine the logarithms on the LHS to get the equivalent equation $\log(3(x-4))$ = $\log(x)$.

(b) Next, we use the fact that log is one-to-one to get the equivalent equation $3x - 12$ = x.

$3(x-4) = 3x - 12 = x \Rightarrow$

$2x = 12$

$x = 6$

(c) Now we find $x =$ 6. .

3 Find the solution of the exponential equation. (Enter your answers as a comma-separated list.)

$3^{x-1} = 27 = 3^3 \Rightarrow$

$x - 1 = 3$

$\boxed{x = 4}$

Find the solution of the exponential equation, as in Example 1. (Enter your answers as a comma-separated list.)

4 $e^{x^2} = e^{81}$

The exponential function is 1-to-1; therefore,

$x^2 = 81$

$\boxed{x = \pm 9}$

Find the solution of the exponential equation. (Enter your answers as a comma-separated list.)

5 $4^{2x-1} = 1 = 4^0 \Longrightarrow$

$2x - 1 = 0$

$2x = 1$

$\boxed{x = \frac{1}{2}}$

Find the solution of the exponential equation. (Enter your answers as a comma-separated list.)

6 $7^{4x-5} = \frac{1}{7} = 7^{-1}$

$4x - 5 = -1$

$4x = 4$

$\boxed{x = 1}$

Consider the following.

7 $200(1.02)^{2t} = 1,900$

(a) Find the exact solution of the exponential equation in terms of logarithms.

$t =$

$$200\,(1.02)^{2t} = 1900$$
$$(1.02)^{2t} = \frac{1900}{200} = \frac{19}{2}$$

(b) Use a calculator to find an approximation to the solution, rounded to six decimal places.

$t =$

$$\log_{1.02}\left((1.02)^{2t}\right) = \log_{1.02}\left(\frac{19}{2}\right)$$
$$2t = \log_{1.02}\left(\frac{19}{2}\right)$$
$$t = \frac{1}{2}\log_{1.02}\left(\frac{19}{2}\right)$$
$$= \frac{1}{2}\left(\frac{\ln(19/2)}{\ln(1.02)}\right)$$

The other way!

$$1.02^{2t} = \frac{19}{2} \Longrightarrow$$
$$\ln\left(1.02^{2t}\right) = \ln\left(\frac{19}{2}\right)$$
$$2t\ln(1.02) = \ln\left(\frac{19}{2}\right)$$
$$2t = \frac{\ln\left(\frac{19}{2}\right)}{\ln(1.02)} \Longrightarrow$$
$$t = \frac{1}{2}\,\frac{\ln\left(\frac{19}{2}\right)}{\ln(1.02)}$$

Consider the following equation.

8 $$5(1 + 10^{9x}) = 11$$

(a) Find the exact solution of the exponential equation in terms of logarithms.

$x =$

(b) Use a calculator to find an approximation to the solution, rounded to six decimal places.

$x =$

$$5(1 + 10^{9x}) = 11$$

$$5 + 5 \cdot 10^{9x} = 11$$

$$5 \cdot 10^{9x} = 6$$

$$10^{9x} = \frac{6}{5}$$

$$\log_{10}(10^{9x}) = 9x = \log\left(\frac{6}{5}\right)$$

$$x = \frac{1}{9}\log\left(\frac{6}{5}\right)$$

```
log(6/5)/9
      .0087979162
```

Consider the following equation.

10 $4^x + 2^{1 + 2x} = 20$

(a) Find the exact solution of the exponential equation in terms of logarithms.

(b) Use a calculator to find an approximation to the solution, rounded to six decimal places.

$$4^x + 2^1\cdot 2^{2x} = 4^x + 2\cdot(2^2)^x = 4^x + 2\cdot 4^x = 3\cdot 4^x = 20 \Rightarrow$$

$$4^x = \frac{20}{3}$$

$$\log_4(4^x) = \log_4\left(\frac{20}{3}\right)$$

$$\boxed{x = \log_4\left(\frac{20}{3}\right)}$$

$$4^x + 2\cdot 2^{2x} = (2^2)^x + 2\cdot 2^{2x} = 2^{2x} + 2\cdot 2^{2x} = 3\cdot 2^{2x} = 20$$

$$\Rightarrow 2^{2x} = \frac{20}{3}$$

$$\Rightarrow \log_2(2^{2x}) = \log_2\left(\frac{20}{3}\right)$$

$$\Rightarrow 2x = \log_2\left(\frac{20}{3}\right)$$

$$x = \frac{1}{2}\log_2\left(\frac{20}{3}\right)$$

```
ln(20/3)/ln(4)
      1.368482797
1/2*ln(20/3)/ln(
2)
      1.368482797
```

Consider the following equation.

