1 The logarithm of a product of two numbers is the same as the [---Select---] of the logarithms of these numbers.

So $\log_4(16 \cdot 64) = \log_4(16) + \boxed{\log_4(64)}$.

$a^b a^c = a^{b+c}$

$\log(ab) = \log(a) + \log(b)$

---

2 The logarithm of a quotient of two numbers is the same as the [---Select---] of the logarithms of these numbers.

So $\log_4\left(\frac{16}{64}\right) = \log_4(16) - \boxed{}$.

$\frac{a^b}{a^c} = a^{b-c}$ $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$

3 The logarithm of a number raised to a power is the same as the [---Select---] times the logarithm of the number. So $\log_4(16^8) = 8 \cdot \boxed{}$.

$(a^b)^c = a^{bc}$ $\log(a^b) = b\log(a)$

$\log(a^b) = \log(\underbrace{a \cdot a \cdot a \cdots a}_{b \text{ factors}})$

$= \underbrace{\log(a) + \log(a) + \cdots + \log(a)}_{b \text{ terms}}$

$= b\log(a)$

4 We can expand $\log\left(\frac{x^4 y}{z}\right)$ to get $\log(x^4) + \log(y) - \log(z)$

$= 4\log(x) + \log(y) - \log(z)$

5 We can combine $4\log(x) + \log(y) - \log(z)$ to get

$\log\left(\frac{x^4 y}{z}\right)$

6 (a) To express $\log_3(8)$ in terms of common logarithms, we use the Change of Base Formula to write the following. (Ro your answers to three decimal places.)

$\log_3(8) = \frac{\log(8)}{\log(3)} = \frac{\ln(8)}{\ln(3)} = \frac{\log_7(8)}{\log_7(3)}$

(b) Do we get the same answer if we perform the calculation in part (a) using ln() in place of log()?

Yes!

$\log_a(x) = y \iff x = a^y \implies$

$\log_b(x) = \log_b(a^y) = y\log_b(a)$

$\implies y = \boxed{\frac{\log_b(x)}{\log_b(a)} = \log_a(x)}$

7 *True or False*? (Let $A > 0$ and $F > 0$.)

(a) $\log(A + F)$ is the same as $\log(A) + \log(F)$. F

(b) $\log(AF)$ is the same as $\log(A) + \log(F)$. T

8 *True or False*? (Let $E > 0$ and $G > 0$.)

(a) $\log\left(\frac{E}{G}\right)$ is the same as $\log(E) - \log(G)$. T

(b) $\frac{\log(E)}{\log(G)}$ is the same as $\log(E) - \log(G)$. F

No.

9 Use the Laws of Logarithms to evaluate the expression.

$\log(5) + \log(200) = \log(1000) = \log(10^3) = 3$

10 Use the Laws of Logarithms to evaluate the expression.

$\log_6(54) + \log_6(24)$

$= \log_6(6^4) = 4$

$6\,\underline{|\,54}$ $\quad 9$ $\qquad 6\,\underline{|\,24}$ $\quad 4$

$54 \cdot 24 = 6 \cdot 6 \cdot 9 \cdot 4$

$= 6 \cdot 6 \cdot 3^2 \cdot 2^2$

$= 6 \cdot 6 \cdot (3 \cdot 2)(3 \cdot 2)$

$= 6^4$

11 Use the Laws of Logarithms to evaluate the expression.

$\log_2(52) - \log_2(13)$

$= \log_2\left(\frac{52}{13}\right) = \log_2(2) = 1$

12 Use the Laws of Logarithms to evaluate the expression.

$\log_3(\sqrt{3}) = \log_3(3^{\frac{1}{2}}) = \frac{1}{2}$

13 Use the Laws of Logarithms to evaluate the expression.

$\log_5\left(\frac{1}{\sqrt{78,125}}\right) = \log_5\left(\frac{1}{(5^7)^{1/2}}\right)$