11

$$27^x + 3^{3x+1} = 400$$

(a) Find the exact solution of the exponential equation in terms of logarithms.

(b) Use a calculator to find an approximation to the solution, rounded to six decimal places.

$$(3^3)^x + 3^{3x}\cdot 3^1 = 3^{3x} + 3^{3x}\cdot 3 = 3^{3x}(1+3) = 400$$

$$3^{3x} = \frac{400}{4} = 100$$

$$\ln(3^{3x}) = 3x\cdot\ln(3) = 3\ln(3)x = \ln(100)$$

$$x = \frac{\ln(100)}{3\ln(3)} \approx$$

```
ln(100)/(3ln(3))
          1.39726885
ln(100)/ln(27)
          1.39726885
```

$$27^x + 3^{3x+1}$$

$$= 27^x + 3^{3(x+\frac{1}{3})}$$

$$= 27^x + (3^3)^{x+\frac{1}{3}}$$

$$= 27^x + 27^{x+\frac{1}{3}} = 27^x + 27^x 27^{\frac{1}{3}}$$

$$= 27^x + 3\cdot 27^x$$

$$= 27^x(1+3) = 4\cdot 27^x = 400$$

$$\Rightarrow 27^x = 100$$

$$\ln(27^x) = \ln(100)$$

$$\ln(27)x = \ln(100)$$

$$x = \frac{\ln(100)}{\ln(27)}$$

12

Consider the following equation.

$$\frac{66}{1 + e^{-x}} = 4$$

(a) Find the exact solution of the exponential equation in terms of logarithms.

(b) Use a calculator to find an approximation to the solution, rounded to six decimal places.

$\Rightarrow 66 = 4(1+e^{-x}) = 4e^{-x}+4 = 66$

$\Rightarrow 4e^{-x} = 62$

$e^{-x} = \frac{62}{4} = \frac{31}{2}$

$\ln(e^{-x}) = \ln\left(\frac{31}{2}\right)$

$-x = \ln\left(\frac{31}{2}\right)$

$x = -\ln\left(\frac{31}{2}\right) = \ln\left(\left(\frac{31}{2}\right)^{-1}\right) = \ln\left(\frac{2}{31}\right)$

$\boxed{x \approx -2.740840}$

```
-ln(31/2)
          -2.740840024
ln(2/31)
          -2.740840024
```

Solve the equation. (Enter your answers as a comma-separated list. Round your answers to four decimal places.)

14 $e^{4x} + 7e^{2x} - 18 = 0$

$\Rightarrow e^{2x \cdot 2} + 7e^{2x} - 18 = 0$

$(e^{2x})^2 + 7e^{2x} - 18 = u^2 + 7u - 18 = 0,$

where $u = e^{2x}$

$\rightarrow (u+9)(u-2) = 0 \Rightarrow$

$u = -9 = e^{2x}$ or $u = 2 = e^{2x}$

No Way!

$e^{2x} = 2$

$\ln(\ ) = \ln(\ )$

$2x = \ln(2)$

$x = \frac{1}{2}\ln(2)$

```
-ln(31/2)
     -2.740840024
ln(2/31)
     -2.740840024
```

Solve the equation. (Enter your answers as a comma-separated list. Round your answers to four decimal places.)

16

$$3^x - 15(3^{-x}) + 2 = 0$$

$3 - 15\left(\frac{1}{3}\right) + 2 = 3 - 5 + 2 = 0$ ✓

$3^x\left(3^x - 15(3^{-x}) + 2\right)$

$= 3^{2x} - 15(3^{x-x}) + 2 \cdot 3^x$

$= (3^x)^2 - 15 + 2 \cdot 3^x$

$= u^2 + 2u - 15 = 0$ where $u = 3^x$

$(u+5)(u-3) = 0$

$u = 3 = 3^x$

$\Rightarrow x = 1$

Solve the equation. (Enter your answers as a comma-separated list.)

17

$$x^2 6^x - 25(6^x) = 0$$

$\Rightarrow 6^x(x^2 - 25) = 0$

$6^x = 0$ Nevah!

OR $x^2 - 25 = 0$

$x^2 = 25$

$\boxed{x = \pm 5}$

19 Solve the logarithmic equation for x. (Enter your answers as a comma-separated list.)

$$2\log(x) = \log(2) + \log(5x - 8)$$

$\log(x^2) = \log(2(5x-8)) = \log(10x-16)$

$\Rightarrow x^2 = 10x - 16$

$x^2 - 10x + 16 = 0$

$x^2 - 8x - 2x + 16 =$

$x(x-8) - 2(x-8) =$

$= (x-8)(x-2) = 0$

$\boxed{x = 2, 8}$

D: Need $x > 0$

& $5x - 8 > 0$

$5x > 8$

$x > \frac{8}{5} = 1.6$

#21 - See what I did for #22.

22 Solve the logarithmic equation for x. (Enter your answers as a comma-separated list.)

$$\log(x) + \log(x - 48) = 2$$

$\log(x(x-48)) = 2$

$\log_{10}(x^2 - 48x) = 2$, 10^2

$x^2 - 48x = 100$

$x^2 - 48x - 100 = 0$

$(x-50)(x+2) = 0$

$x = -2, 50$ Nope

$x \in \{-2, 50\}$ = Solution Set

Check Domain:

Need $x > 0$ for $\log(x)$

$x - 48 > 0$ for $\log(x-48)$

$x = -2$ is Extraneous Solution.

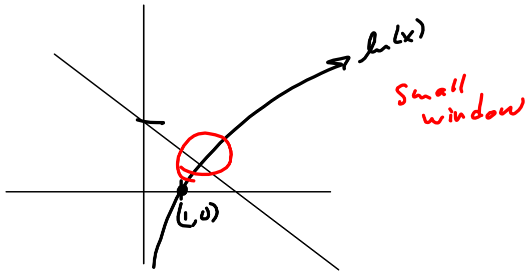
$$\boxed{\begin{array}{c} x = 50 \\ x \in \{50\} \end{array}}$$

23 A graphing device is recommended.

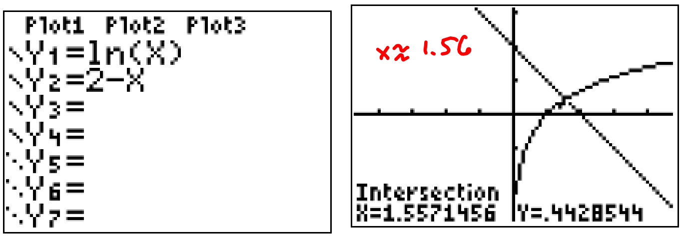
Use a graphing device to find all solutions of the equation, rounded to two decimal places. (Enter your answers as a comma-separated list.)

$$\ln(x) = 2 - x$$

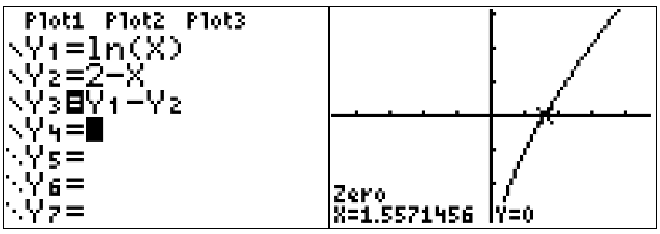
4 Methods. All methods begin with a rough hand sketch, for perspective.