$= \log_5(5^{-7/2}) = -\frac{7}{2}$

$5\,|\,78125$
$5\,|\,15625$
$5\,|\,3125$
$5\,|\,625$
$5\,|\,125$
$5\,|\,25$
5

14 Use the Laws of Logarithms to evaluate the expression.

$\log_3(15) - \log_3(30) + \log_3(486) = \log_3\left(\frac{15(486)}{30}\right)$ $\qquad 243^{\frac{1}{5}} = ?$

$= \log_3\left(\frac{486}{2}\right) = \log_3(243) = \log_3(3^5) = 5$

15 Use the Laws of Logarithms to evaluate the expression.

$\log_3(9^{200}) = \log_3((3^2)^{200}) = \log_3(3^{400}) = 400$

16 Use the Laws of Logarithms to evaluate the expression.

$\log(\log(10^{100})) = \log(100) = 2$

Use the Laws of Logarithms to expand the expression.

22 $\log_5\left(\dfrac{\sqrt{5x^9}}{y}\right)$

$= \log_5\left(\sqrt{5x^9}\right) - \log_5(y)$

$= \frac{1}{2}\log_5(5x^9) - \log_5(y)$

$= \frac{1}{2}\left[\log_5(5) + \log_5(x^9)\right] - \log_5(y)$

$= \frac{1}{2}\left[1 + 9\log_5(x)\right] - \log_5(y)$

$= \frac{1}{2} + \frac{9}{2}\log_5(x) - \log_5(y)$

Use the Laws of Logarithms to expand the expression.

26 $\log\left(\sqrt{\dfrac{x^2+2}{(x^2+3)(x^3-2)^2}}\right)$

$= \frac{1}{2}\left[\log(x^2+2) - \log(x^2+3) - 2\log(x^3-2)\right]$

Use the Laws of Logarithms to expand the expression.

27 $\log\left(\sqrt{x^5\sqrt{y^3\sqrt{z}}}\right)$

$= \frac{1}{2}\left[5\log(x) + \frac{3}{2}\log(y) + \frac{1}{4}\log(z)\right]$

28 Use the Laws of Logarithms to combine the expression.

$7\log(x) - 8\log(x + 1)$

30 Use the Change of Base Formula to express the given logarithm in terms of common or natural logarithms, and then evaluate. State your answer rounded to six decimal places.

$\log_3(2.25)$

$= \dfrac{\ln(2.25)}{\ln(3)} \approx 0.738140$

```
ln(2.25)/ln(3)
      .7381404929
```

Vilfredo Pareto (1848–1923) observed that most of the wealth of a country is owned by a few members of the population. **Pareto's Principle** is

32 $\log(P) = \log(c) - k\log(W)$

where W is the wealth level (how much money a person has) and P is the number of people in the population having that much money.

(a) Solve the equation for P.

(b) Assume $k = 2.5$ and $c = 6{,}000$ and that W is measured in millions of dollars. Use part (a) to find the number of people who have \$2 million or more. (Round your answer to the nearest whole number.)

How many people have \$10 million or more? (Round your answer to the nearest whole number.)

(a) $\log_{10}(P) = \log_{10}(c) - k\log_{10}(w) = \log_{10}\left(\frac{c}{w^k}\right)$

$P = c \cdot 10^{\log(w^{-k})}$

$= c \cdot w^{-k} = \frac{c}{w^k}$

$\frac{c}{w^k}$

```
     .7381404929
6000*(2*10^6)^-2
.5
  1.06066017E-12
6000*(10^7)^-2.5
   1.8973666E-14
```

$= 6000(2\,000\,000)^{-2.5}$ \$2 million

$= 6000(10\,000\,000)^{-2.5}$ \$10 million

```
6000*(10^7)^-2.5
   1.8973666E-14
6000*(2)^-2.5
     1060.660172
6000*(10)^-2.5
     18.97366596
```

≈ 1061

≈ 19