Method 1



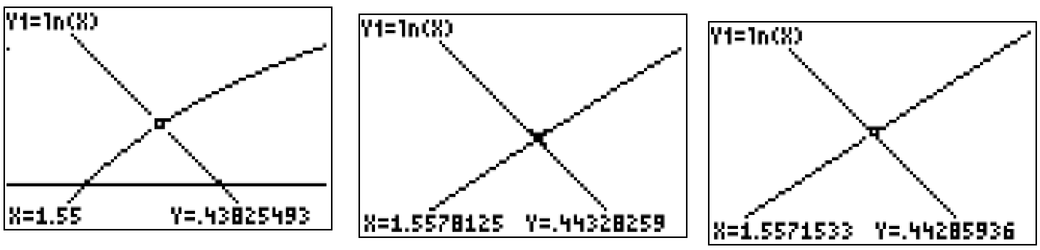
Method 2



This method, you're

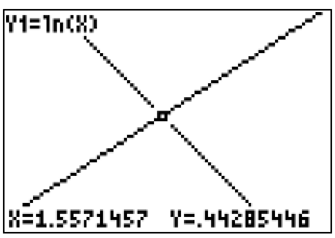
just looking for *x*-intercepts!

Method 3: Zoom and Trace (Old-School Graphing Calculator).



Keep zooming and tracing back to the intersection point.

The digits slowly fill out.



Click Here for Wolfram Alpha

23 A graphing device is recommended.

Use a graphing device to find all solutions of the equation, rounded to two decimal places. (Enter your answers as a comma-separated list.)

$$\ln(x) = 2 - x$$

Method 4

Clobber it with Wolfram Alpha

solve ln(x)=2-x

NATURAL LANGUAGE MATH INPUT EXTENDED KEYBOARD EXAMPLES UPLOAD RANDOM

Input interpretation

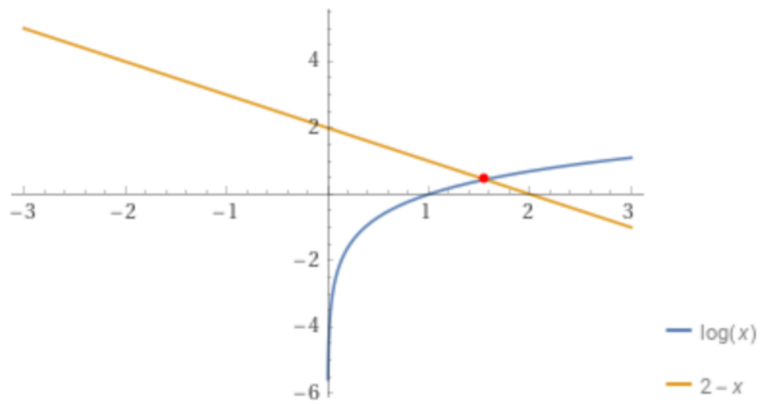
solve	$\log(x) = 2 - x$

$\log(x)$ is the natural logarithm

Result More digits

$x = W(e^2) \approx 1.55715$

Plot



25 A graphing device is recommended.

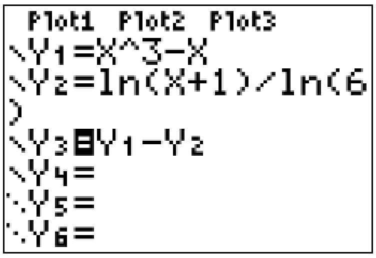
Use a graphing device to find all solutions of the equation, rounded to two decimal places. (Enter your answers as a comma-separated list.)

→ Just want x-values

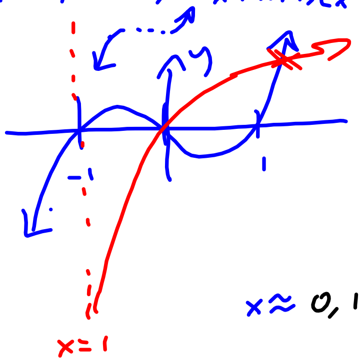
$x^3 - x = \log_6(x + 1)$

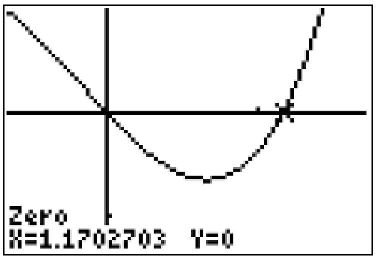
$f(x) = g(x) \Longrightarrow x^3 - x - \log_6(x+1) = 0$

$f(x) - g(x) = 0$ x-ints easier than (x,y) intersections.



$x^3 - x = x(x^2-1) = x(x+1)(x-1)$





$x \approx 0, 1.1702703$

27 Solve the inequality. (Enter your answer using interval notation.)

$x^2e^x - 7e^x < 0$

$e^x(x^2-7) = e^x(x-\sqrt{7})(x+\sqrt{7}) < 0$

Always "+", ">0"

N N Y N N

\+ − +

$-\sqrt{7}$ $\sqrt{7}$

$=0$ $=0$

want "<0" "−"

Test:

$(-\infty, -\sqrt{7})$ -3 $e^3((-3)^2-7) > 0$ "+"

$(-\sqrt{7}, \sqrt{7})$ 0 $e^0(0^2-7) = -7 < 0$ "−"

$(\sqrt{7}, \infty)$ 3 $e^4(3^2-7) = e^4(2) > 0$ "+"

$\Rightarrow x \in (-\sqrt{7}, \sqrt{7})$

28 Find the inverse function of f.

$f(x) = 5^{4x}$

Swap variables and solve for *y*.

$5^{4y} = x$

$4y = \log_5(5^{4y}) = \log_5(x)$

$4y = \log_5(x)$

$\boxed{y = \frac{\log_5(x)}{4} = f^{-1}(x)}$

29 Find the inverse function of f.

$f(x) = \log_6(x - 2)$

$6^{\log_6(y-2)} = 6^x$

$y - 2 = 6^x$

$y = \boxed{6^x + 2 = f^{-1}(x)}$

Find the value(s) of x for which the equation is true. (Round your answers to four decimal places. Enter your answers from smallest to largest.)

30 $(\log(x))^3 = 5\log(x)$.

smallest value x = ______

x = ______

largest value x = ______

Let $u = \log(x)$. Then

$u^3 = 5u \implies$

$u^3 - 5u = u(u^2-5) = u(u-\sqrt{5})(u+\sqrt{5}) \overset{SET}{=} 0$

$u = -\sqrt{5}, 0, \sqrt{5}$

$\log(x) = -\sqrt{5}$ $\quad$ $\log(x) = 0$ $\quad$ $\log(x) = \sqrt{5}$

$10^{\log(x)} = x = 10^{-\sqrt{5}}$ $\quad$ $x = 1$ $\quad$ $10^{\sim} = 10^{\sim}$

$x = 10^{\sqrt{5}}$

31 Suppose you invest \$2,000 in an account that pays 7.25% interest per year, compounded quarterly.

(a) Find the amount (in \$) after 3 years. (Round your answer to the nearest cent.)

(b) How long (in yr) will it take for the investment to double? (Round your answer to two decimal places.)

(a) P = Principal Amount = \$2000

r = Annual percentage rate as a decimal = 0.0725

t = 3 yrs

m = # periods per year = 4

$A(3)$ = Future Amount = $P\left(1+\frac{r}{m}\right)^{mt}\Big|_{t=3} = 2000\left(1+\frac{.0725}{4}\right)^{(4)(3)}$

$A(t) = P\left(1+\frac{r}{m}\right)^{mt}$

(a) $A(3) =$

(b) How Long to double?

$2000\left(1+\frac{r}{m}\right)^{mt} = 2(2000) = 4000$

Doubling Time: $P\left(1+\frac{r}{m}\right)^{mt} = 2P$

$\left(1+\frac{r}{m}\right)^{mt} = 2$

$\ln\left(\left(1+\frac{r}{m}\right)^{mt}\right) = mt\ln\left(1+\frac{r}{m}\right) = \ln(2)$

$\implies t = \dfrac{\ln(2)}{m\ln\left(1+\frac{r}{m}\right)} = \dfrac{\ln(2)}{4\ln\left(1+\frac{.0725}{4}\right)}$

```
2000(1+.0725/4)^
12
        2481.093983
ln(2)/(4ln(1+.07
25/4))
        9.647034774
```

32 Suppose you invest \$4,000 in an account that pays 4.25% interest per year, compounded quarterly.

(a) Find the amount (in \$) after 3 years. (Round your answer to the nearest cent.)

(b) Write a formula in terms of t that can be used to find how long (in yr) it will take for the investment to triple.

How long (in yr) will it take for the investment to triple? (Round your answer to two decimal places.)

________ yr

See #31

33 Suppose you invest \$6,200 in an account that pays 4.5% interest per year, compounded continuously.

(a) What is the amount (in \$) after 2 years? (Round your answer to the nearest cent.)

(b) How long (in yr) will it take for the amount to be \$10,000? (Round your answer to two decimal places.)

$A(t) = Pe^{rt}$

$= 6200e^{.045t}$

(a) Find $A(2) = 6200e^{.045(2)} \approx \boxed{\$6783.88 \approx A(2)}$

(b) How long to hit \$10000

$6200e^{.045t} = 10000$

$e^{.045t} = \frac{10000}{6200}$

$.045t = \ln\left(\frac{100}{62}\right)$

$t = \frac{\ln\left(\frac{50}{31}\right)}{.045} \approx \boxed{10.62 \text{ yrs} \approx t}$

```
6200e^(.045*2)
         6783.880559
ln(50/31)/.045
          10.6230178
```

34 Find the time (in yr) required for an investment of \$3,000 to grow to \$9,000 at an interest rate of 5.5% per year, compounded quarterly. (Round your answer to two decimal places.)

See Previous

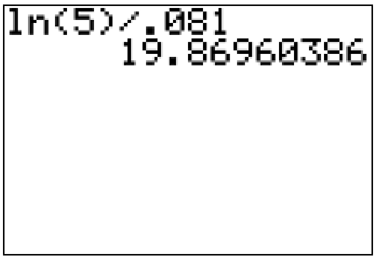
35 A 15-gram sample of radioactive iodine decays in such a way that the mass remaining after t days is given by

$$m(t) = 15e^{-0.081t}$$

where $m(t)$ is measured in grams. After how many days is there only 3 g remaining? (Round your answer to the near whole number.)

t= time, in days.

Want to know when



$$15e^{-0.081t} = 3$$

$$e^{-.081t} = \frac{3}{15} = \frac{1}{5}$$

$$-.081t = \ln(1/5) = \ln(1) - \ln(5) = -\ln(5)$$

$$t = \frac{-\ln(5)}{-.081} \approx 19.86960386 \approx \boxed{20 \text{ yrs} \approx t}$$

36 A small lake is stocked with a certain species of fish. The fish population is modeled by the function

$$P = \frac{10}{1 + 4e^{-0.6t}}$$

where P is the number of fish (in thousands) and t is measured in years since the lake was stocked.

(a) Find the fish population after 2 years. (Round your answer to the nearest whole fish.)

(b) After how many years will the fish population reach 5,000 fish? (Round your answer to two decimal places.)

$P(2) = \frac{10}{1+4e^{-.6(2)}}$

(b) want $P(t) = 5$

$\frac{10}{1+4e^{-.6t}} = 5$

$10 = 5(1+4e^{-.6t})$

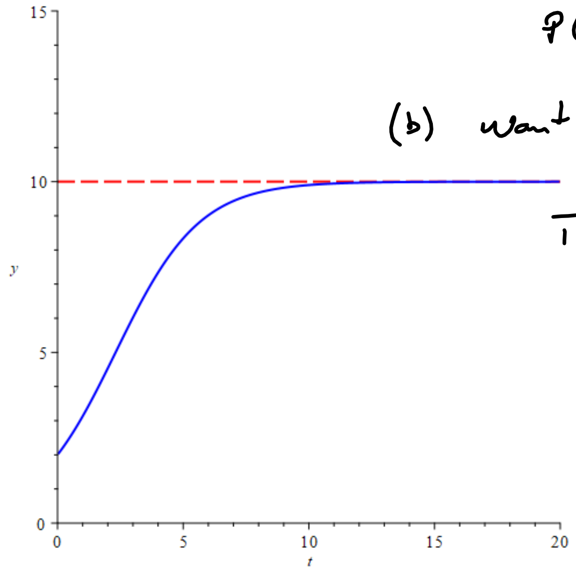
$2 = 1+4e^{-.6t}$

$1 = 4e^{-.6t} = 1$

$e^{-.6t} = \frac{1}{4}$

$-.6t = \ln(4^{-1}) = -\ln(4)$

$t = \frac{-\ln(4)}{-.6} \approx$




```
10/(1+4e^(-.6*2)
)
        4.535606409
ln(4)/.6
        2.310490602
```

≈ 4536 fish (a)
≈ 2.31 yrs (b)